

#### QUESTION 0

Hmmm. I do not really have any questions that I can ask in a detailed manner. I am not very well acquainted with Tensors. I have used them etc, but am not used to them well enough to make good sense of the part of 8.2 that uses them. So I currently have questions about it. But seeing as I am having problems with it in general, I cannot ask any "Good" questions until I get a better understanding of that section.

#### ANSWER

I don't know what to say except perhaps it will make more sense after doing the problems.

#### QUESTION 1

In section 8.2.1 we learn that momentum conservation holds in electrodynamics because the fields gain momentum lost by the particles. This is confusing to me. Since fields are due to the particles, how can they have a different momentum from the particles? We learned in classical mechanics that momentum is equal to mass times velocity, and it seems to me that a field would have to move at the same velocity as the particle it originates from. Since electric fields have momentum, angular momentum, and energy, does this also mean that they have some equivalent to mass? If they do, it seems this would account for the difference in momentum between the particle and field, but I am not even sure that the mass times velocity relationship from classical mechanics applies to this situation.

#### ANSWER

This is an excellent set of questions. It was this line of questioning that helped lead to quantum electrodynamics and the particle picture of electromagnetism. Indeed electric and magnetic fields are created by photons. These are particles with zero mass, so the energy momentum relationship becomes

$$E = (m^2 c^4 + p^2 c^2)^{1/2} = pc.$$

For a particle with a large mass, one has

$$E = (m^2 c^4 + p^2 c^2)^{1/2} \approx mc^2 + \frac{p^2}{2m}$$

The  $mc^2$  term is just a constant that is left out of classical theory (but important to those watching North Korea.) The  $\frac{p^2}{2m} = K$  is the classical energy-momentum relationship.

#### QUESTION 2

I reread chapter 8 and went over the derivations once again. For the most part I feel like I get the gist of the chapter. I think I just need to work some

problems at this point. The only thing that sounds fishy is the bit about hidden momentum (p. 357). I assumed that when we talked about the fields storing momentum it meant that the electromagnetic waves propagating through the system carried a momentum. However, in example 8.3 the fields are static. Similarly, the paragraph at the end of the chapter says that a system can have nothing moving within it and yet still have angular momentum. Griffith says that he is waiting to explain these phenomenons in later chapters, but could you tell us a little bit about it now. Perhaps give us just a qualitative explanation. I understand the desire to have conservation laws hold within the theory, but this all seems a little contrived.

ANSWER:

Conservation laws, as contrived as they seem, are essential to a working theory. First off let me debunk the notion that electromagnetic motion is only associated with a changing magnetic field. This is a problem with the two hats the pointing vector has in conservation laws. In conservation of energy we have

$$\frac{\partial(u_{mech} + u_{elec})}{\partial t} = -\vec{\nabla} \cdot \vec{S}$$

Here  $\vec{S}$  is associated with the flow of energy out of a surface and we might expect

$$\frac{d(U_{mech} + U_{elec})}{dt} = - \int \int_{closed} \vec{S} \cdot d\vec{a}$$

to be zero for a static system. However, in the role of conservation of momentum we have

$$\frac{\partial(\vec{p}_{mech} + \vec{p}_{elec})}{\partial t} = \vec{\nabla} \cdot \overleftarrow{T}$$

with  $\vec{p}_{mech}$  and  $\vec{p}_{elec}$  being the electromagnetic and mechanical momentum per unit volume. Here is the other hat for  $\vec{S}$ :

$$\vec{p}_{mech} = \frac{1}{c^2} \vec{S}$$

which is the momentum density of a field which can be nonzero, even for a static situation. Said another way, the flux of  $\vec{S}$  over a closed surface is zero for a static system, but that does not mean  $\vec{S}$  must be zero.

Ok, where is the hidden momentum? Classically we have

$$\vec{p} = m\vec{v}$$

but relativistically,

$$\begin{aligned} \vec{p} &= \gamma m\vec{v} = \frac{mN\vec{v}}{(1 - v^2/c^2)^{1/2}}, N_{\pm} = ((m/e)\lambda_{\pm})\ell \\ \vec{p}_+ + \vec{p}_- &= \left( \frac{(m\ell/e)\lambda_+ v_+}{(1 - v_+^2/c^2)^{1/2}} - \frac{(m\ell/e)\lambda_- v_-}{(1 - v_-^2/c^2)^{1/2}} \right) \hat{z} \end{aligned}$$

Because the current in the middle and outside tube are the same, we have

$$|\lambda_- v_-| = |\lambda_+ v_+| = I$$

and where  $\lambda_{\pm}$  is the charge per unit length on the outside and inside wires respectively. Note if the laws of classical physics were to apply, this would imply that  $\vec{p}_+ + \vec{p}_- = \vec{0}$  because the total momentum would be the same if a few particles carried a lot of momentum or a lot of particles carried a little momentum. There is, however, a difference in relativistic mechanics and this leads to the hidden momentum:

$$\begin{aligned} \vec{P}_{mech} &= \vec{P}_{inside} + \vec{P}_{outside} = \frac{m\ell I}{e} \left( \frac{1}{(1 - (\frac{I}{\lambda_+ c})^2)^{1/2}} - \frac{1}{(1 - (\frac{I}{\lambda_- c})^2)^{1/2}} \right) \hat{z} \\ &\approx \frac{m\ell I^3}{2ec^2} \left( \frac{1}{\lambda_+^2} - \frac{1}{\lambda_-^2} \right) \hat{z} \end{aligned}$$

Thus a different charge per unit length is set up on the inside and outside that cancels the difference in momentum.

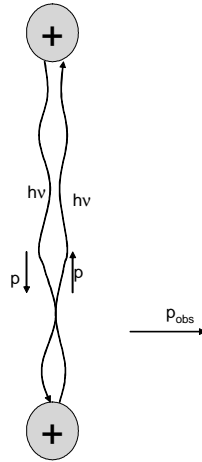
### QUESTION 3

First question, I know we touched on it in class, but I still can't picture the idea of momentum carried in a field (unless the field is moving). Energy in a field makes sense, because that's mostly what fields are anyway, right? Something like patterned energy? When you usually think of momentum, you think of the physical manifestation of momentum, you see it in the velocity of the object. I guess part of my problem is that with  $p=mv$ , if the object isn't moving, it doesn't have momentum, so I'm thinking if the field isn't moving how does it have momentum, much less "store" it as Griffiths says?

### ANSWER 3

I think what you want is some intuition for why even stationary electromagnetic fields carry momentum. First I emphasize moving charge is required to create electromagnetic momentum, as moving charges are required to create a magnetic field. (My point is that time-independent magnetic fields are created by steady state, but still non-zero currents.) You still might wonder why we must have electromagnetic momentum at all. Imagine two charged objects at rest. Perhaps they are glued to a table. In any case, they experience equal and opposite forces. Surely action at a distance must be unphysical. What is propagating this force? Let us assume a steady stream of objects called virtual photons do the job. These objects could carry a momentum that pushes against the charges. If the objects are not moving, the total momentum in the field is zero (because  $F_{12}=F_{21}$ .) But to an observer moving at a constant velocity to the left, each photon has an additional momentum  $\vec{p}_{obs}$  moving to the right. This momentum does not add up to zero. Is this consistent with the picture of momentum we get from Maxwell's equations? Absolutely. This observer sees

not only an electric field, but a magnetic field. The product of  $(1/c^2)E \times B$  is thus not zero.



#### QUESTION 4

Second question, has to do with when Griffiths talks about the "dual roles" of S and T. Are the dual roles actually exactly the same quantities? That is to say, since  $S = \text{energy}/d^2/t$ , if I plug that in to

$\mu_o * \epsilon_o * S$ , shouldn't the units at least make sense as the momentum/volume that it says it does? I tried and got that  $\mu_o * \epsilon_o * S = \text{energy} * \text{time}/d^4$ .

That doesn't look like momentum/volume to me. Same goes for T. It says T is both force/area and  $dp/dt/\text{volume}$ .

#### ANSWER 4

Ok, Griffiths was being a bit too much the theorist, that tends to work in units with  $e = c = \pi = 10 = \hbar = h = 1$ . The dual roles come from conservation of energy

$$\frac{d(u_{EM} + u_{mech})}{dt} = -\vec{\nabla} \cdot \vec{S}$$

$$u_{EM} = \frac{\epsilon_o}{2} E^2 + \frac{1}{2\mu_o} B^2$$

and conservation of momentum:

$$\frac{d(\vec{p}_{EM} + \vec{p}_{mech})}{dt} = \vec{\nabla} \cdot \vec{T}$$

$$\vec{p}_{EM} = \frac{1}{c^2} \vec{S}$$

You should see that  $\vec{S}$  has units of energy/area/time which makes  $\frac{1}{c^2}\vec{S}$  have units of energy time, or (mass) (dist)/(time)/(dist<sup>3</sup>)=momentum/volume. Griffith's point is that the energy flux  $\vec{S}$  has a very simple relationship to the momentum density  $\frac{1}{c^2}\vec{S}$ .