Chapter 27 Lecture Notes

Physics 2424 - Strauss

Formulas:

$\lambda_p T = 2.80 \times 10^{-3}$ m·K
$E = nhf = nhc/\lambda$
$f\lambda = c$
$hf = K_{max} + W_0$
$\lambda = h/p$
$\lambda' - \lambda = (h/mc)(1 - \cos\theta)$
$1/\lambda = R(1/n_f^{\,2} - 1/n_i^{\,2})$
Lyman Series $n_f = 1$, $n_i = 2,3,4...$ Balmer Series $n_f = 2$, $n_i = 3,4,5...$
Paschen Series $n_f = 3$, $n_i = 4,5,6...$

Constants:

$h = 6.6260755 \times 10^{-34}$ J·s
$h/(mc) = 2.43 \times 10^{-12}$ m (Compton Wavelength of electron)
1 eV = 1.602×10^{-19} J
$R = 1.097 \times 10^{7}$ m^{-1}

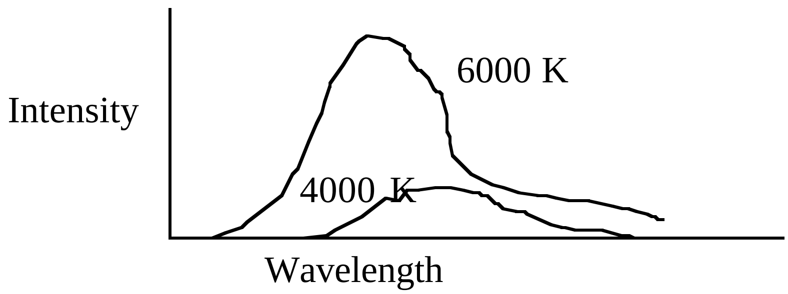
1. INTRODUCTION

At the end of the 19th century, physicists thought they pretty much had the world figured out. Newton's laws explained gravity and motion. Maxwell's equations explained electricity and magnetism and light. Then some amazing discoveries occurred which led to what can only be called a "revolution." This revolution was so large in scope that we now call the physics done before 1900 "classical physics," and the physics after 1900 "modern physics." The revolution came in our understanding of nature at very fast speeds (the special theory of relativity), and at very small sizes (quantum theory). It turns out that Newton's laws are good approximations of quantum theory and relativity at large sizes and at slow speeds.

2. LIGHT

2.1 Blackbody Radiation

We have seen that light acts like a wave. It interferes and diffracts. In the early 20th century a number of scientists were studying electromagnetic (em) radiation from objects. Every single object radiates em energy, no matter how cold or hot it is. At certain wavelengths we see it (like a fire or the sun), and at other wavelengths, our eyes don't see it. For instance, our bodies radiate infrared radiation (heat). Night goggles work by detecting this infrared radiation Even what appears to be a cold object is actually radiating some em radiation. All objects radiate. A body which absorbs all the energy on it is called a perfect absorber. An object which radiates an energy spectrum equivalent to a perfect absorber is called a "black body." Black body radiators do not radiate one wavelength, but rather, a spectrum of wavelengths.



The wavelength at the peak of the blackbody spectrum is related to the temperature of the body by the equation

$$\lambda_p T = 2.80 \times 10^{-3} \text{ m}\cdot\text{K}$$

where λ_p is the wavelength and T is the temperature of the body in Kelvin.

At the turn of the century physicists believed that they should be able to calculate that spectrum, but whenever they tried to, they got the wrong answer. Finally a German physicist, Max Planck, figured out a way to calculate the black body spectrum. He made an *ad hoc* assumption that they energy of the atomic oscillators which create the black body spectrum only comes in packets with specified energies.

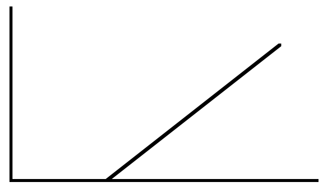
$$E=nhf$$

where h is Planck's constant (h = 6.6260755 $\times$ 10^{-34} J·s), and f is the frequency of vibration. When he did this the blackbody spectrum was explained. He didn't see the ramifications of his prediction. It was just a mathematical trick. But Einstein did see the ramifications and applied them to the photoelectric effect.

<u>Problem:</u> The temperature of skin is about 35° C. At what wavelength does the radiation from skin reach its peak?

2.2 Photoelectric Effect

Another experiment that no one could explain was called the photoelectric effect. When light shines on an object it can free electrons from the object. So we see the electrons. There were aspects of the photoelectric effect which could not be explained if light were a wave. The electrons were ejected with some kinetic energy (K) or motion. Let's see what we would expect to happen if light were a wave, and if light were a particle.



For Waves:

1. If the light intensity is increased, the number of electrons ejected and their maximum kinetic energy should increase because the higher intensity means a greater electric field amplitude. ($S = c\varepsilon_0 E^2$).
2. The frequency of the light should not affect the maximum kinetic energy (K_{MAX}) of the ejected electrons, only the intensity.

Einstein suggested that light acted like a particle (we now call a light particle a photon) with a discrete energy that was related to the frequency of the light. He said that.

$E = hf$ (Energy of a Photon) Remember that $\lambda f = c$

Then with this hypothesis, the predictions for the photoelectric effect would be

1. An increase in intensity means more photons are incident on the material, so more electrons would be ejected, but the energy of each photon is not changed so the maximum K of the electrons would not change.
2. If the frequency of the light is increased, the maximum kinetic energy of the electrons increases linearly.
   E(initial) = Energy (final)
   $hf = K_{max} + W_0$
3. If the frequency is less than the cutoff, f_0, where $f_0 = W_0$ no electrons will be emited, no matter how intense the light is. W_0 is called the work function and is the minimum energy necessary to eject electrons from the material.

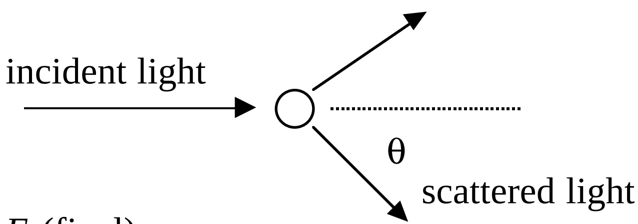
This is exactly what was seen. So Einstein said that light was a particle. If we have n photons, then the total energy is just $E = nhf$.

Problem: Radiation with a wavelength of 281 nm shines on a metal surface and ejects electrons that have a maximum speed of 3.48×10^5 m/s. We put a negative electrode across from the metal and note that a current flows (Figure 27-7). We increase the voltage on the electrode until no current flows. (a) What is the

minimum voltage (called the stopping potential) that will stop the current flow? (b) What is the metal?
(Work functions of K: 2.24 eV, Ca: 2.71 eV, U: 3.63 eV, Al: 4.08 eV, Au: 4.82 eV)

2.3 Photon Momentum

In 1923, the American Arthur Compton was doing an experiment scattering X-rays off of electrons in graphite. Again, his results were not consistent with light being a wave, but if light were like a particle, then collisions with electrons would be like collisions with billiard balls.



E (initial) = E (final)
$hf_0 = hf + K$ (electron recoil)
p (initial) = p (final)

It turned out that everything was fine if light like acted like particles with $E=hf$, and with $p=h/\lambda$. (The book gives the derivation of this using special relativity). Then Compton calculated that light would scatter off of electrons with the angular distribution of

$$\lambda - \lambda_0 = (h/mc)(1 - \cos\theta)$$

where h/mc is called the Compton wavelength of the electron. Note that λ_0 is less than λ, so that f_0 is greater than f, or the energy of the initial photon is greater than the energy of the final photon, since some energy goes into the final particle's kinetic energy.

So light acts like a wave (remember all of the stuff from Chapter 24) and it acts like a particle. What is it? It is really neither and it is both. This is the principle of duality. We use what we know (waves and particles) to describe properties of things we don't know, like light. But light is a different entity altogether from anything in our normal experience.

This brings up one final point. If photons have energy and energy is a form of mass ($E = mc^2$), is it possible for the photon energy to create mass? The answer is yes. A photon can convert all of its energy into a particle and its antiparticle. This is called **pair production**.

Problem: What is the minimum energy a photon needs in order to produce two particles called muons, each with a mass 207 times the electron mass.

(See example 27-5). What would happen if the muons also had kinetic energy?

3. ELECTRONS AND THE DE BROGLIE WAVELENGTH

In 1923 Louis de Broglie made the bold assertion that if light were both a wave and a particle, then particles were also both particles and waves, and should also show wave properties. He said that the relationship between the wavelength of a particle and its momentum would be given by same equation as for light, $\lambda = h/p$. This wavelength is called the de Broglie wavelength. We have seen in many experiments that particles do exhibit wave behavior. For example, electrons can undergo diffraction and produce diffraction patterns.

Let's consider a double slit experiment with an electron. If I have two slits, particle can only go through one slit at a time, but waves go through both slits simultaneously. What happens if I do a double slit experiment with electrons? I see a diffraction pattern (page 806). Each electron hits only one place on the screen but all the electrons together give me a diffraction pattern. This is what I would expect if electrons were a wave. It's like each electron know about both slits. Now let's say I close one slit for a while, then I close the other so that each electron can only go through only one slit. I don't see a diffraction pattern anymore. I see what I would expect if electrons were a particle. I see two bright spots on the screen and no diffraction pattern. Finally, I do an experiment where I leave both slits open, but I set up an apparatus so that I can observe which slit the electron goes through. I have both slits open, but I will observe if the electron goes through the first or second slit. What do I see? You may be amazed to find out I do not see a diffraction pattern. By observing the electron as it goes through one and only one slit, I have forced the electron to behave like a particle. (After all particles go through one and only one slit and waves go through both slits). Then the electron looks like a particle.

So what is an electron: a wave or a particle? Like electromagnetic radiation, we label things as waves and particles, but nature is strange. The electron is in some ways neither a wave or a particle. We make models of the electron saying it acts like a wave sometimes and it acts like a particle sometimes but our model is only to describe something we really don't experience in our macroscopic world.

Very large momentums have very small wavelengths and since $p = mv$, anything with any appreciable mass will have a very short wavelength.

Problem: What is the de Broglie wavelength of an electron in a 5000 volt X-ray tube?

The mass of the electron is 9.11×10^{-31} kg. q=1.60×10^{-19} C.

Remember that we have learned that we can only "see" something if the wavelength of the light is about the same size as the object we are trying to see. Well since electrons have a wavelength, we can bounce them off of objects and observe the objects. (Our eyes can't see these wavelengths, but other instruments can). That is what an electron microscope is. Electrons are bounced off of an object and then detected so that we can observe the object. The electron's small wavelength at high kinetic energies allows us to probe very small objects.

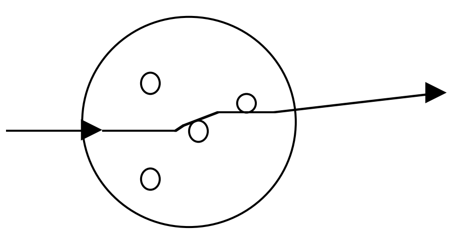
Problem: What is the de Broglie wavelength of a 141 gram frisbee thrown at 30 miles/hour on a frisbee golf course.

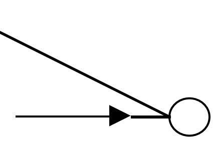
This is so small we will never be able to observe the wave properties of a frisbee. But if Planck's constant was a much larger number, we could observe wave properties of frisbees and baseballs like we do for electrons. There are many ways of thinking about what the wavelength of a particle means. One of the best ways is as a probability wave. The places where the wave has a large amplitude means there is a large probability the particle will be there. Actually we use the amplitude of the wave squared. If the wave has a small squared amplitude at a certain place, then there is a small probability the particle will be there. If it has a large squared amplitude, there is a large probability the particle is there.

4. EARLY MODELS OF THE ATOM

4.1 The Nuclear Atom (Rutherford's)

What does the atom look like and how do we know? An early model of the atom thought that the atom was a "pudding" of positive charge with negatively charged "plums" distributed throughout. The Plum Pudding model was discredited by Ernest Rutherford in 1911. Rutherford fired alpha particles (helium nucleus) into gold foil and some of the alpha particles bounced back. "It was almost as incredible as if you had fired a fifteen inch shell at a piece of tissue and it came back and hit you."





Rutherford surmised that the atom had a hard center, a nucleus. This is called the nuclear atom. The electrons orbit like planets (the planetary model). There is a problem, however, with this model. Remember that the source of em radiation is accelerating charges. An electron in orbit around a nucleus is accelerating (centripetal acceleration) so it is creating em radiation which carries energy away from the atom. Where is the energy coming from? In this model, energy is not conserved. Or if it was, the electron would lose energy and spiral into the nucleus. What is going on here?

4.2 The Quantized Planetary Model (Bohr's)

The answer to what was going on came from an experimental understanding of line spectra. If we excite a low pressure gas and look at the spectra of colors that come out we see characteristic patterns or bands. (See page 788 and 789). In the early 20th century, it was not known what the cause of the bands were. (By the way, if we shine light on these various gases, they will absorb the same areas of the spectrum that they emit when excited). It was observed that for the lines from a hydrogen gas, the distance between the lines followed various mathematical formulas.

Lyman Series $1/\lambda = R(1/1^2 - 1/n^2)$ $n = 2,3,4$
Balmer Series $1/\lambda = R(1/2^2 - 1/n^2)$ $n = 3,4,5$
Paschen Series $1/\lambda = R(1/3^2 - 1/n^2)$ $n = 4,5,6$

$R = 1.097 \times 10^7 \text{ m}^{-1}$ (R is called the Rydberg constant).

Note there is a long and short wavelength limit. When n is ∞ we get the shortest wavelength, and when n is the smallest value, we get the longest wavelength.

A Dutch physicist, Neils Bohr, used observations from line spectra and Rutherford's model to propose a model of the hydrogen atom in 1913. He said that electrons followed **stationary orbits** which did not radiate any energy. These were *ad hoc* orbits to fit the experimental evidence.

These orbits were quantized, meaning they had specific values, or radii. Electrons could not orbit at arbitrary radii. As they jumped from a higher orbit to a lower orbit, the change in energy would be a specific amount, and a photon of a certain wavelength (color) was emitted. Because the orbit energies are only at specific values, the energy (wavelength) of the photon would also only be a specific value, and we would see a specific color of light. Absorbing a specific photon would cause the electron to jump from a lower to a higher orbit.

Bohr postulated the energy of the atom was equal to

$E = K + U_E = 1/2mv^2 - kZe^2/r.$ (1)

Also the force holding the electron is

$F = mv^2/r = kZe^2/r^2 \Rightarrow mv^2 = kZe^2/r$ (2)

So plugging this into (1) gives

$E = (1/2)kZe^2/r - kZe^2/r = -(1/2)kZe^2/r.$ (3)

So far this is all material we have covered in class. It is kinetic plus potential energy, Coulomb's law, and centripetal acceleration. But now Bohr made a bold prediction. He predicted that the angular momentum of the electron spinning around the nucleus was quantized. Why? Because it worked! Really that was his only reason!

$L_n = mvr_n = nh/2\pi$ $\quad n = 1,2,3...$

So $v = nh/(2\pi mr_n)$ and from (2)

$m(nh/(2\pi mr_n))^2 = kZe^2/r_n \quad \Rightarrow \quad r_n = \{h^2/(4\pi^2 mke^2)\}\, n^2/Z$

Plugging in for all the numbers we know we get

$r_n = (5.29 \times 10^{-11}\text{ m})\, n^2/Z$ $\quad n = 1,2,3...$

and plugging into (3) gives

$E_n = \{2\pi^2 mk^2e^4/h^2\}\, Z^2/n^2 = -(2.18 \times 10^{-18}\text{ J})\, Z^2/n^2$
$= -(13.6\text{ eV})\, Z^2/\text{n}^2$ $\quad n = 1,2,3...$
(1 ev = 1.602×10^{-19} J)

Then the transition from one energy level to another is

$E_f - E_i = hf = hc/\lambda = -(2.18 \times 10^{-18}\text{ J})\,(1/n_f^{\,2} - 1/n_i^{\,2})$

$1/\lambda = R(1/n_f^{\,2} - 1/n_i^{\,2})$ which is exactly the series from the line spectra when n_f equals 1 or 2 or 3.

The Line Spectra observed occur when electrons jump from one energy level to another. When the electron is in its lowest energy level, this is called the ground state (n=1). When it is in higher energy levels it is called an excited state. The first excited state is (n=2). So the spectral lines we see are the atoms jumping from an excited state to the ground state. Absorption lines are when the electrons jump from a lower state to a higher one.

In general, when all electrons in an atom are in the lowest state that is called the ground state.

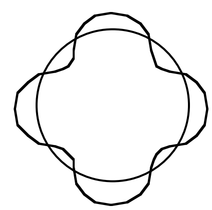
Problem: Find the (a) longest and (b) shortest wavelength photon emitted from the Balmer series, and determine their energies.

4.3 de Broglie's synthesis

Ten years after Bohr proposed this, de Broglie came up with an explanation why the orbital angular momentum was quantized. He said that you could only put an integral number of wavelengths around the orbit of the atom so that

$2\pi r = n\lambda$ n = 1,2,3...

$\lambda = h/p$ and $p=mv$ so



$\boxed{mvr = nh/2\pi}$ which is Bohr's hypothesis.