

## 1 Question

Griffiths says on page 347 that "the work done on the charges by the electromagnetic force is equal to the decrease in energy stored in the field, less the energy that flowed out through the surface." I would just like to clarify this a bit, because I think I understand this section pretty well. According to my understanding, and also according to equation 8.14 if I read it correctly, energy is conserved in that the decrease in the energy stored in the field is exactly countered by the increase in mechanical energy of the charges involved, given that there is no net "energy flux" in or out of the system. If  $\vec{\nabla} \cdot \vec{S} = 0$ , then  $u_{em}$  is constant. Am I interpreting this statement more or less correctly?

*You have got this exactly right.*

## 2 Question

1. This one might seem a little absurd to you, partly because it seems a little absurd to me. On page 322, Griffiths suggests that using a "balloon-shaped surface" is different from using the "simplest surface" that "lies in the plane of the loop." I can understand that they probably should be different, and for the most part can see that there is no current flowing through the balloon. But later on, after equation 7.38, he states that the integral of  $\int \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{a} = \frac{I}{\epsilon_0}$ . I know he is implying that partial E/partial t is equal to I over epsilon nought times A, and so doing the integration just multiplies this by A,

*Actually he is not implying  $IA/\epsilon_0$ . Remember  $E = Q/A\epsilon_0$  so the A from the area integral cancels the A from the value of E.*

## 3 Question

but I can't help noticing the fact that the balloon does not reside only between the plates, where the field is as suggested, but also outside them where, methinks, the field is not so simple. Why doesn't he include the field outside the plates? Am I just misunderstanding the concept of the balloon in the first place?

*You are right to worry about the field outside the plates. However, you will get the same result if you include this field and do everything right. In the limit of two plates very close together (the limit that Griffith claims he is using) then a large portion of the surface integral will have  $E = Q/A\epsilon_0$  and fraction that is vanishingly small, (contributing an amount that shrinks as d). Of course I*

*welcome you to solve the problem including edge effects. Might I suggest you use a disk capacitor and give yourself a lot of time, for the problem is hard.*

## 4 Question

2. I worked on this one for a long while and haven't come up with anything, and it doesn't really have much to do with Chapter 7. Griffiths says that equation 7.41 is the "mathematical expression of conservation of charge," but I just can't see it. How is this the mathematical expression for conservation of charge? Maybe I should just come talk to you about this one some time.

Equation 7.41 is

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

To visualize, it is often better to go to the integral form of fundamental laws. Here we have

$$\begin{aligned} \iint \vec{J} \cdot d\vec{a} &= -\frac{\partial}{\partial t} \iiint \rho d\tau \\ &= -\frac{dQ_{enc}}{dt} \end{aligned}$$

Remember that  $\vec{J}$  is a charge flux, that is charge per area per time. This statement is that the net charge passing through a closed surface per unit time is simply the charge leaving (or entering) per unit time. This implies charge is neither created or destroyed inside the closed surface.

## 5 Question

1. Is  $\vec{H}$  (eq. 7.53, 7.54 and onward) just a mathematical quantity, or does it have some physical representation?

$\vec{H}$  is as real as the concept of a continuous mass density. Sure we are really made up of discrete nuclei and electrons, but  $\rho_m$  sure is a useful quantity.

## 6 Question

2. Why is the direction of  $\vec{D}$  in Figure 7.46 towards the surface, rather than away from it?

We have to (somewhat arbitrarily for open surfaces) define a positive direction for our area. Once this is done,  $\vec{D}$  must be defined as positive. In other words, let  $d\vec{a} = \hat{n} da$ . Then  $D^\perp = \hat{n} \cdot \vec{D}$ . For  $\vec{D}$  above the surface, this points away, for  $\vec{D}$  below, this points toward the surface

## 7 Question

3. I'm becoming confused with the parallel and perpendicular "reasoning" that is done in section 7.3.6. Perhaps is it the way it is explained, but it seems counter intuitive, especially given the figures 7.46 and 7.47.

These equations are a direct result of Maxwell's equations in matter. Let's review them in class.

## 8 Question

4. I understand a "free" charge, but what do they mean by a free current? As opposed to a bound(?) current?

Actually there are three different types of current we need to go through. Let us talk about all of them in class.

## 9 Question/discussion of conservation of angular momentum.

I hesitate to ask a question that is more math than physics, but the discussion on page 352 escapes my comprehension for mathematical reasons. For starters, I have conceptual difficulties with tensors. How is a tensor different from a matrix or an array?

*A vector that depends on position is called a vector field. A matrix or vector that depends on position is called a tensor. A tensor can have, however, many indices. More over, tensors usually have very specific ways in which they transform under rotation (in 3-D) or under Lorentz transformations (in 4-D space time.)*

Griffiths introduces the Maxwell stress tensor with the implicit assumption that the reader has some familiarity using tensors.

*At some point we have to dive in!*

In addition, I've never seen a Kronecher delta and I don't understand the role it plays in the tensor.

*The Kronecher delta function is very simple:*

$$\delta_{ik} = \begin{cases} 0 & i \neq k \\ 1 & i = k \end{cases}$$

Given my ignorance of these math concepts, you can probably guess that example 8.2 confuses me.

If you understand

$$\vec{F} = \int_V (\vec{E} + \vec{v} \times \vec{B}) \rho d\tau$$

then the rest is just math and with care and time, we will get to the answer. Do not be frightened by the Tensors! It is just convenient notation to clean up the result, which is written in conventional notation in Equation 8.18. The question is what do you prefer

$$\vec{f} = \epsilon_o((\vec{\nabla} \cdot \vec{E})\vec{E} + (\vec{E} \cdot \vec{\nabla})\vec{E}) + \frac{1}{\mu_o}((\vec{\nabla} \cdot \vec{B})\vec{B} + (\vec{B} \cdot \vec{\nabla})\vec{B}) - \frac{1}{2}\vec{\nabla}(\epsilon_o E^2 + \frac{1}{\mu_o} B^2) - \epsilon_o \frac{\partial}{\partial t}(\vec{E} \times \vec{B})$$

or

$$\vec{f} = \vec{\nabla} \cdot \overleftrightarrow{T} - \frac{1}{c^2} \frac{\partial \vec{S}}{\partial t}$$

with

$$\vec{S} = \frac{1}{\mu_o} \vec{E} \times \vec{B}$$

and

$$T_{ij} = \epsilon_o(E_i E_j - \frac{1}{2} \delta_{ij} E^2) + \frac{1}{\mu_o}(B_i B_j - \frac{1}{2} \delta_{ij} B^2)$$

I think it is MUCH easier to remember this in terms of the Maxwell stress tensor. We then have something similar to the force on rocket particle that moves in a gravitational potential  $U$  while spewing out fuel with momentum  $\vec{P}$ :

$$\vec{F} = -\vec{\nabla}U + \frac{d\vec{P}}{dt}$$

For instance, why are there only three components to the stress tensor (equation 8.23)? On page 352 Griffiths says that the stress tensor has nine components.

Recall that a  $3 \times 3$  matrix has 9 components.

The later discussion in ch.8 is fairly lucid, but it is hard for me to follow the math because the stress tensor is used in a number of parts.

## 10 Question

(1) Just as a changing magnetic field induces an electric field, so a changing electric field induces a magnetic field. Consider a magnetic field that is changing in such a way as to induce an electric field, which is also changing in time. The induced changing electric field will induce a new magnetic field. How quickly will this interaction die off, and how would you make such calculations?

This question is very similar to the problems with mutual inductance and back emf. The fields are always such that they solve Maxwell's equations. Nature does not get to this solution by ignoring the induced magnetic field, then adjusting. Consider a long solenoid with a magnetic field increasing so that

$$\vec{B} = B(t)\hat{z} \tag{1}$$

then the electric field is

$$\begin{aligned} 2\pi E s &= -\frac{\partial B}{\partial t} \pi s^2 \\ \vec{E} &= -\frac{1}{2} \frac{\partial B}{\partial t} \hat{\phi} \end{aligned}$$

Now lets worry about

$$\vec{\nabla} \times \vec{B} = \mu_o \epsilon_o \frac{\partial \vec{E}}{\partial t} \tag{2}$$

If  $B(t)$  is growing linearly with time, then all is ok, for  $\vec{\nabla} \times \vec{B} = \vec{0}$  if we consider either Eq 1 or Eq 2. If, however  $\vec{B}$  is not growing linearly then the simultaneous solution to Maxwell's equation is not so simple and finding the correct solution is harder.

## 11 Question

(2) Is Maxwell's displacement current really a separate source of magnetic fields, or just a mathematical contrivance to deal with the Amperian loop around the capacitor problem? After all, the changing electric field in the problem is caused by the current piling up on the capacitor plate. The current is ultimately responsible for creating the magnetic field, no matter which Amperian surface you use.

*It is really, really a source of magnetic field. Suppose for example you move a charge. Then a magnetic field will appear that satisfies*

$$\vec{\nabla} \times \vec{B} = \mu_o \vec{J} + \mu_o \epsilon_o \frac{\partial \vec{E}}{\partial t}$$

## 12 Question

(3) I'm not smoking pot, but my last question got my mind wandering. Why should electric and magnetic fields be so different? After all, electric and magnetic fields are ultimately due to charges and currents, and currents are just moving charges. One man's current is another man's stationary line of charge.

It just depends on your frame of reference. Why shouldn't one man's magnetic field be another man's electric field?

*Consider a non-zero electric and magnetic field in a given frame of reference. Unless  $\vec{E}$  is perpendicular to  $\vec{B}$ , there will be an electric and magnetic field  $\vec{E}'$  and  $\vec{B}'$  in every Lorentz frame. Thus, although the relative magnitude of  $\vec{E}$  and  $\vec{B}$  might be different in different frames, the theory does require both  $\vec{E}$  and  $\vec{B}$ .*

### 13 Question

I've used tensors a little bit from classical mechanics, but I couldn't say that I'm really used to them yet. Griffiths uses the dot product of a vector and a tensor, and the divergence of a tensor. Will we use other vector like operators with tensors (cross product and curl).

*Usually to describe these other operations, we use the antisymmetric tensor  $\varepsilon_{ijk}$  to make it very clear. Let's cross this bridge when we get there.*

### 14 Question

For the maxwell stress tensor, the diagonal elements are described as pressures and the off diagonals are described as shears. Do the shears act parallel to the surface? Which direction would the Txy component act?

*Consider a block sliding down an incline plane. The force of friction can be thought of as a shear force. It opposes the slip of the block. Let us consider the force per unit charge per unit volume in the y direction. We have*

$$f_y = \partial_x T_{xy} + \partial_y T_{yy} + \partial_z T_{zy}$$

*Thus a gradient in  $T_{xy}$  in the x direction leads to a force in the y direction, much like a force on a block in the  $\hat{z}$  direction leads to a force in the  $\hat{x}$  direction.*

### 15 Question

The hidden momentum that balances the apparent momentum in the example 8.3 has me curious. Griffiths says that we will cover it in chapter 12, but can you give us a qualitative explanation to hold us over. Griffiths just says that it is from relativistic effects.

*I am not 100% sure on this, but I think the charges in the center and the outside carry different amount of momentum because they are traveling in a different electric field. More later.*

## 16 Question

In class you mentioned that there is a ( very large ) minimum mass for a magnetic monopole. While I suspect that the physics involved may be too subtle to explain rigorously in this class, can you tell us some more about how we can come to this conclusion. Does it rule out the possibility of a massless magnetic monopole?

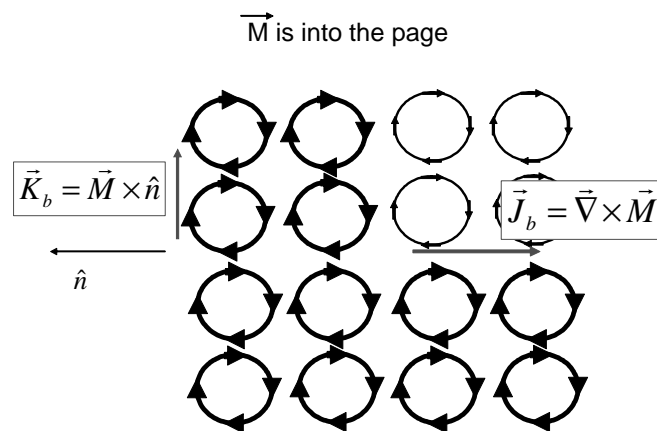
have a good weekend

*I am going to tell Kim Milton (a world expert in monopoles) answer this question on February 18 because I am clueless.*

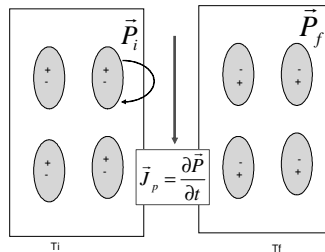
## 17 Question

1) Could you explain the difference between polarization current, bound current, and displacement current? What I interpreted from the reading was that polarization current leads to something similar to a dipole, but I thought this was the same type of effect that bound current has. I also didn't find a very good description of displacement current. I would like to see some example problems with these currents to help me learn the difference between them.

*This is a great question. Bound current is due material made up of magnetic moments that are for some reason aligned. This can be a static situation:*



The polarization current is on the other hand due to the motion of charges required to change the polarization of a system:



2) I am having difficulty understanding what the electric displacement  $\vec{D}$  is, and what the quantity called  $\vec{H}$  is. These seem to have similar properties that are useful for developing simplicity in calculations, but is there any other significance to them? They appear many times in the book, but I am still having trouble grasping the concepts behind these quantities and the physical interpretations of them.

Let us go back to

$$\begin{aligned}\vec{D} &= \epsilon_0 \vec{E} + \vec{P} \\ \vec{H} &= \frac{1}{\mu_0} \vec{B} - \vec{M}\end{aligned}$$

The polarization  $\vec{P}$  and the magnetization  $\vec{M}$  have a physical meaning as real as the charge density  $\rho$  and current density  $\vec{J}$ . We can think of  $\vec{D}$  and  $\vec{H}$  as a convenient intermediate quantity for finding  $\vec{E}$ ,  $\vec{B}$ ,  $\vec{M}$  and  $\vec{P}$ .

## 18 Question

### 8.1.2

I do not seem to understand what the Poynting Vector ( $\vec{S}$ ) is too well. It is  $1/u * \vec{E} \times \vec{B}$ . It seems to be similar to an EM wave in that it is about the relationship between the two fields at a right angle. But, in Example 8.1, it seems to be a result of the  $\vec{E}$  field at the surface of the wire and the  $\vec{B}$  field just outside the wire. Then the two are not in the same space and not interacting, thus no  $\vec{E} \times \vec{B}$ . Unless it is using the volume charge density, in which case it makes more sense. But that is not how I am interpreting what they are saying.

Thank you.

As we shall see in class, the Poynting vector has two meanings. First, it can be seen as the flux of energy giving the rate that e.m. energy leaves a

volume.  $\frac{1}{c^2}\vec{S}$  has another meaning altogether. It is the momentum per unit volume associated with an e&#246;m field.

## 19 Question

Poynting vector. First of all, it seems like a very non-intuitive quantity, and secondly, the direction has me confused. Figure 8.1 displays the  $\mathbf{E} \times \mathbf{B}$  relation of the Poynting vector, but it seems like this implies that the energy transported is coming from without? Where does the energy that is transferred to the material really come from if it is produced by the material? I know this is supposed to be proving conservation of energy, but it seems not only very unproven, but doesn't really explain the energy source at all.

*In the quantum picture, the energy source is moving photons. Moving a charge creates photons which then take the energy away. Does this help?*

## 20 Question

One comment as well, I found the magnetic monopole discussion in class very intriguing. It seems like such a basic concept that it should have been definitively proven one way or the other by now.

*Do not overestimate how much we know. There are a lot of open questions, including what is mass, what is the right quantum picture of gravity (if there is one) and does a magnetic monopole exist.*