1 READING QUESTIONS, 9.4-9.5

QUESTION
If the dot product is communitive, why does eq. 8.18 (etc.) show del dot E and E dot del seperately?

ANSWER

The Laplacian operator is not communitive. Let's see this is 1-D and 2-D:

1-D

Consider functions $B(x)$ and $C(x)$. $B\frac{d}{dx}C$ is not normally equal to $\frac{dB}{dx}C$.

2-D

Let's do this one by example. Let $\vec{B} = y\hat{x} + x\hat{y}$. Then

$(\vec{\nabla} \cdot \vec{B})\vec{B} = \vec{0}$

and
$(\vec{B} \cdot \vec{\nabla})\vec{B} = (y\partial_x + x\partial_y)\vec{B} = y\hat{y} + x\hat{x} \neq \vec{0}$
QUESTION
In example 8.2 how do you know what direction the net force is in? Also, why, for the disk, is only Tzz calculated? I guess I'm confused about what components of the T tensor to use and why.
ANSWER

Imagine sawing a uniformly charged sphere in half. Then we would expect the two halves to fly apart in opposite directions, one along the $+\hat{z}$ and the other along the $-\hat{z}$ direction. It is not a surprise then that the force is in the $\hat{z}$ direction. Conservation of momentum is

$$\frac{d\vec{p}_{em} + \vec{p}_{mech}}{dt} = \frac{d\vec{p}_{mech}}{dt} = \vec{\nabla} \cdot \overleftrightarrow{T}$$

which is really three conservation laws, one for p_x, p_y and p_z. We also know $\vec{p}_{em} = \frac{1}{c^2\mu_o}\vec{E} \times \vec{B} = \vec{0}$ because $\vec{B} = \vec{0}$. To use this conservation law to find the net force on the upper hemisphere, we recall that

$$\begin{aligned} F_{z,mech} &= \frac{dP_{z,mech}}{dt} \\ &= \frac{d}{dt}\int\int\int p_{z,mech} d\tau \\ &= \int \vec{\nabla} \cdot \vec{T}_z d\tau \\ &= \oint \vec{T}_z \cdot d\vec{a} \end{aligned}$$

Here we have let

$$\vec{T}_z = (\overleftrightarrow{T}_{zx}, \overleftrightarrow{T}_{zy}, \overleftrightarrow{T}_{zz})$$
$$= \varepsilon_o(E_zE_x, E_zE_y, \frac{1}{2}(E_z^2 - E_x^2 - E_y^2)$$

The volume is taken to be the hemisphere exactly containing the charged distribution. Another approach would be to use an infinitely big hemisphere. In this case the entire integral would be in the plane because the integral of $\vec{T}_z$ as r goes to infinity would vanish.

QUESTION
In section 9.1.3 Griffiths mentions that the "knot" on the string must be of negligible mass, or else there would be a net force on the knot and infinite acceleration. Could you explain this reasoning? Specifically, the "infinite acceleration"?

ANSWER.

I think you have this a bit wrong. Griffith says that if we assume the knot has zero mass, we must also assume that the net force on the knot must also be zero, or it would have infinite acceleration.

QUESTION
In equation 9.90, the reflected wave is shown to have positive kr-wt in the exponent, whereas in equation 9.76 the exponent is -kz-wt. What is the reason for this difference? Does it have anything to do with the phase of the reflected wave?

ANSWER

In this analysis, we always assume that the plane wave is something multiplied by $e^{-i\omega t}$. We then write $\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ with $\hat{k}$ being the direction of the beam. The reflected beam travels in the $-\hat{z}$ direction so $\vec{k} = -k\hat{z}$ and $\vec{k}\cdot\vec{r} = -kz$. Thus the general form for the reflected beam is $\vec{E} = \vec{E}_R e^{i(-kz-\omega t)}$.

QUESTION
On page 399, Griffiths tries to explain wave velocity and group velocity, and then goes on to say that we may sometimes find a wave velocity v that is greater than c (or even a group velocity, according to the footnote). How is this possible? Can you explain the circumstances in which we might find this result? (ANOTHER STUDENT WRITES: In the footnote on page 399, Griffiths states that there are up to eight different speeds of light. What on earth is he talking about? On this same page it is mentioned that we might find velocities greater

than the speed of light. I understand why a single wave can travel faster than the group velocity, but how can it exceed the speed of light?)

ANSWER

Special relativity tells us that no physical object can move faster than the speed of light. But in what way is light a physical object? This is the million dollar question first answered correctly by Brillouin. An electromagnetic disturbance that is analytic (that is that can be wirtten as a Taylor series) can move faster than the speed of light. Any discontinuous disturbance can not, however, travel faster than the speed of light. Sending information always involves a decision point, and hence a discontinuity. We can conclude that information can not move faster than the speed of light in a vacuum. For a really good discussion, try

http://www.optics.arizona.edu/Wright/images/SlowLight.pdf

QUESTION

On page 393, Griffiths states in the footnote that Ohm's law breaks down on time scales shorter than the time between collisions. Why is this, and does it have any significant consequences?

ANSWER

You misinterpret what Griffith says a bit. He says that the time scale one gets by applying Ohm's law to get charge relaxation (that is $\tau = \varepsilon/\sigma$) is unphysically short in a good conductor. That actually the time scale can not be faster than the $\approx 10^{-14}$ seconds between collisions and is actually even slower (presumably due to LC time constancts of the macroscopic material.) Let us consider a 100kHz radio station ($\frac{1}{\nu} = 10^{-5}$sec) and visible light at 600nm ($\frac{1}{\nu} = \frac{\lambda}{c} = \frac{6\times10^{-7}}{3\times10^{8}} = 2 \times 10^{-15}$sec). We see that our model that charges rearrange quickly to keep $\rho_f = 0$ is a valid approximation for radio signals, but not so good for light.

QUESTION

I know you're pretty busy this week, but I have a random question about magnetism that I thought you might be able to answer.

How can a refrigerator magnet hold something up without an expenditure of energy? After all, if you used your finger to keep a piece of paper off the floor you would be expending energy to press the paper against the door with enough force so that the friction between the paper and door was sufficient to overcome gravity. On the other hand, once in place, chemical (tape) or mechanical (thumb tack) bonding will also do the trick with no outside help.

ANSWER

Imagine holding a book at arms length for two hours. Your arms hurt and you expend a lot of energy. The point is that you still have done no work. If you had just placed the book on your head, you would not be nearly so tired. Imagine you place a book on the top of a metal cylinder. Now you want to somehow raise the book by 1mm and hold it there. You could do this by running a current through the cylinder until it heats up enough to expand by 1mm. In order to keep the book elevated, you would have to maintain the temperature, and hence run a constant current. In the end, you would have expended a lot of energy, but done no work. Remember that thermodynamics tells us the maximum efficiency of a process, but there is no limit to how much energy we can waste, even if we do no work.

Now consider the magnet. It turns out that electrons at rest have a magnetic moment. This is absolutely not classical. You would think that if a charge moved in a loop or spun, it would radiate its power away. But this does not happen. Because of the non-zero magnetic moment of the ground state of ferromagnetic materials, it is possible to get a magnetic attraction to hold up the refrigerator magnetic with no work being done.

1. This is a mathematical question, but how did Griffiths derive equation 9.120 from the equation previous to it? This is from page 393.

$$\frac{\partial \rho_f}{\partial t} = -\sigma \vec{\nabla} \cdot \vec{E} = -\frac{\sigma}{\varepsilon} \rho_f$$

Assume

$$\rho_f = \rho(\vec{r}) f(t)$$

then

$$\begin{aligned}
\frac{df}{dt} &= -\frac{\sigma}{\varepsilon} f \\
\frac{1}{f} df &= \frac{-\sigma}{\varepsilon} dt \\
\int \frac{1}{f} df &= -\frac{\sigma}{\varepsilon} \int dt \\
\ln f &= \frac{-\sigma}{\varepsilon} t + C \\
f &= e^C e^{-\sigma t/\varepsilon} \\
&= f_o e^{-\sigma t/\varepsilon}
\end{aligned}$$

so

$$\rho_f = \rho_o e^{-\sigma t/\varepsilon}$$

QUESTION

2. In the footnote on page 393, Griffiths states that Ohm's law breaks down on time scales shorter than 10^-14s. Why does this occur?

ANSWER

Ohm's law assumes the charge have a smooth drift velocity so that $\vec{J} = en\vec{v} = \sigma\vec{E}$. The model is that continuous collisions occuring on a time scale τ_c create a velocity bias equal to the mean velocity of a particle accelerated by a field $\vec{E}$ for a time τ_c. (That is $\vec{v} \approx \frac{e}{2m}\tau_c\vec{E}$ so $\sigma = \frac{e^2 n \tau_c}{2m}$.) If we consider charges on a time less than τ_c we must consider how they accelerate, and not just their mean velocity.

QUESTION

3. I had difficulty following the derivation of the wave equation in conductors on page 394. I am particularly unsure about why the wavenumber is complex. I would like to see more details on this derivation.

ANSWER

Remember that throughout all of this we are assuming that $\vec{E}$ is complex. That is we have

$$\vec{E} = \vec{E}_o e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

If we can justify this, we certainyl can justify the fact that $\vec{k}$ is complex. But how is $\vec{E}$ complex physical? The point is it is not, but by considering complex electric fields, we can get to an answer rather quickly. The "answer" is the response of our system to this complex disturbance. Let us assume (to make this more concrete) that the response is the reflection of the electric field off of a surface. We do a bunch of boundary condition matching to find

$$\vec{E}_R(\vec{E})$$

Then we repeat the problem, this time not with $\vec{E}$, but rather with $\vec{E}^*$ with

$$\vec{E}^* = \vec{E}_o^* e^{-i(\vec{k}\cdot\vec{r}-\omega t)}$$

we then repeat our math to find

$$\vec{E}_R(\vec{E}^*)$$

Because we are finding the solutions to a linear set of differential equations, the response to the sum $\frac{1}{2}(\ \vec{E} + \vec{E}^*)$ (that is the real part of $\vec{E}$) must be given by the sum of the solutions. Specifically we have

$$\vec{E}_R(\frac{1}{2}(\vec{E} + \vec{E}^*)) = \frac{1}{2}(\vec{E}_R(\vec{E}) + \vec{E}_R(\vec{E}^*))$$

So in the end, we find the response to a physical (real) disturbance, but we avoid a lot of ugly algebra.

QUESTION

5. Figure 9.22 on page 403 seems unclear to me. I am not sure of the point Griffiths is trying to make about anomalous dispersion with this graph. Can you go over this?

Anomalous dispersion is nothing more than a situation for which the index of refrection falls when the frequency increases. It is nothing that strange, just it is not the usual case, so it is given the name anomalous. This is common in physics. People like to title papers "On the anomalous behavior of blah when exposed to light at blah frequency." The name anomalous sticks, even after the physics is figured out.

QUESTION

Dr. Shafer-Ray, I know this is from 9.3, but it has really been bugging me. In the Figure 9.13, the reflected wave has the velocity flipped (for obvious reasons), the E field remaining (seems normal) and the B field's polarization flipped (???). I understand that the cross product demands this, but it REALLY seems that if the B field flips, the E field should have to do so as well. What's the physical explanation for this? Why are B fields different?

ANSWER

This is a direct result of the boundary conditions. The boundary contions are

$$\begin{aligned} E_R^{||} + E_0^{||} &= E_T^{||} \\ E_R^{||} + E_O^{||} &= \frac{n_2}{n_1}\frac{\mu_1}{\mu_2} E_T \end{aligned}$$

Which has the solution

$$\begin{aligned} E_R^{||} &= \frac{1-\beta}{1+\beta} E_o^{||} \\ \beta &= \frac{n_2\mu_1}{n_1\mu_2} \end{aligned}$$

Here's the key. If $\beta > 1$ (going from, say air to glass) the electric field vector of the reflected wave is opposite to the incident wave. If $\beta < 1$ (from glass to air) thenthe electric field vector does not. Thus there is nothing to special about $\vec{E}_R$ flipping or not. We must remember that energy always flows in the direction of the pointing vector so one and only one of $\vec{B}$ and $\vec{E}$ must flip to create a reflected beam.

QUESTION

As for 9.4, once again, why are B fields different? Griffiths spends literally one sentence saying the B field lags behind the E field in a dispersive medium. Why? How? I see it mathematically, but it makes no sense to me physically. What makes B fields different?

ANSWER

This is a really good question because it brings home at once the power of the plane wave picture, as well as its limitations. Remember the plane wave solution is just that, a solution to Maxwell's equations in the vacuum. The solution to the electric field for a stationary charge located at the origin is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{\vec{r}}{r^3}$$

Here their is no magnetic field. Why are the electric and magnetic fields so different? The answer is obvious. We have a source of electric fields, but none of magnetic fields. In a conductor there is a source of magnetic fields (currents brought about by $\sigma\vec{E}$) but there is no local source of electric field. The fact that the electric and magnetic fields are related at all is because of the simple assumption that the current is directly proportional to the electric field.