

Chapter 21

Faraday's Law of Electromagnetic Induction



Magnetic Flux

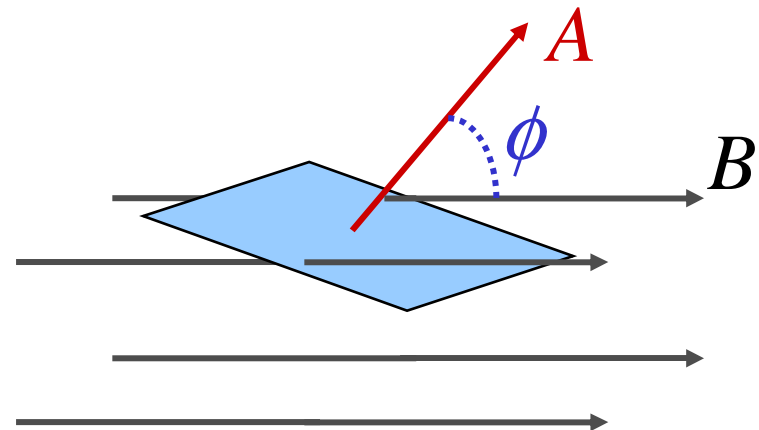
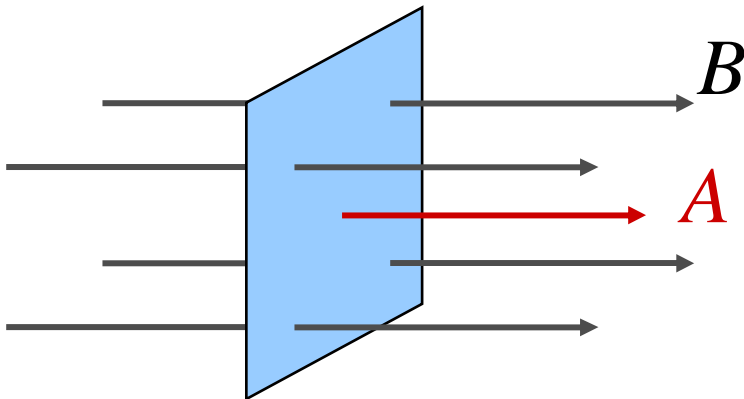
$$\Phi_B = \oint \mathbf{B} \cdot d\mathbf{A}$$

Φ_B : Magnetic Flux

\mathbf{B} : Magnetic Field

\mathbf{A} : Area vector of enclosed current (perpendicular to area)

SI Units: Weber, $\text{Wb} = \text{T} \cdot \text{m}^2$



Problem: A wire is wound in a circular coil of 10 turns with radius of 5.0 cm.

- a) When this coil is placed in an external magnetic field of 0.75 T at an angle of 60° between the face and the magnetic field, what is the flux passing through the coil?
- b) The coil is taken out of the magnetic field and a current of 2.5 A is generated in the coil. Assuming that the magnetic field throughout the whole coil is the same as that at its center, what is the flux through the coil now?

A changing magnetic flux induces an emf (voltage difference).

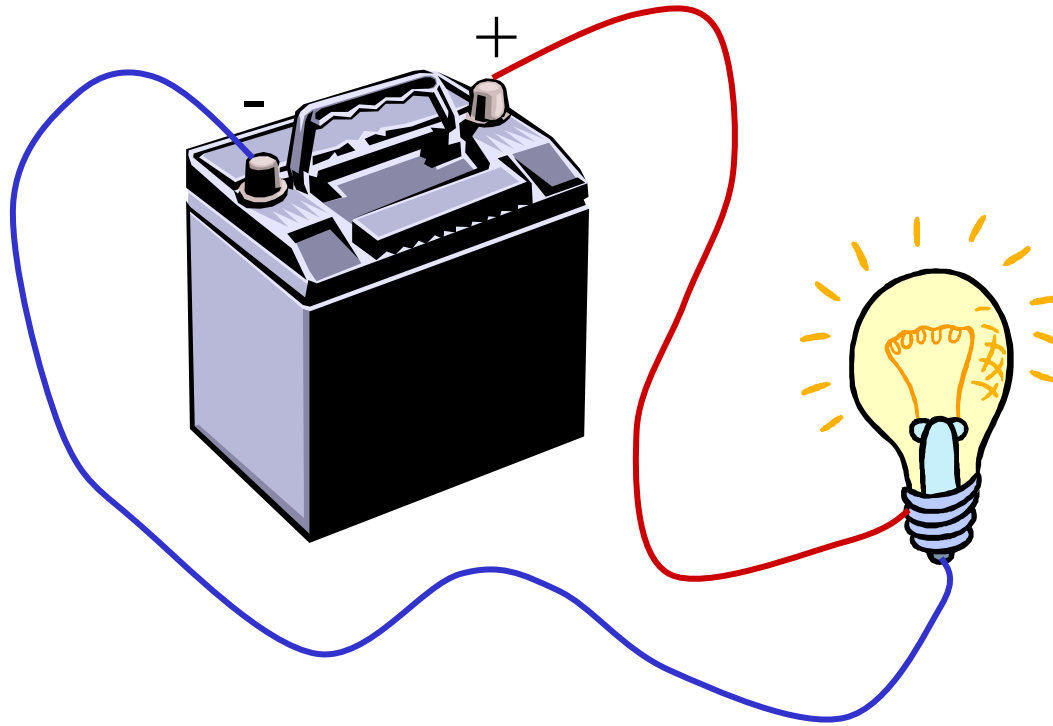
1. How do you change it?
2. What are the consequences of the induced emf?

How can we change the magnetic flux?

$$\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA \cos \phi$$

1. Change the area, A
2. Change the angle, ϕ
3. Change the field, B

What are the consequences of the induced emf
(or potential difference)?



So simply changing the magnetic flux creates a
potential difference that can produce electricity!

Faraday's Law gives the magnitude of the induced emf

$$\mathcal{E} = - d\Phi_B/dt$$

\mathcal{E} : The induced potential difference

$d\Phi_B/dt$ is the changing magnetic flux in a certain time

—: A reminder to use Lenz's law

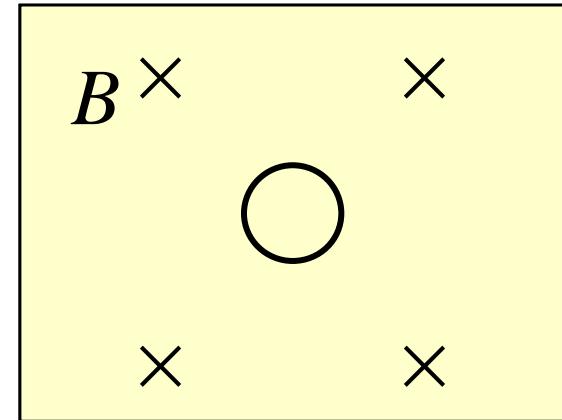
Lenz's Law gives the direction of the induced emf

The induced emf always acts to oppose the changing magnetic flux.

Three steps to solving problems with Lenz's law.

1. Determine the direction of the *external* magnetic field (B), *inside* the designated area (often a loop of wire).
2. Determine if the flux (Φ) is increasing or decreasing inside the designated area. (Look at how B , A , and $\cos\phi$ are changing).
3. Choose the current direction to create an *induced* magnetic field (B_I) that opposes the change in Φ . If flux is decreasing choose current to increase B . If flux is increasing, choose current to decrease B .

Consider a loop of wire in a magnetic field. What happens if the diameter of the wire is suddenly decreased?



1. Determine the direction of the *external* magnetic field (B), *inside* the designated area (often a loop of wire).

Into the page.

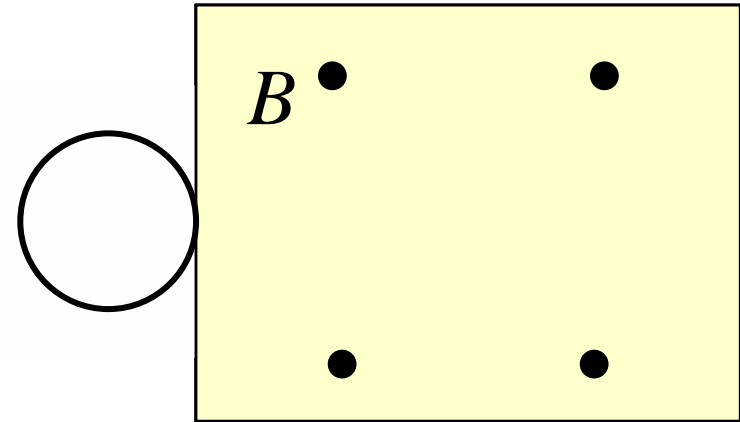
2. Determine if the flux (Φ) is increasing or decreasing inside the designated area.

Area is decreasing so flux is decreasing.

3. Choose the current direction to create an *induced* magnetic field (B_I) that opposes the change in Φ .

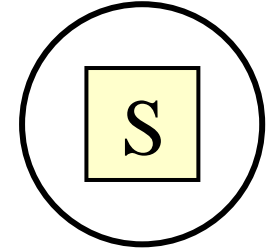
If current is clockwise, then inside the loop there will be an *induced* magnetic field into the page. This will add to the *external* magnetic field to try to increase the flux.

Consider pulling a loop of wire out of a region containing a magnetic field



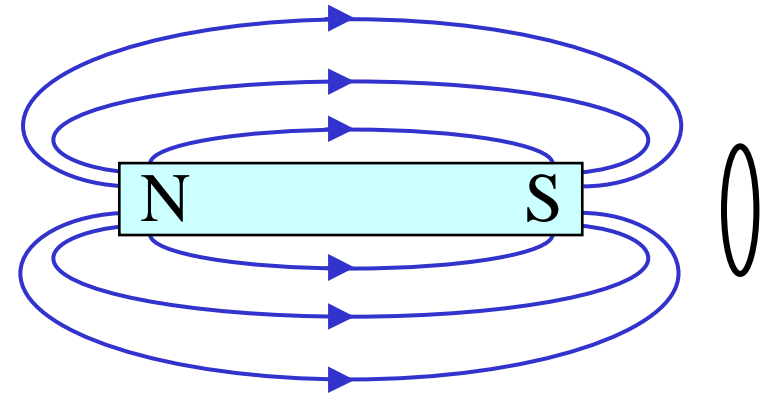
1. Determine the direction of the *external* magnetic field (B), *inside* the designated area.
Out of the page.
2. Determine if the flux (Φ) is increasing or decreasing.
 B inside the loop is decreasing so flux is decreasing.
3. Choose the current direction to create an *induced* magnetic field (B_I) that opposes the change in Φ .
A counterclockwise current will create an *induced* magnetic field inside the loop that points out of the page. This will add to the *external* magnetic field to increase the flux compared to what it would be if there were no current.

A bar magnet is pushed from behind the page through a loop of wire, south end first. What direction does current flow in the wire while the magnet is moving toward the wire?



Direction of external magnetic field inside the loop?

Into the page.



Flux increasing or decreasing?

B inside the loop is increasing as magnet gets closer, so flux is increasing.

Current that opposes this change?

We must reduce the flux. To do this we should create an *induced* magnetic field that points *out of* the page so it will *subtract* from the *external* magnetic field that points into the page. A counterclockwise current will create such a field.

Interactive Question

A long straight wire is in the plane of a rectangular conducting loop. The wire carries a constant current I as suggested by the figure. The wire is held in one place. Which statement is true?



- A) There is no magnetic flux inside the loop
- B) There is a magnetic flux inside the loop, but no current in the loop
- C) There is an induced clockwise current in the loop
- D) There is an induced counterclockwise current in the loop

Interactive Question

A long straight wire is in the plane of a rectangular conducting loop. The wire carries a constant current I

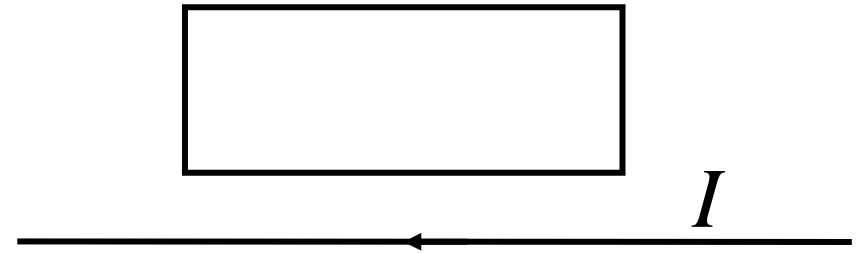


as suggested by the figure. Which is true if the wire is suddenly moved toward the loop?

- A) There will be no induced emf
- B) There will be an induced emf, but no current.
- C) There will be an induced current which is clockwise around the loop
- D) There will be an induced current which is counterclockwise around the loop

Interactive Question

A long straight wire is in the plane of a rectangular non-conducting loop of wood. The wire carries a

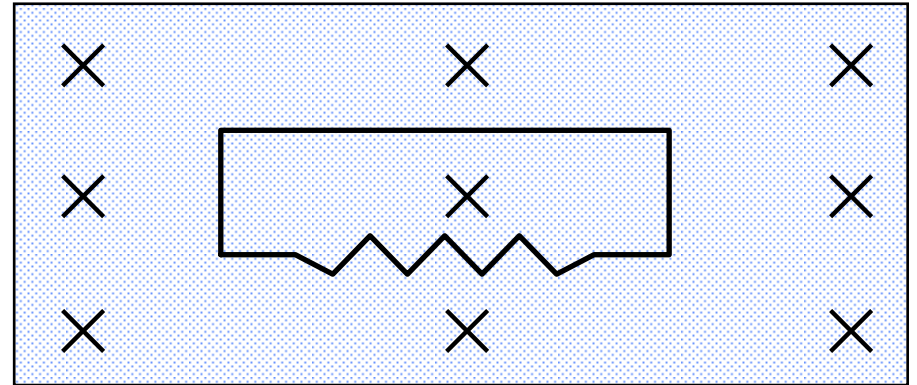


constant current I as suggested by the figure. Which is true if the wire is suddenly moved toward the loop?

- A) There will be no induced emf
- B) There will be an induced emf, but no current.
- C) There will be an induced current which is clockwise around the loop
- D) There will be an induced current which is counterclockwise around the loop

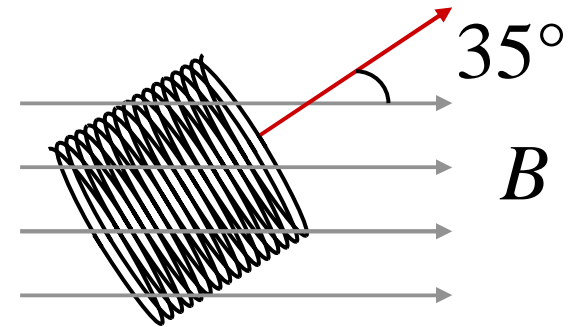
Interactive Question

The figure shows a uniform magnetic field which is normal to the plane of a conducting loop of resistance R . Which of the following changes will cause an induced current to flow from left to right through the resistor?



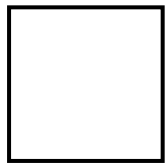
- A) decreasing the area of the loop
- B) decreasing the magnitude of the magnetic field
- C) increasing the magnitude of the magnetic field
- D) rotating the loop 90° into the plane of the paper
- E) rotating the loop 90° out of the plane of the paper

Problem: A coil of 200 turns is wrapped on an 18.0 cm by 18.0 cm frame. The total resistance of the coil is $2.00\ \Omega$. A magnetic field at an angle of 35° as shown to the plane of the coil is change from 0 to 0.500 T in 0.800 s. (a) Find the magnitude of the induced emf. (b) What is the current in the wire?



Interactive Question

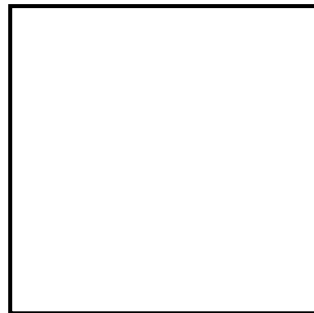
The figure shows four wire loops with edge lengths of either L or $2L$ entering a uniform magnetic field at the same velocity. Rank the four loops according to the magnitude of the emf induced as they enter the field, greatest first.



(a)



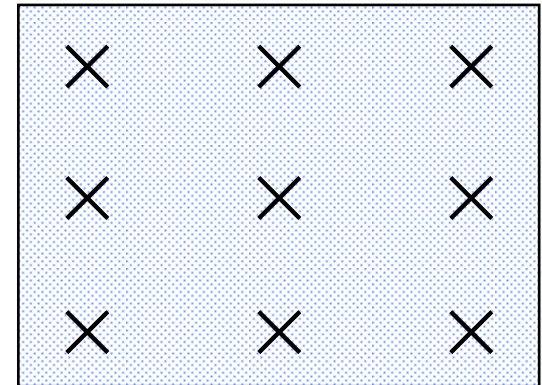
(b)



(c)



(d)



A) $a=b>c=d$

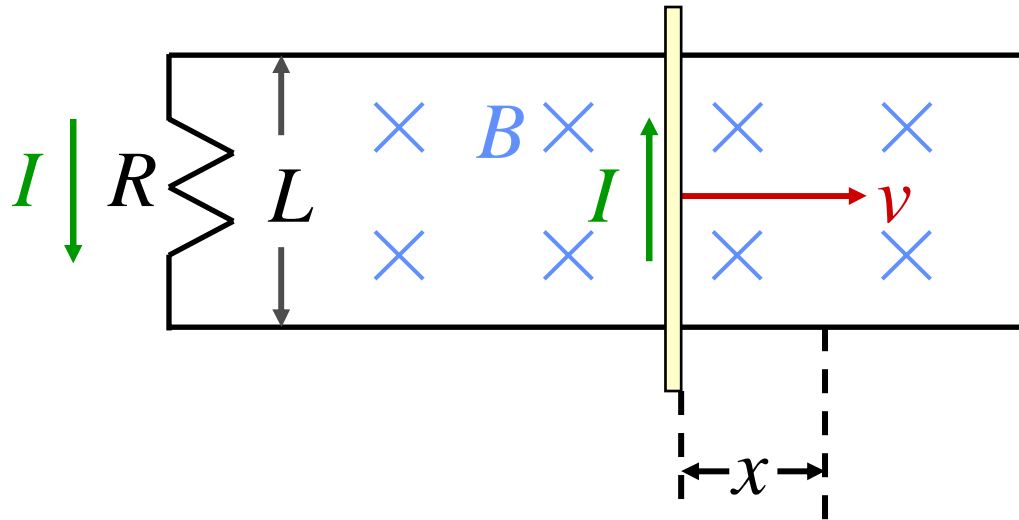
B) $c=d>a=b$

C) $c>d=b>a$

D) $a>b=d>c$

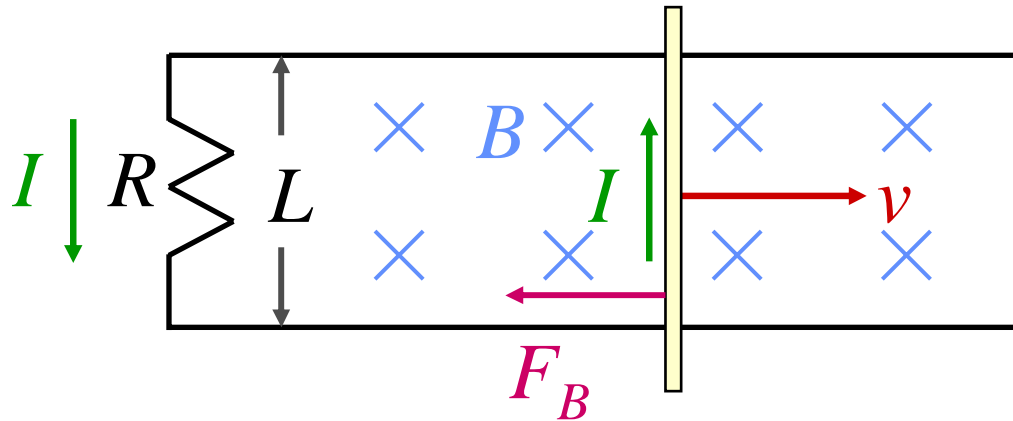
E) $c>d>b>a$

Motional emf



$$\mathcal{E} = d\Phi/dt = d(BA)/dt = B dA/dt = BL dx/dt$$

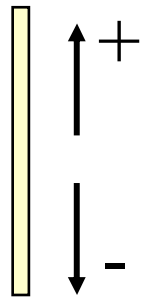
$$\mathcal{E} = Blv$$



Three things to look at:

1) Consider the force on charged particles in the rod:

$$F = qvB$$



2) Consider the magnetic force on the rod: $F_B = ILB \sin \theta$

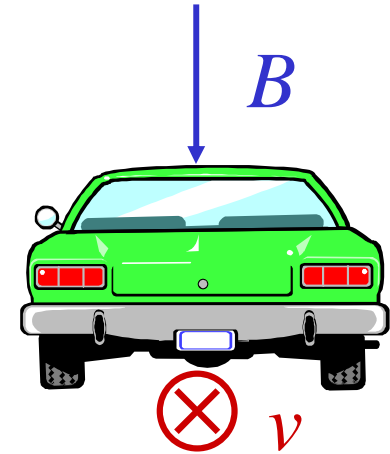
How does the rod continue to the right?

3) Consider $vB = F/q = E$

A general principle:

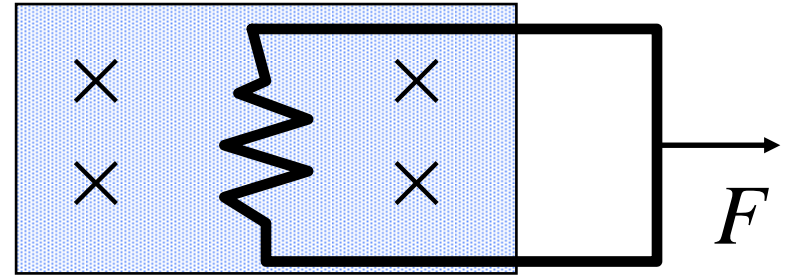
A changing magnetic flux creates an electric field.

Problem: You are driving along at 75 mph within the earth's magnetic field that has a vertical component downward of 5.0×10^{-5} T. The car is about 1.5 meters wide. What is the induced emf and is it positive on driver's or passenger's side?



Interactive Question

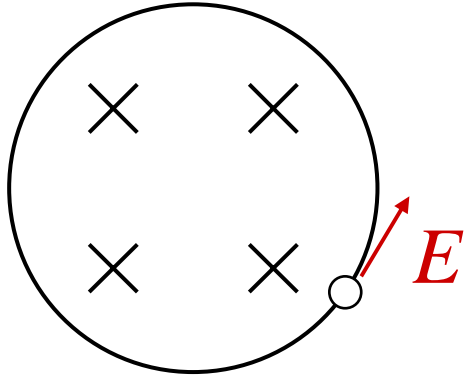
The circuit shown is pulled to the right with a constant velocity by a force directed to the right. Which statement is true regarding this situation?



- A) The induced current flows clockwise around the circuit.
- B) Since the circuit moves with constant speed, the force F does no work.
- C) If the circuit were replaced with a wooden loop, there would be no emf.
- D) As the circuit moves through the field, the field does work to produce the current.

Induced emf and Electric Fields

Consider B increasing



An emf implies there is a potential difference, which implies there is an electric field as discussed before.

Recall: $\mathbf{E} = \oint \mathbf{E} \cdot d\mathbf{s}$

$$q\mathbf{E} = W = \oint \mathbf{F} \cdot d\mathbf{s} = \oint q\mathbf{E} \cdot d\mathbf{s} = -q d\Phi_B / dt$$

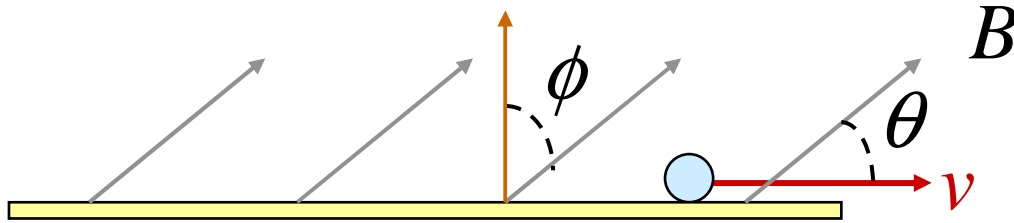
We can write Faraday's law as: $\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$

A changing magnetic flux creates an electric field.

For a conservative electric field created by charged particles:

$$\oint \mathbf{E} \cdot d\mathbf{s} = 0$$

Motional emf at an Angle

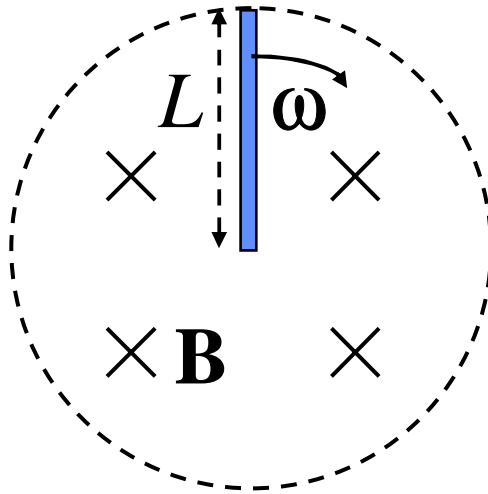


$$\mathcal{E} = d\Phi/dt = d(\mathbf{B} \cdot \mathbf{A})/dt = B \cos \phi \, dA/dt = BL \cos \phi \, dx/dt$$

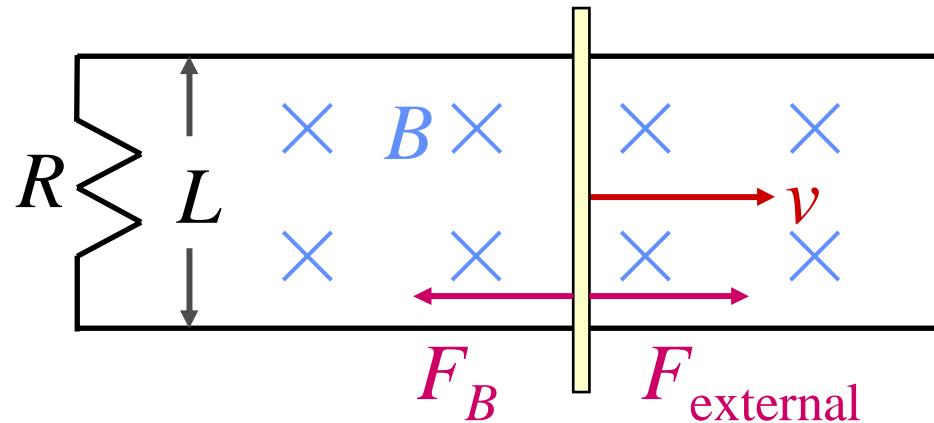
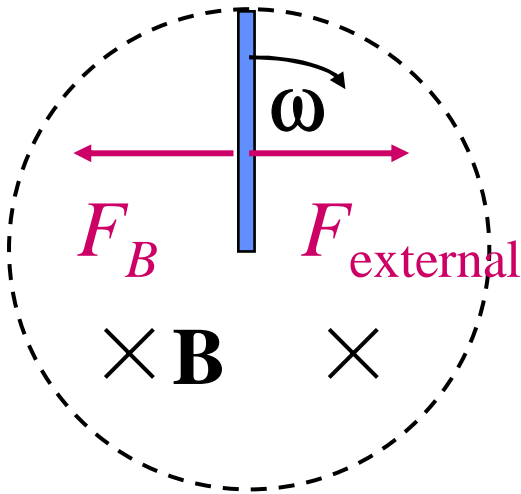
$$\mathcal{E} = Blv \sin \theta$$

$$\mathcal{E} = Blv_{\perp}$$

Problem: A metal rod of length L is rotated at an angular velocity ω about an end in a uniform magnetic field \mathbf{B} , that is parallel to ω . (a) Find the magnitude of the induced emf (b) find the end of the rod that is at the higher electric potential (c) Find the induced electric field.

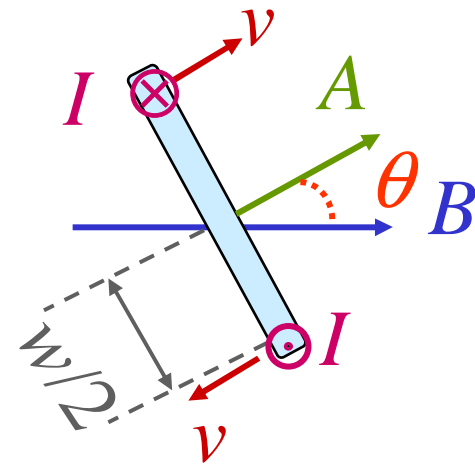
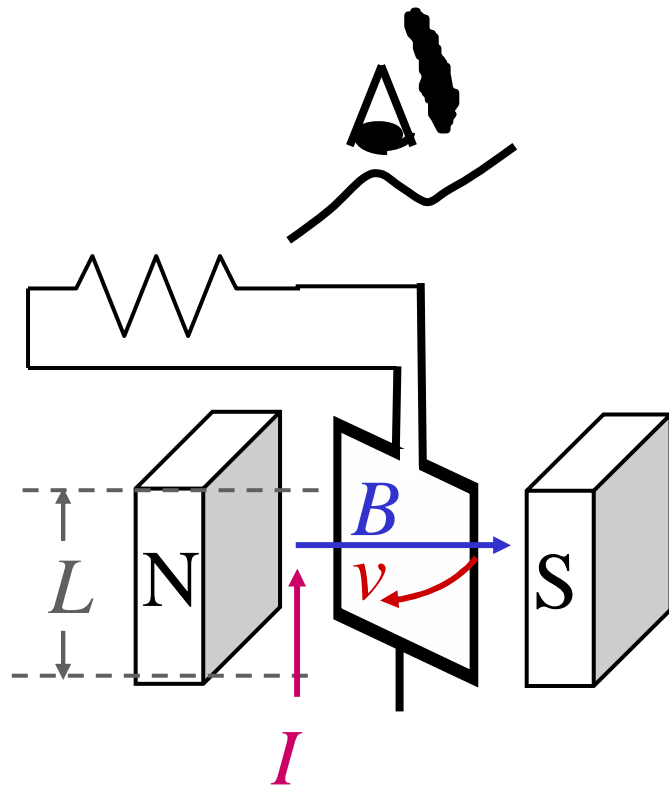


These are both motional emf and require an external force to keep the motion going.



They can both be explained using magnetic forces on moving charged particles or by Faraday's Law.

The Electric Generator



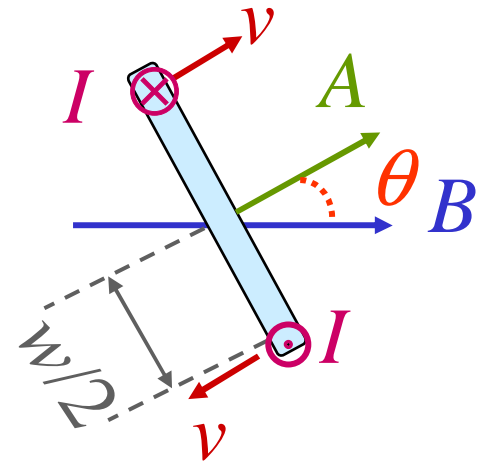
$$\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA \cos \theta$$

$$E = -N d\Phi_B/dt = N BA \sin \theta d\theta/dt$$

$$E = NBA \omega \sin \omega t$$

$$E = NBA \omega \sin(\omega t + \delta)$$

$$E = E_0 \sin(\omega t + \delta) \quad \text{with } E_0 = NBA \omega$$



Or we can do this by looking at motional emf:

For each wire: $E = BLv_{\perp} = BLv \sin \theta$

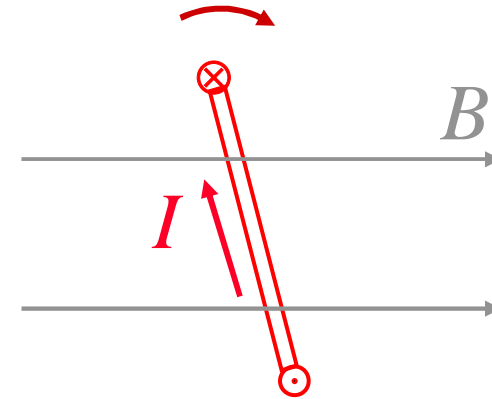
For both wires, and many turns: $E = 2NBLv \sin \theta$

$\theta = \omega t$ and $v = \omega r = \omega w/2$

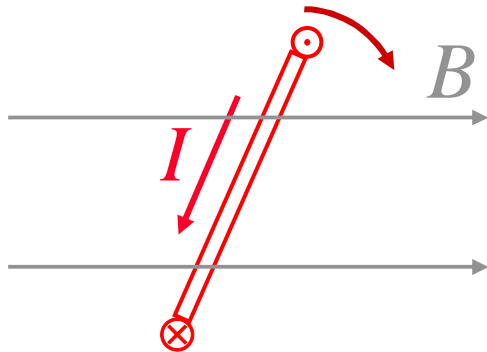
$E = 2NBL\omega(w/2) \sin \omega t = 2NBLw \omega \sin \omega t = NBA \omega \sin \omega t$

The Electric Generator Cycle

Flux is increasing



Flux is decreasing



Repeat the cycle

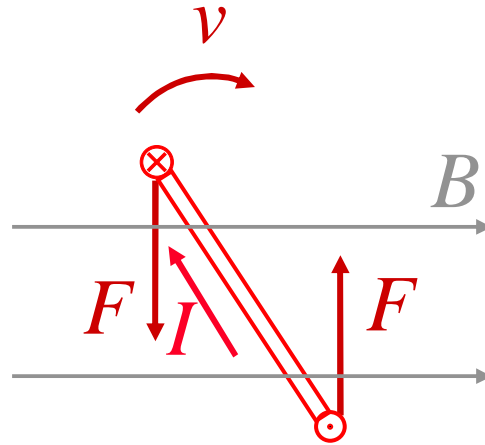
Current changes direction

Notice the similarity

Electric Generator: Using some mechanism, turn a loop of wire in a magnetic field to produce a current.

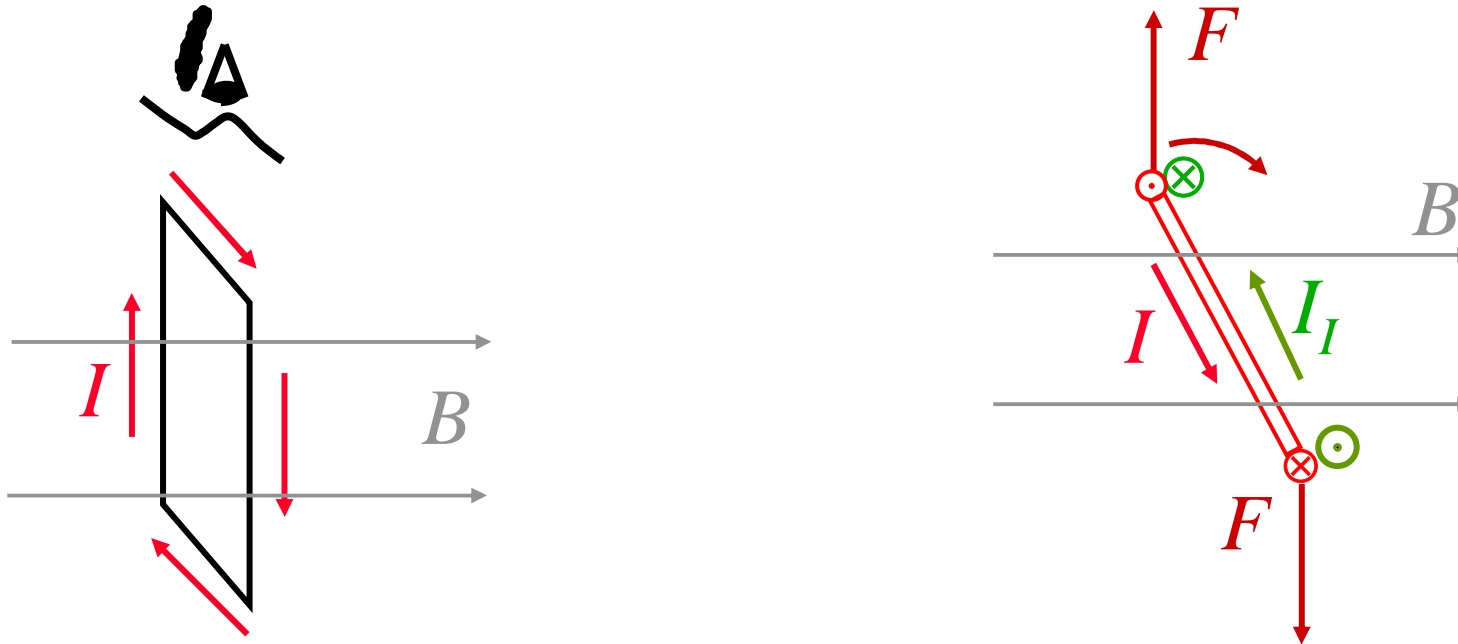
Electric Motor: Run a current through a loop of wire in a magnetic field to make the loop turn.

Consider a generator spinning and creating current.



The current created by the generator creates a torque in the opposite direction from the rotating loop. This is called “countertorque.” So it takes energy to keep the loop spinning.

Now consider an electric motor spinning because a current is running through it.



When a motor is spinning, the *induced* current (I_I) is in the opposite direction from the applied current. This is called “counter emf.” It only occurs while the motor is spinning and makes the total current less than the applied current.

V : Voltage applied to a motor

R : Resistance of the motor

E : Counter emf (potential difference) due to changing magnetic flux.

I : Current flowing when the motor is running.

I_I : Induced Current from changing magnetic flux, caused by E .

With no induced current, $I = V/R$

With an induced current:

$$I = V/R - I_I$$

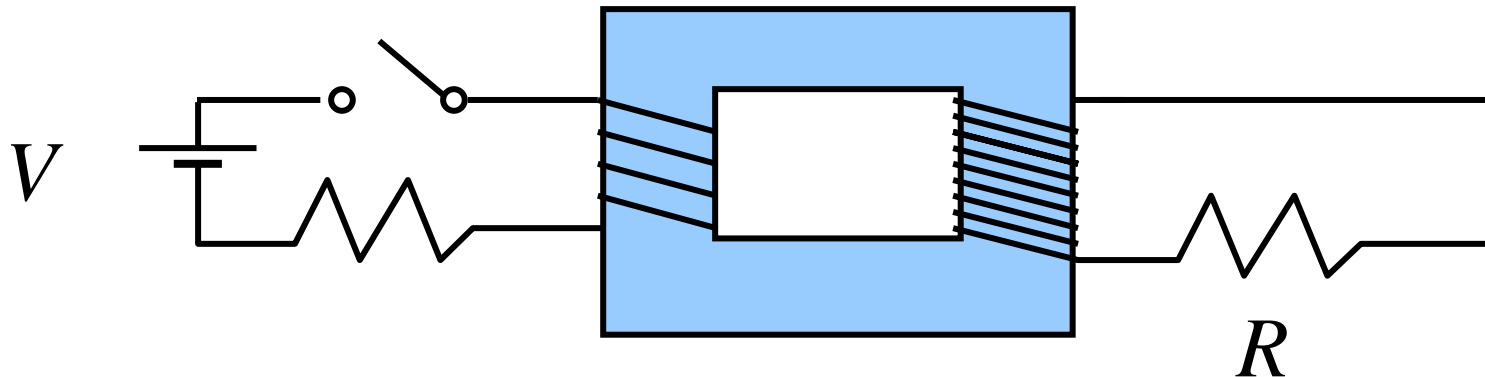
$$I = V/R - E/R$$

$$I = (V - E)/R$$

Problem: Suppose that a motor running at 1200 rpm has a counter emf of 50 V. What is the counter emf when the motor is running at 2000 rpm?

Interactive Question

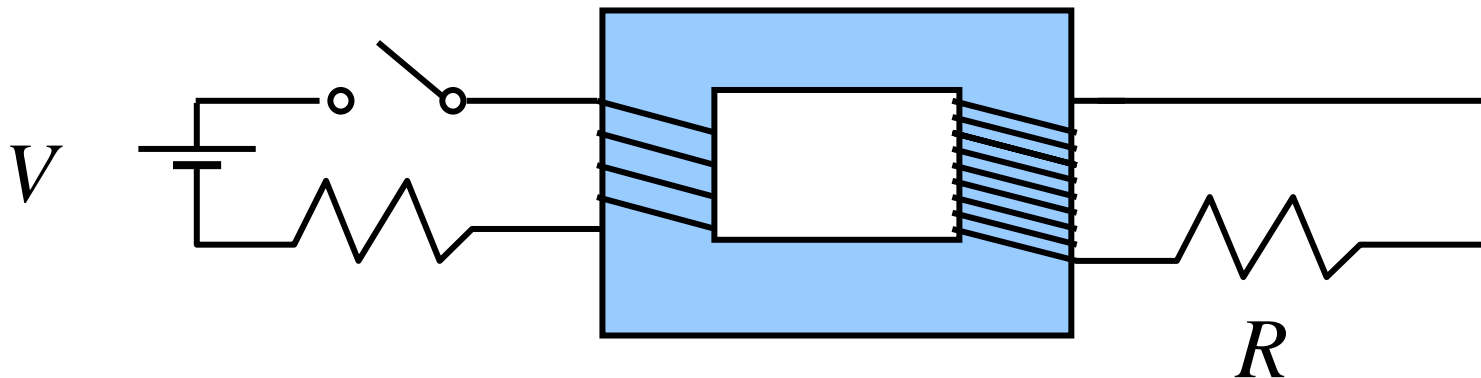
A dc battery, a switch and two resistors are shown. When the switch in the figure is closed, what will happen to the resistor R ?



- A) There will be zero current through R
- B) There will be a steady current through R
- C) There will be a current for a short instant through R

This cannot be explained using magnetic forces on moving charged particles

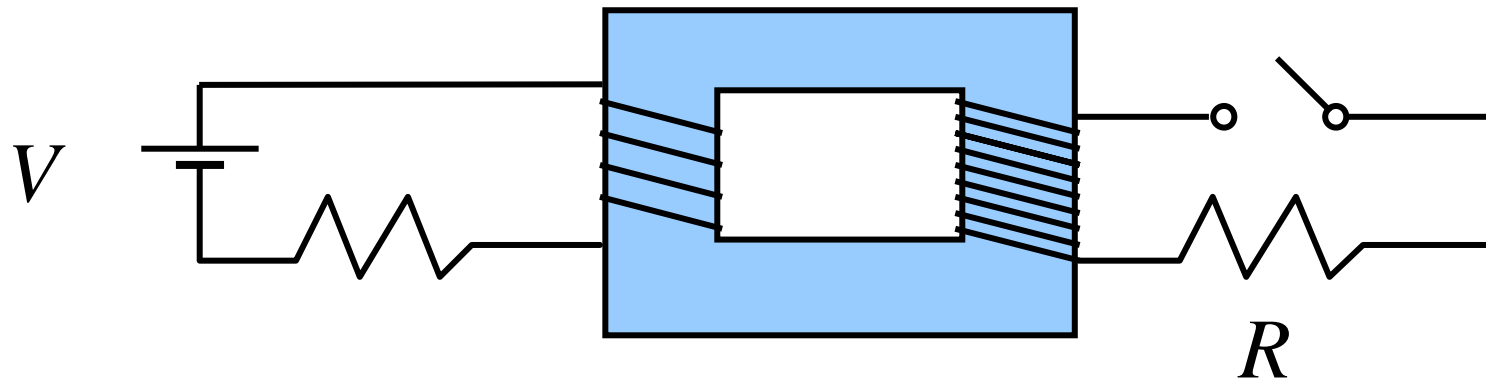
A dc battery, a switch and two resistors are shown. When the switch in the figure is closed there will be a current for a short instant through R ?



It is something entirely different and can only be explained by Faraday's law: $\mathcal{E} = -N d\Phi_B/dt$

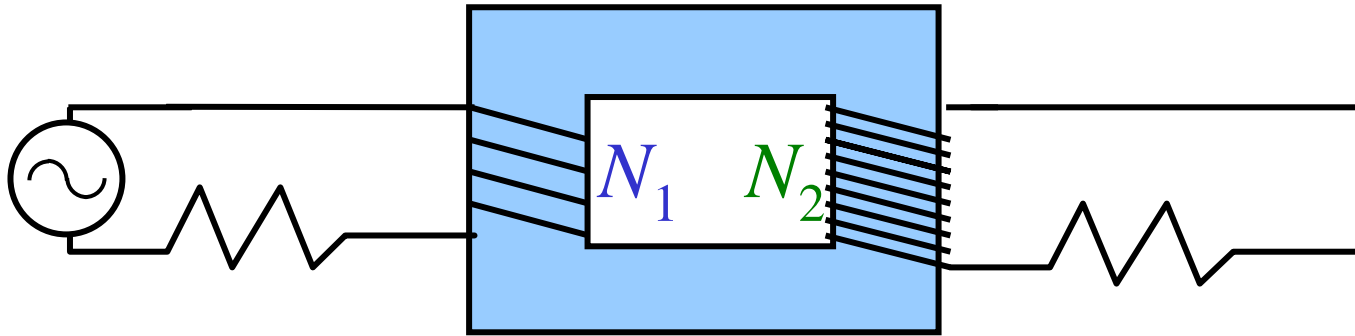
Interactive Question

A dc battery, a switch and two resistors are shown. When the switch in the figure is closed, what will happen to the resistor R ?



- A) There will be zero current through R
- B) There will be a steady current through R
- C) There will be a current for a short instant through R

Transformers



Faraday's Law for coil 2: $E_2 = d\Phi_2/dt = N_2 d(A_2 B_2)/dt$

Faraday's Law for coil 1: $E_1 = d\Phi_1/dt = N_1 d(A_1 B_1)/dt$

Designed so $A_1 B_1 = A_2 B_2$

$$E_2/E_1 = N_2/N_1$$

Often written as: $V_s/V_p = N_s/N_p$.

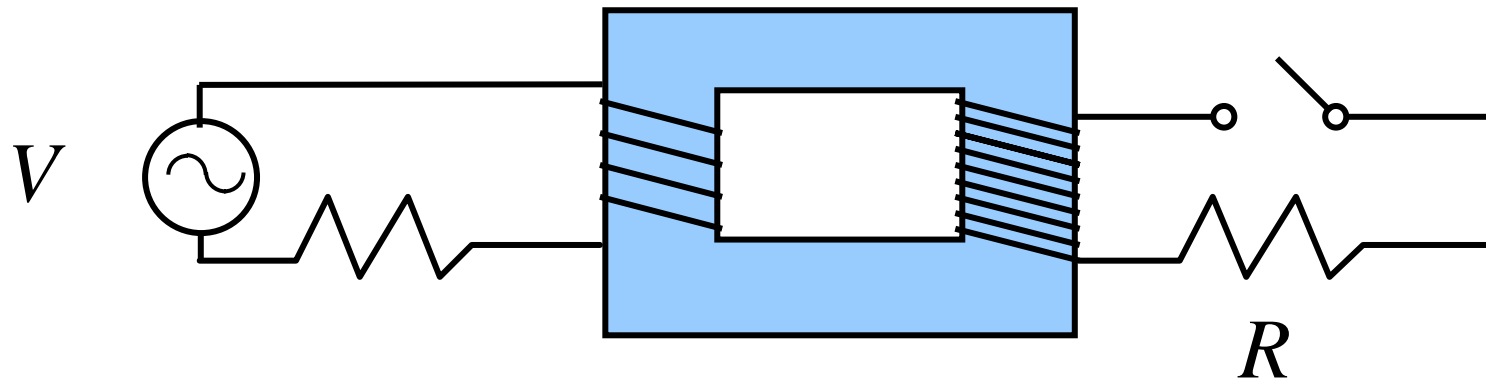
Energy must be conserved: $P_1 = P_2$

$$I_1 V_1 = I_2 V_2$$

Often written as: $I_p/I_s = V_s/V_p = N_s/N_p$

Interactive Question

An ac source, a switch and two resistors are shown. When the switch in the figure is closed, what will happen to the resistor R ?



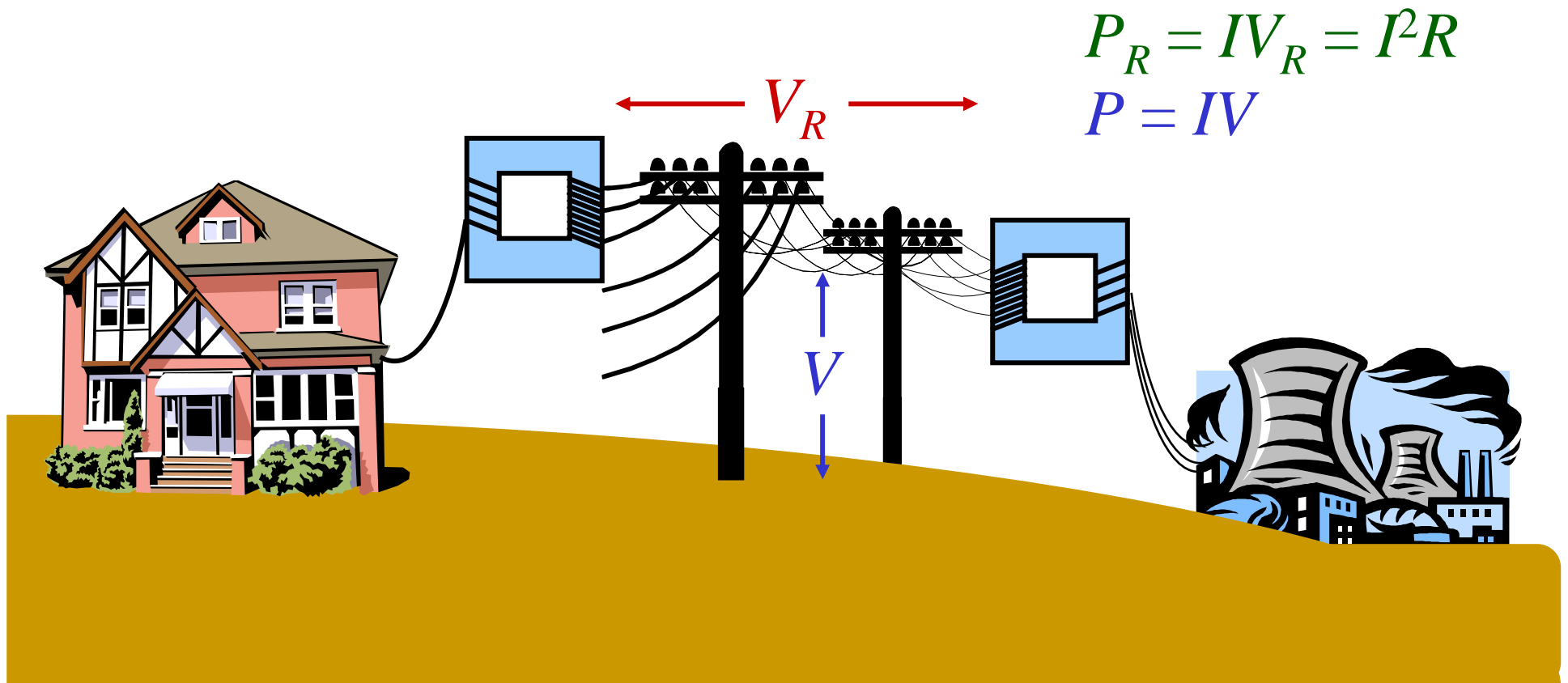
- A) There will be zero current through R
- B) There will be an ac current through R
- C) There will be a current for a short instant through R

Problem: The input to the primary coil of a transformer is 120 V while the current in the secondary coil is 0.10 A.

(a) When 60.0 W of power are delivered to the circuit attached to the secondary coil, what is the voltage across this coil?

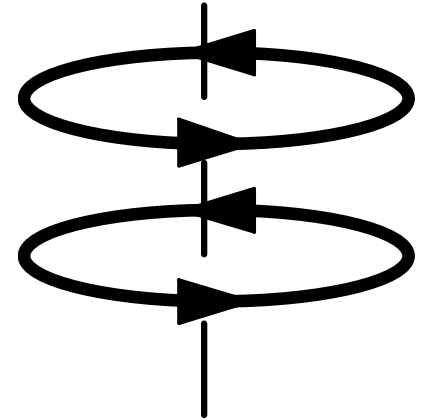
(b) If the primary coil has 20 turns, how many does the secondary coil have?

Problem: Suppose 1 MW of power is being transported over a power line that has a resistance of $20.0\ \Omega$. How much power is lost along the line if the voltage of the line is (a) 240 V (b) 24,000 V?



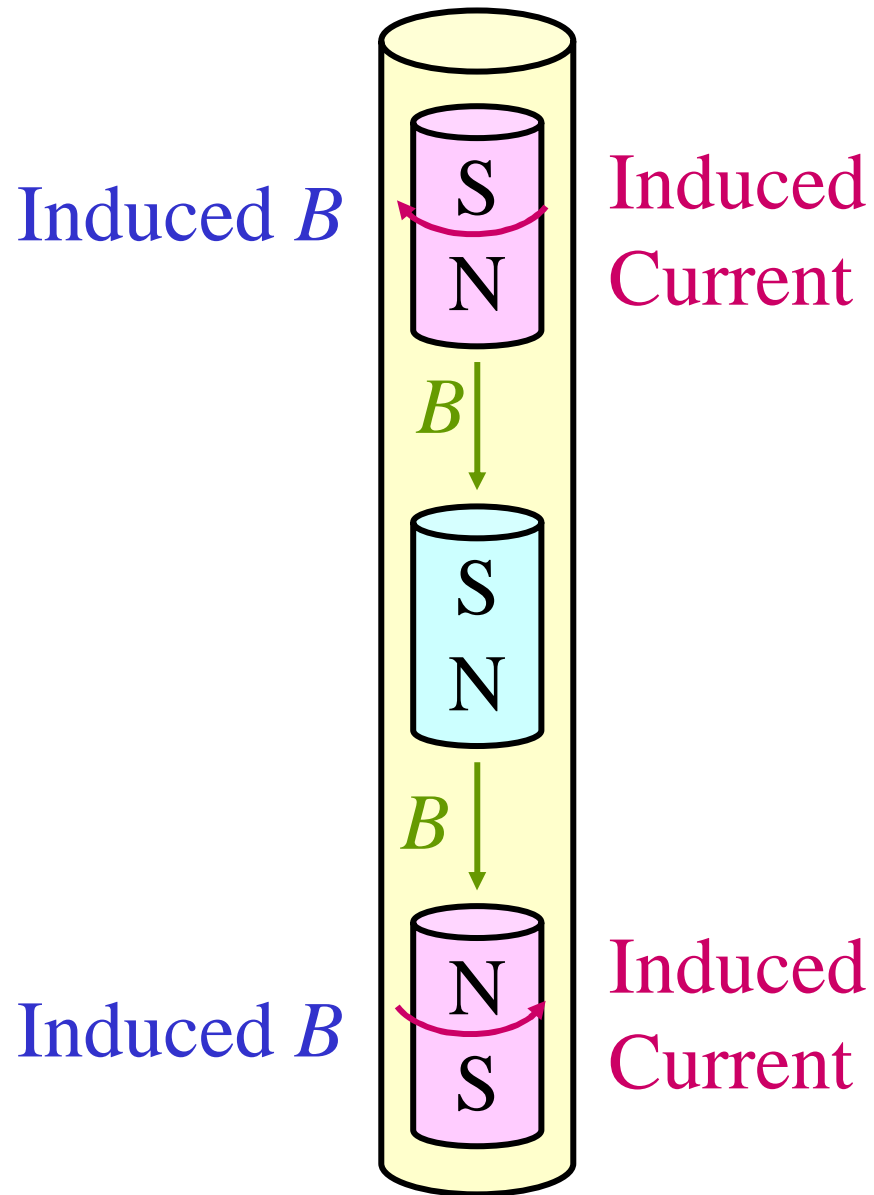
Interactive Question

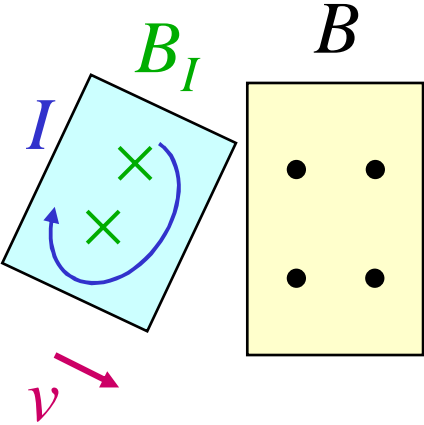
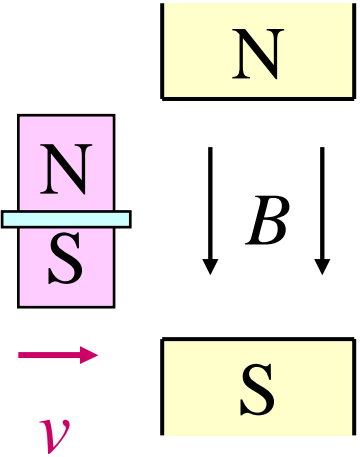
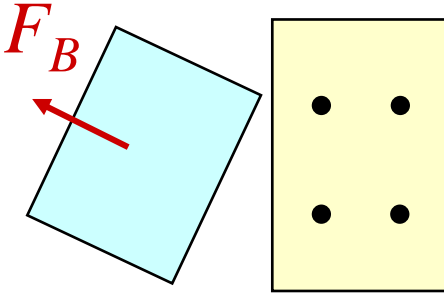
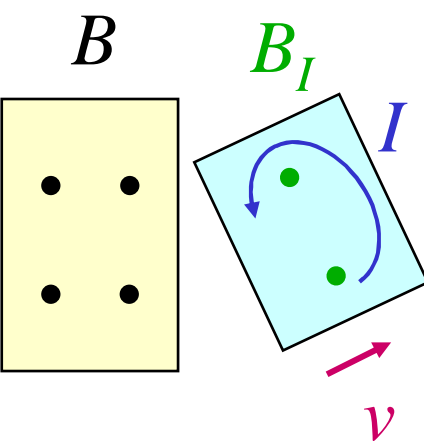
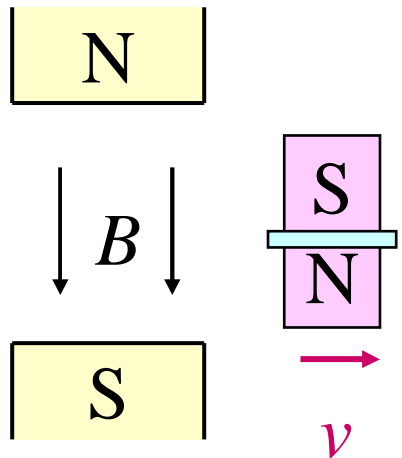
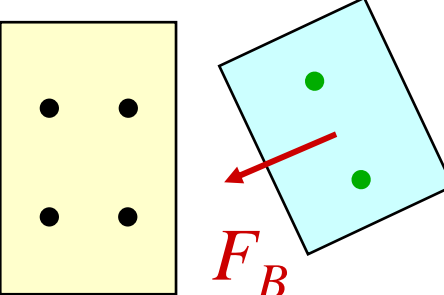
Two loops carry equal currents I in the same direction. They are held in the positions shown in the figure. The current in the top loop suddenly drops to zero. According to Lenz's law, what will happen to the current in the bottom loop?



- A) It will decrease
- B) It will increase
- C) It will not change
- D) It will stop
- E) It will reverse direction

Eddy Currents



| Front View | Top View | Forces | Energy |
|--|--|---|--|
|  |  |  <p>F_B slows the velocity</p> | <p>Potential energy (mgh) changes to kinetic ($mv^2/2$), then to thermal (I^2Rt)</p> <p>$R = \rho L/A$</p> <p>Longer paths for current (L) gives larger R, thus more power dissipated.</p> |
|  |  |  <p>F_B slows the velocity</p> | <p>More surface area allows more current to flow giving larger B, and thus more F_B.</p> |

Maxwell's Equations

| | Integral Form | Differential Form |
|----------------------------|--|--|
| Faraday's Law | $\oint \mathbf{E} \cdot d\mathbf{L} = -d\Phi_B/dt$ | $\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$ |
| Ampere-Maxwell Equation | $\oint \mathbf{B} \cdot d\mathbf{L} = \mu_0 I + \mu_0 \epsilon_0 d\Phi_E/dt$ | $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + (1/c^2) \partial \mathbf{E} / \partial t$ |
| Gauss' Law for Electricity | $\oint \mathbf{E} \cdot d\mathbf{A} = q/\epsilon_0$ | $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$ |
| Gauss' Law for Magnetism | $\oint \mathbf{B} \cdot d\mathbf{A} = 0$ | $\nabla \cdot \mathbf{B} = 0$ |

Interactive Question

Consider the four Maxwell equations:

1) $\oint \mathbf{E} \cdot d\mathbf{A} = q/\epsilon_0$

2) $\oint \mathbf{B} \cdot d\mathbf{A} = 0$

3) $\oint \mathbf{E} \cdot d\mathbf{L} = -d\Phi_B/dt$

4) $\oint \mathbf{B} \cdot d\mathbf{L} = \mu_0 I + \mu_0 \epsilon_0 d\Phi_E/dt$

Which of these must be modified if magnetic monopoles are discovered?

A) Only 1

B) Only 2

C) Only 2 and 3

D) Only 3 and 4

E) Only 2, 3, and 4

Interactive Question

Consider the four Maxwell equations:

1) $\oint \mathbf{E} \cdot d\mathbf{A} = q/\epsilon_0$

2) $\oint \mathbf{B} \cdot d\mathbf{A} = 0$

3) $\oint \mathbf{E} \cdot d\mathbf{L} = -d\Phi_B/dt$

4) $\oint \mathbf{B} \cdot d\mathbf{L} = \mu_0 I + \mu_0 \epsilon_0 d\Phi_E/dt$

Which of these can be used to calculate the electric field from a time varying magnetic field?

A) 1

B) 2

C) 3

D) 4

E) None of these

Interactive Question

Consider the four Maxwell equations:

1) $\oint \mathbf{E} \cdot d\mathbf{A} = q/\epsilon_0$

2) $\oint \mathbf{B} \cdot d\mathbf{A} = 0$

3) $\oint \mathbf{E} \cdot d\mathbf{L} = -d\Phi_B/dt$

4) $\oint \mathbf{B} \cdot d\mathbf{L} = \mu_0 I + \mu_0 \epsilon_0 d\Phi_E/dt$

These equations can be used to calculate:

A) The electric force on a given charge.

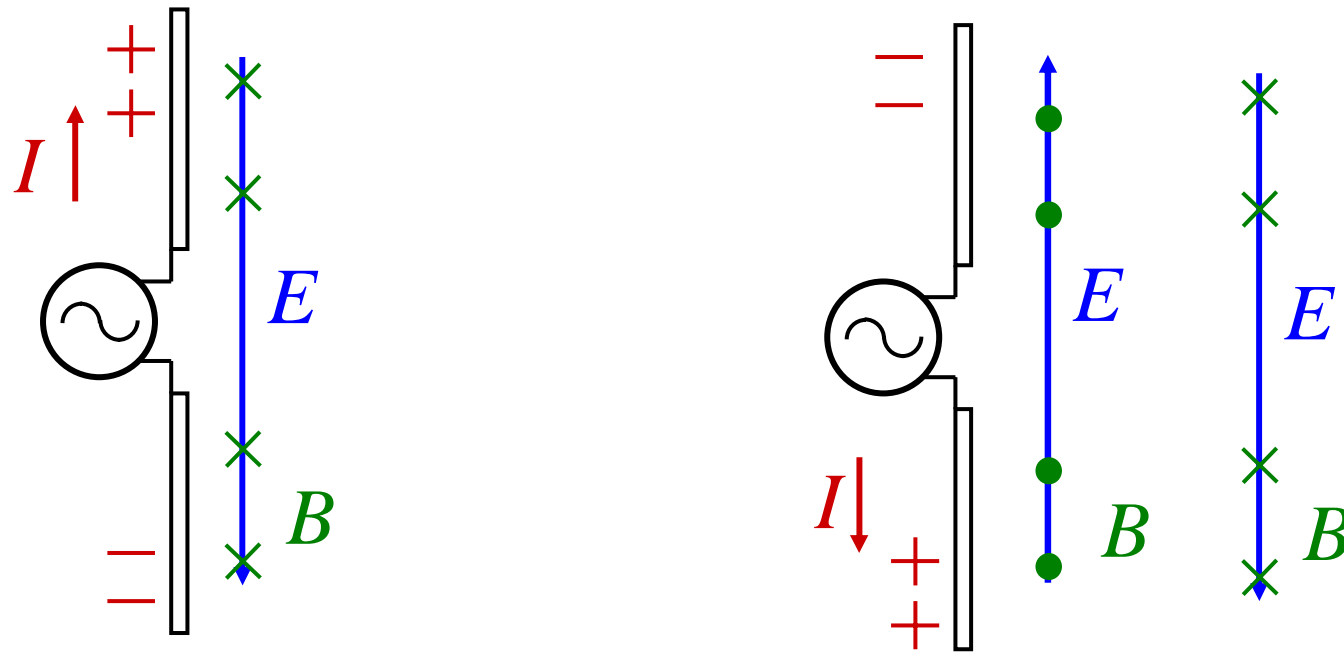
B) The magnetic force on a given moving charge.

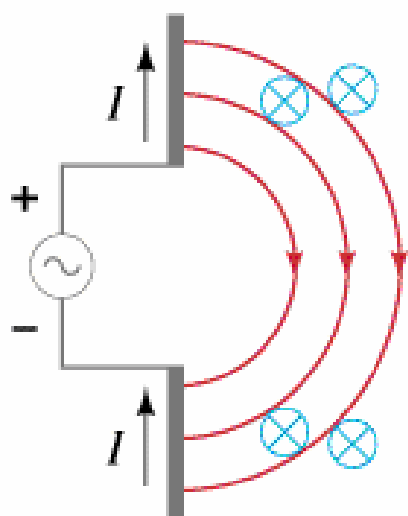
C) The flux of a given electric field.

D) The flux of a given magnetic field.

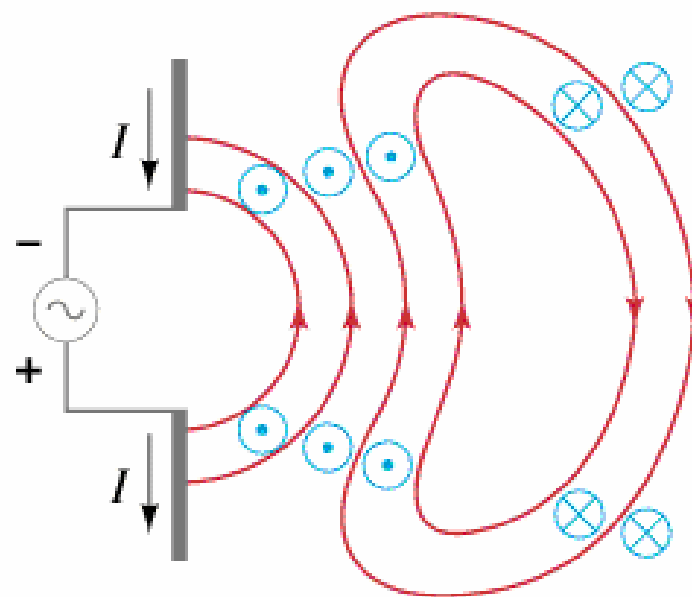
E) None of these.

Producing Electromagnetic Waves





(a)

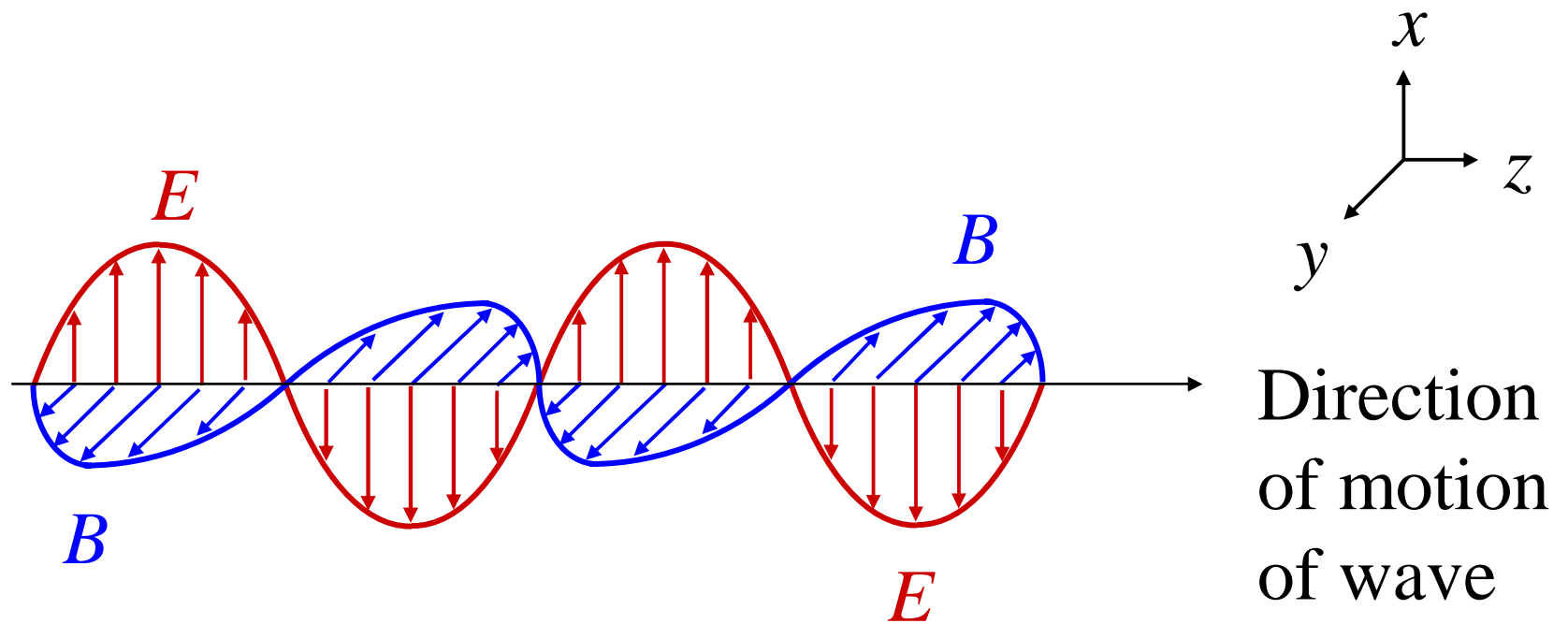


(b)

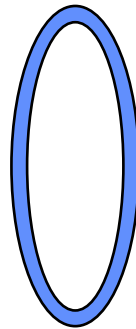
Interactive Question

Which of the following will not generate electromagnetic waves or pulses?

- A) a steady direct current
- B) an accelerated electron
- C) a proton in simple harmonic motion
- D) a bar magnet vibrating about its center
- E) charged particles traveling in a circular path
in a mass spectrometer



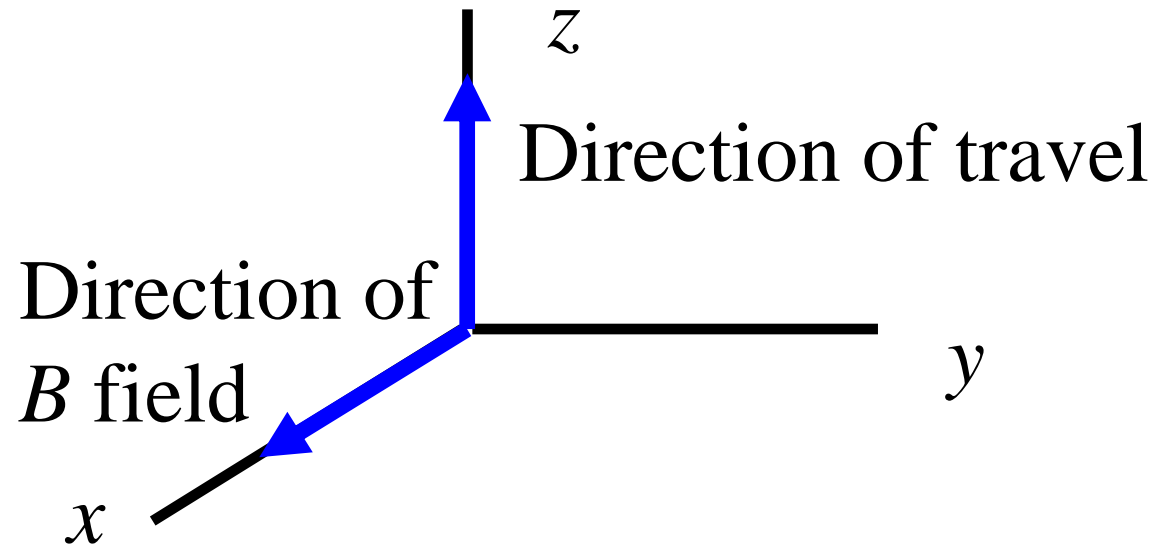
Antenna to
detect E field



Antenna to
detect B field

Interactive Question

An electromagnetic wave is traveling along the $+z$ axis of a right handed coordinate system. At a certain moment the magnetic field is in the $+x$ direction. What direction is the electric field pointing?



A) $+y$

B) $-y$

C) $+x$

D) $-x$

E) $-z$

The Electromagnetic Spectrum

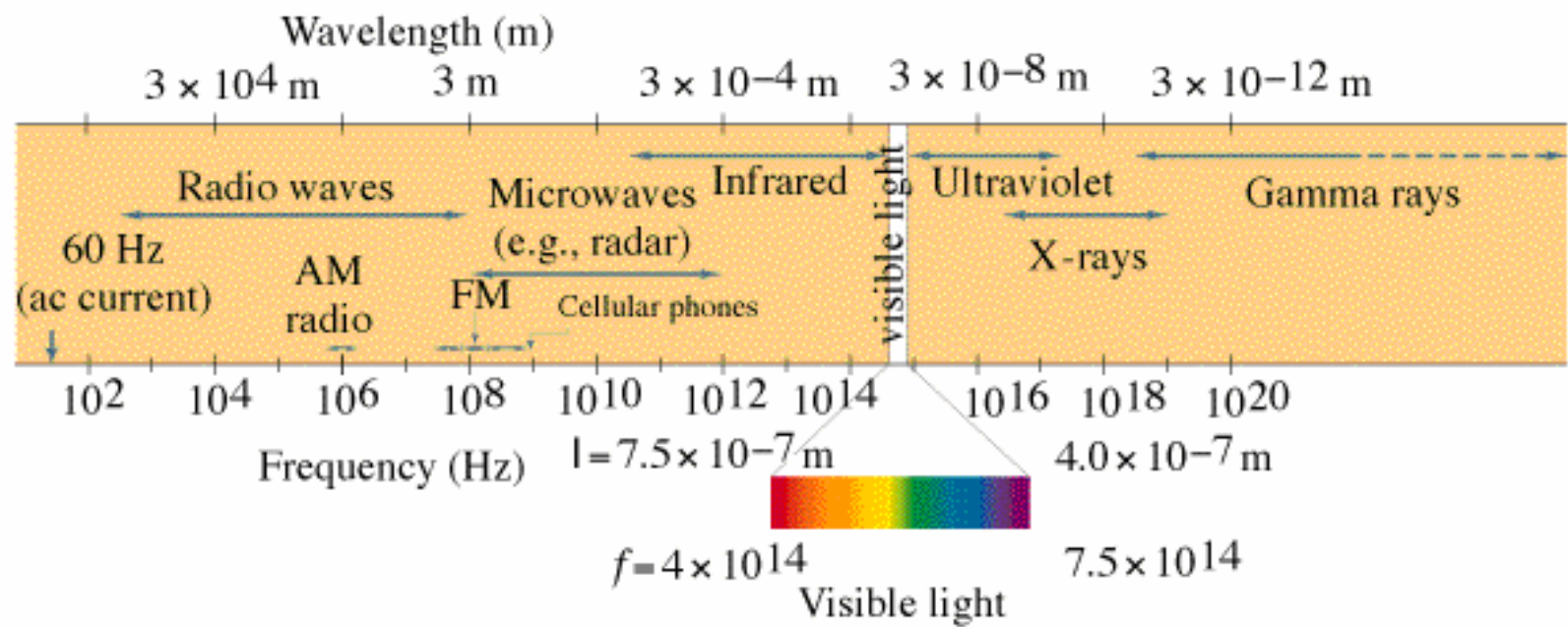
$$v = c = \lambda f$$

| <u>Name</u> | <u>f (Hz)</u> | <u>λ (m)</u> | <u>Time per λ (s)</u> |
|-----------------|----------------------------|---------------------------------|--|
| Extra Low Freq | 60 | 5×10^6 | 1.7×10^{-2} |
| Audio Frequency | 1×10^4 | 3×10^4 | 1×10^{-4} |
| Radio Frequency | 2×10^8 | 1.4 | 4.5×10^{-9} |
| Microwave | 1×10^{10} | 3×10^{-2} | 1×10^{-10} |
| Infrared (Heat) | 1×10^{13} | 3×10^{-5} | 1×10^{-13} |
| Visible | 6×10^{14} | 5×10^{-7} | 1.7×10^{-15} |
| Ultraviolet | 1×10^{16} | 3×10^{-8} | 1×10^{-16} |
| X-ray | 1×10^{18} | 3×10^{-10} | 1×10^{-18} |
| Gamma-ray | 1×10^{20} | 3×10^{-12} | 1×10^{-20} |

Interactive Question

The various colors of visible light differ in

- A) frequency only.
- B) wavelength only.
- C) their speeds in a vacuum.
- D) frequency and wavelength.
- E) frequency and their speed in a vacuum.



Maxwell Predicted...

$$c = 1/(\epsilon_0 \mu_0)^{1/2}$$

$$c = 1/[(8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)(4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A})]^{1/2}$$
$$= 3.00 \times 10^8 \text{ m/s}$$

We check that the units are okay:

$$\text{Since } F = ILB \Rightarrow \text{N} = \text{A} \cdot \text{m} \cdot \text{T}$$

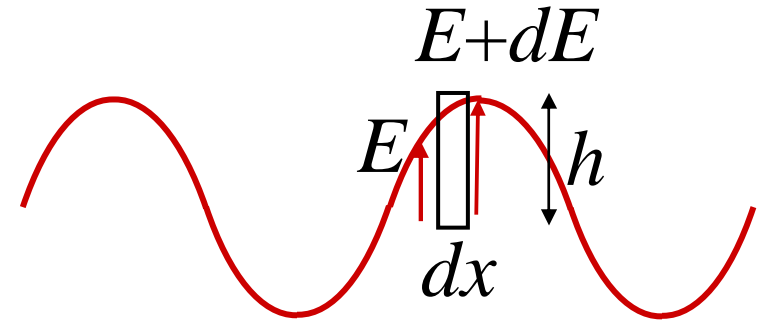
$$\text{T} = \text{N}/\text{A} \cdot \text{m} = \text{N} \cdot \text{s}/\text{C} \cdot \text{m}$$

And the units for c are:

$$\{(\text{N} \cdot \text{m}^2/\text{C}^2)(\text{C} \cdot \text{m}/\text{N} \cdot \text{s})(1/\text{m})(\text{C}/\text{s})\}^{1/2} = \{\text{m}^2/\text{s}^2\}^{1/2} = \text{m/s}$$

EM waves and Speed

Consider an oscillating electromagnetic wave moving to the right.



$$\oint \mathbf{E} \cdot d\mathbf{L} = -d\Phi_B/dt$$

$$h(E + dE) - hE = -h dx dB/dt$$

$$h dE = -h dx dB/dt$$

$$dE/dx = -dB/dt$$

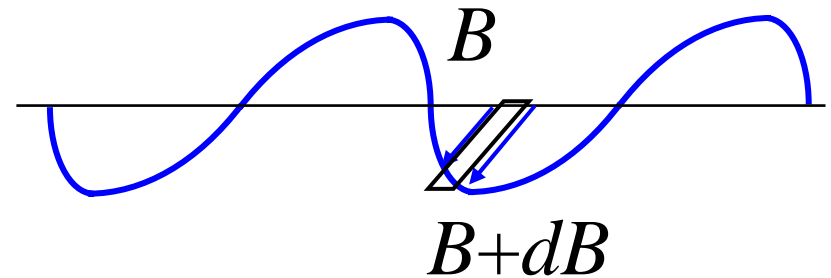
Similarly looping cc,

$$\oint \mathbf{B} \cdot d\mathbf{L} = \mu_0 \epsilon_0 d\Phi_E/dt$$

$$-h(B + dB) + hB = h \mu_0 \epsilon_0 dx dE/dt$$

$$-h dB = h \mu_0 \epsilon_0 dx dE/dt$$

$$-dB/dx = \mu_0 \epsilon_0 dE/dt$$



$$dE/dx = -dB/dt$$

$$\partial^2 E / \partial x^2 = -\partial^2 B / \partial x \partial t$$

$$-dB/dx = \mu_0 \epsilon_0 dE/dt$$

$$-\partial^2 B / \partial x \partial t = \mu_0 \epsilon_0 \partial^2 E / \partial t^2$$

Setting these equal:

$$\partial^2 E / \partial x^2 = \mu_0 \epsilon_0 \partial^2 E / \partial t^2$$

Taking derivative with respect to t in the left equation and with respect to x in the right equation, and setting them equal gives:

$$\partial^2 E / \partial x \partial t = -\partial^2 B / \partial t^2$$

$$-\partial^2 B / \partial x^2 = \mu_0 \epsilon_0 \partial^2 E / \partial x \partial t$$

$$\partial^2 B / \partial x^2 = \mu_0 \epsilon_0 \partial^2 B / \partial t^2$$

Let's look at the solution to one of these (identical) wave equations.

This equation describes a wave traveling in the $+x$ direction:

$$\partial^2 E / \partial x^2 = \mu_0 \epsilon_0 \partial^2 E / \partial t^2$$

The solution is:

$$E = E_0 \sin(kx - \omega t)$$

Let's show it satisfies the equation and solve for k and ω .

$$\partial^2 E / \partial x^2 = -E_0 k^2 \sin(kx - \omega t)$$

$$\partial^2 E / \partial t^2 = -E_0 \omega^2 \sin(kx - \omega t)$$

This satisfies the equation if $\mu_0 \epsilon_0 \omega^2 = k^2$

$$\mu_0 \epsilon_0 = k^2 / \omega^2$$

$$c^2 = \omega^2 / k^2$$

$$v = c = \omega / k$$

Similarly, of course: $B = B_0 \sin(kx - \omega t)$

So using $\partial E / \partial x = -\partial B / \partial t$

$$E_0 k \cos(kx - \omega t) = B_0 \omega \cos(kx - \omega t)$$

$$E_0 = B_0 \omega / k$$

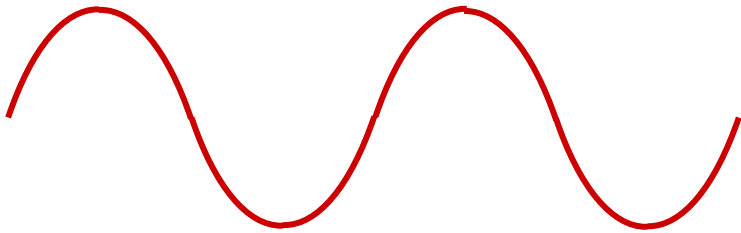
$$E_0 = c B_0$$

Since the two waves are in phase:

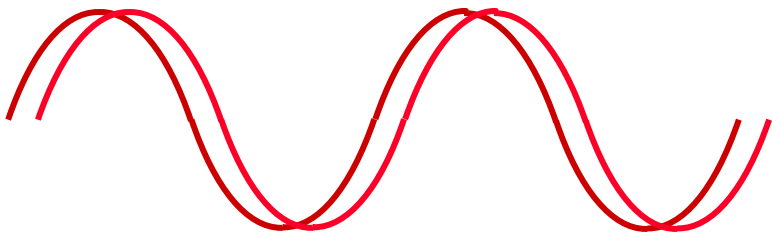
$$E = cB$$

Consider a wave traveling to the right.

$$B = B_0 \sin(kx - \omega t)$$



A short time later, dt , $B = B_0 \sin(kx - \omega(t+dt))$



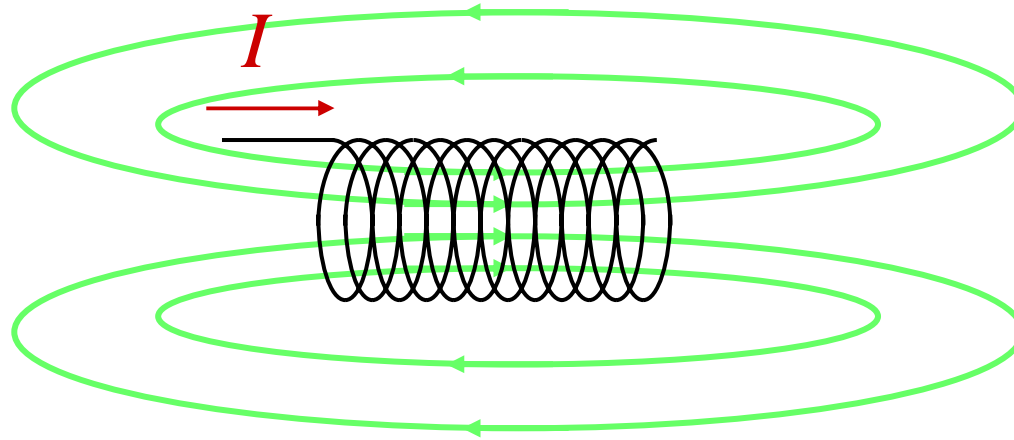
After one period, the wave has moved one wavelength to the right

Interactive Question

The electric field in a plane electromagnetic wave is along the y axis and its component is given by $E_m \sin(kx + \omega t)$ in SI units. The magnetic field is along the z axis and its component is given by:

- A) $(E_m/c) \cos(kx + \omega t)$
- B) $-(E_m/c) \cos(kx + \omega t)$
- C) $-(E_m/c) \sin(kx + \omega t)$
- D) $-E_m \cos(kx + \omega t)$
- E) $(E_m/c) \sin(kx + \omega t)$

Self Inductance



The flux in a circuit is related to the current in the circuit.

$$\Phi_B = LI$$

L is the self-inductance

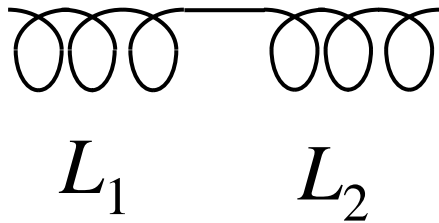
SI Units of inductance are $\text{Wb/A} = \text{V}\cdot\text{s/A} = \text{Henry (H)}$

$$d\Phi_B/dt = L dI/dt$$

$$\mathbf{E} = -L dI/dt$$

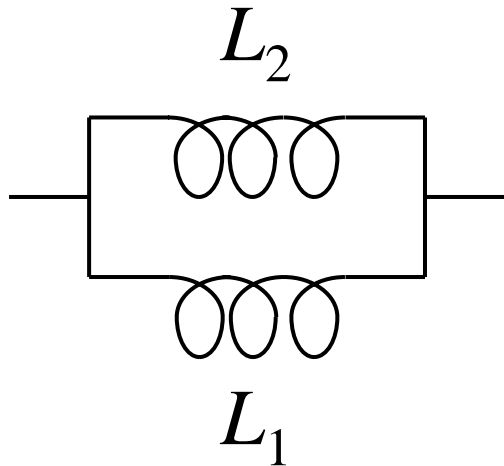
Problem: What is the self inductance of a solenoid?

Inductors in Series and Parallel



$$\begin{aligned}\mathbf{E} &= \mathbf{E}_1 + \mathbf{E}_2 = L_1 dI/dt + L_2 dI/dt \\ &= (L_1 + L_2) dI/dt\end{aligned}$$

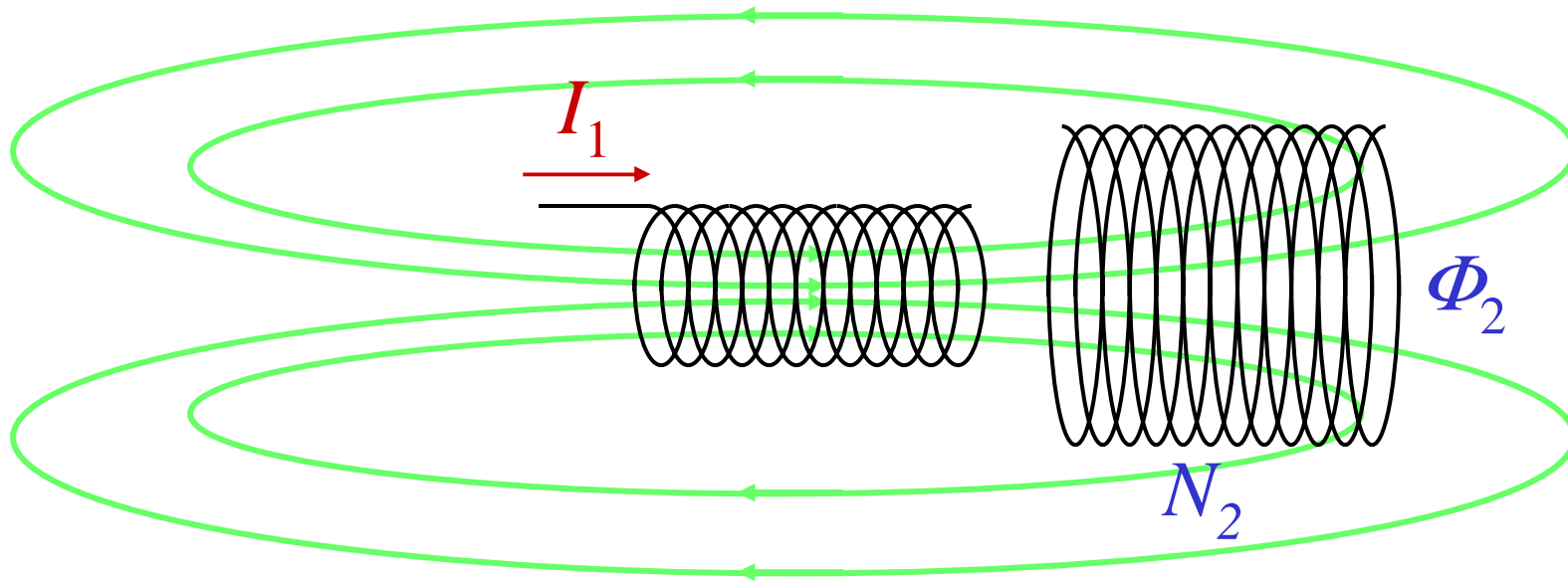
In series: $L_{\text{eq}} = \sum L_i$



$$\begin{aligned}dI/dt &= dI_1/dt + dI_2/dt = \mathbf{E}/L_1 + \mathbf{E}/L_2 \\ &= \mathbf{E}(1/L_1 + 1/L_2)\end{aligned}$$

In parallel: $1/L_{\text{eq}} = \sum 1/L_i$

Mutual Inductance



$$d\Phi_2 = M dI_1$$

$$M = d\Phi_2/dI_1$$

(1)

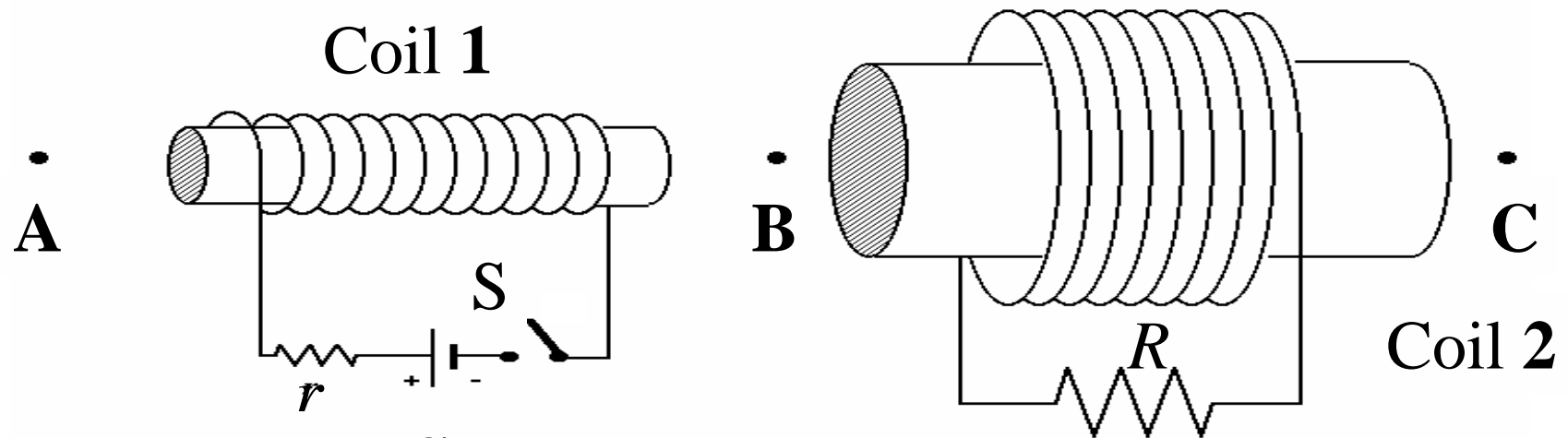
Faraday's law for 2nd coil: $\mathbf{E}_2 = -d\Phi_2/dt$

(2)

Combining (1) and (2): $\mathbf{E}_2 = -M dI_1/dt$

Interactive Question

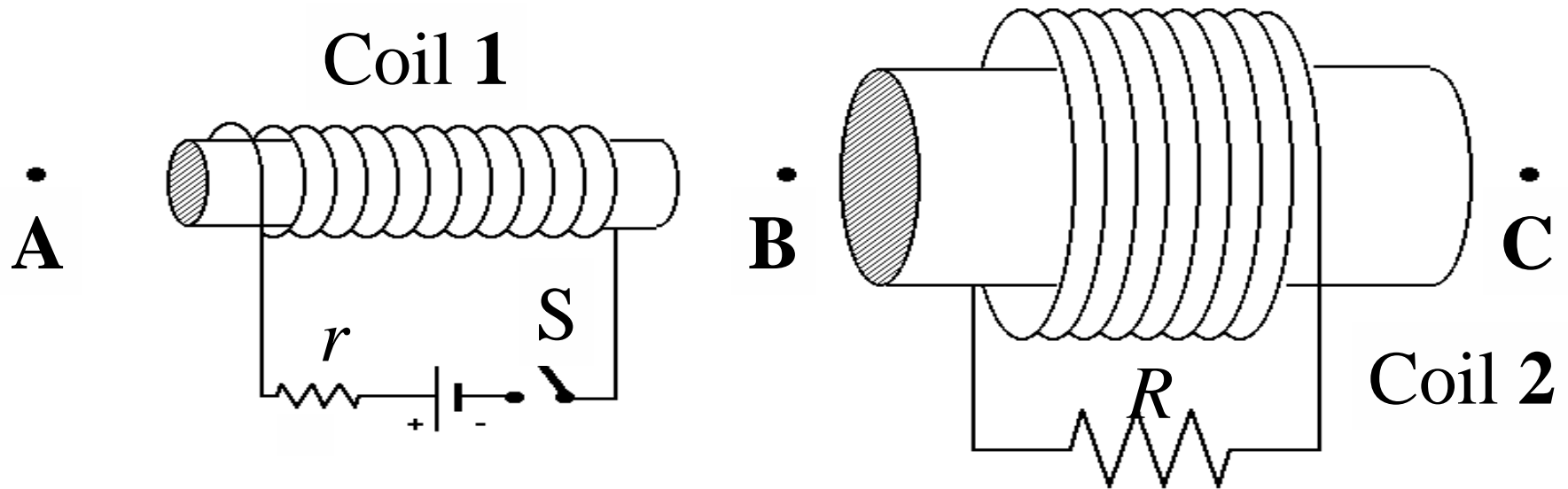
Two coils, **1** and **2**, with iron cores are positioned as shown. Coil **1** is connected to a battery and a switch.



When the switch **S** is initially closed

- A) a current will flow from left to right in r
- B) an induced current will flow from right to left in R
- C) a magnetic field that points toward **B** appears in **1**
- D) an induced magnetic field that points toward **B** appears in **2**
- E) a current flows through r , but not through R

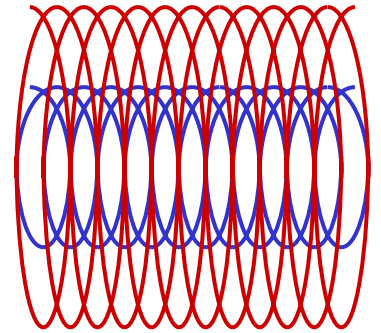
Interactive Question



After the switch **S** is closed *for a long time*

- A) a current will flow from left to right in r
- B) an induced current will flow from right to left in R
- C) a magnetic field that points toward **B** appears in **1**
- D) an induced magnetic field that points toward **B** appears in **2**
- E) a current flows through r , but not through R

Problem: Consider two solenoids of equal length L , placed coaxially, one inside the other. The inner one has a radius R_1 , current I_1 , and N_1 coils, and the outer one has radius R_2 , current I_2 , with N_2 coils. What is the mutual inductance of this setup?



Magnetic Power and Energy

For any inductor:

$$P = dU/dt = IV = I L dI/dt$$

$$dU = LI dI$$

$$U = (1/2) LI^2 \big|_0^I$$

$$U = (1/2) LI^2$$

For a solenoid:

$$U = (1/2) \mu_0 n^2 A l I^2$$

Since, $B = \mu_0 nI$,

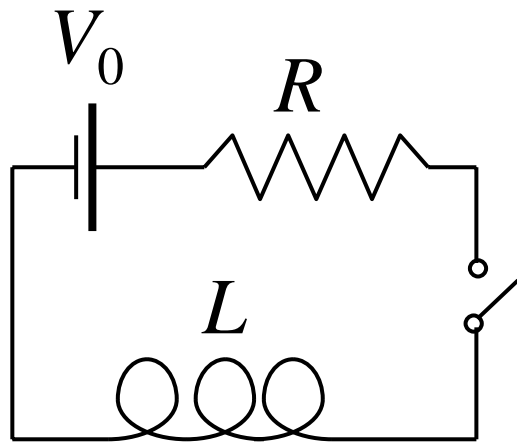
$$U = (1/2\mu_0) B^2 A l$$

Energy density:

$$u = U/V = U/Al$$

$$u = (1/2\mu_0) B^2$$

This is a general equation for the energy density of a magnetic field.



RL Circuits

$$V_0 - V_R - V_L = 0$$

$$V_0 - IR - L \, dI/dt = 0$$

We saw this differential equation for a capacitor.

Let's try the solution, $I = I_0[1 - e^{-t/\tau}]$

With $I_0 = V_0/R$, and $\tau = L/R$

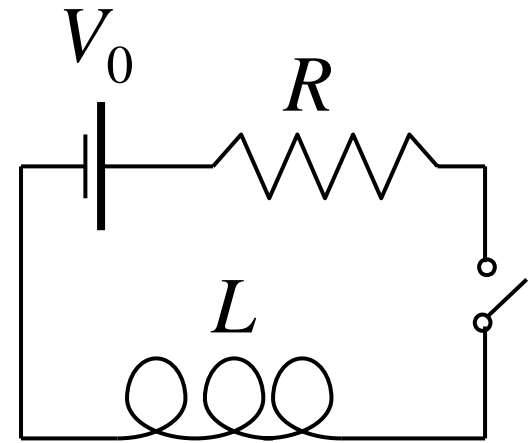
$$dI/dt = (I_0/\tau)e^{-t/\tau}$$

$$V_0 - I_0[1 - e^{-t/\tau}]R - L(I_0/\tau)e^{-t/\tau} =$$

$$V_0 - V_0 + V_0e^{-t/\tau} - L(I_0R/L)e^{-t/\tau} =$$

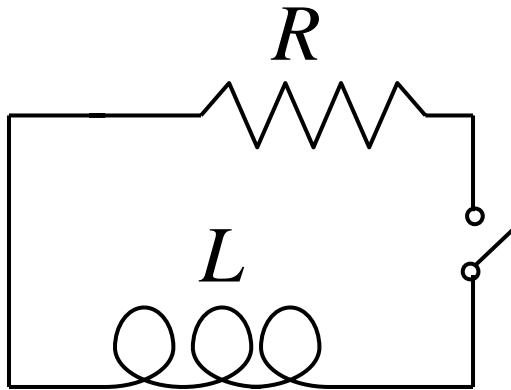
$$V_0 - V_0 + V_0e^{-t/\tau} - V_0R e^{-t/\tau} = 0$$

Problem: An inductor and a resistor are attached to a battery. After a time t , how much power has been supplied to this circuit by the battery?



RL Circuits

Now consider the situation where there is already a magnetic field in the inductor and current flowing, but no battery. Energy is stored in the magnetic field.



$$-IR - L \, dI/dt = 0$$

$$dI/I = - (R/L) \, dt$$

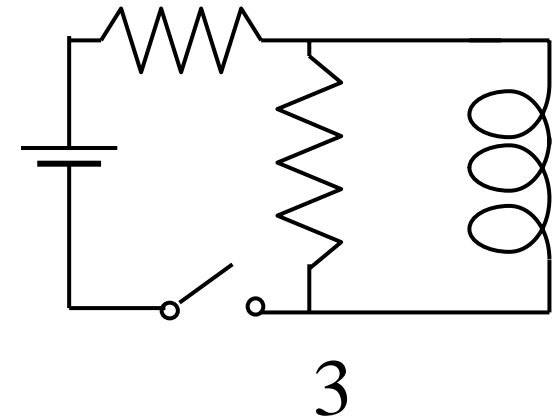
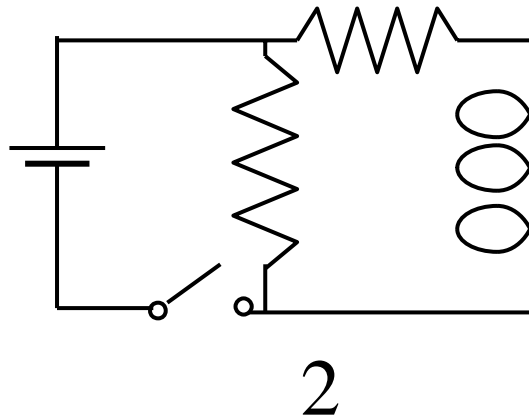
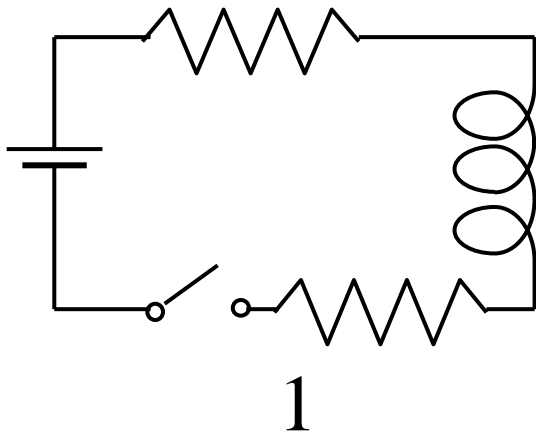
$$\ln I/I_0 = - (R/L) \, t$$

$$I/I_0 = e^{-(R/L)t} = e^{-t/\tau}$$

$$I = I_0 e^{-t/\tau}$$

Interactive Question

The diagram shows three circuits with identical batteries, inductors and resistors. Rank these according to the current through the battery just after the switch is closed, least to greatest.

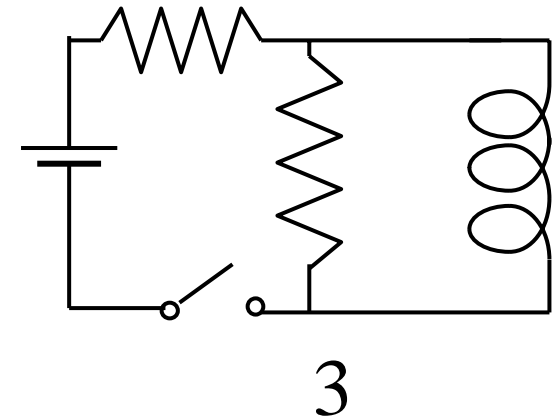
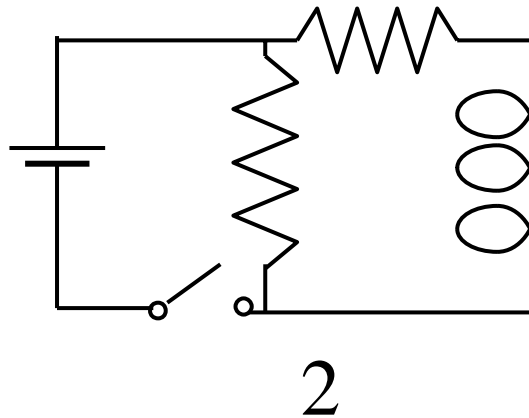
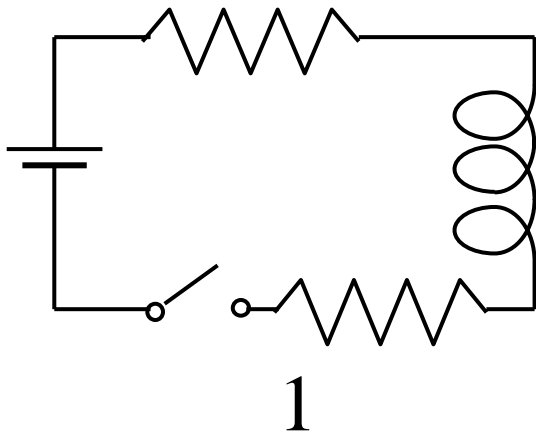


- A) 3, 2, 1
- C) 1, 3, 2
- E) 3, 1, 2

- B) 2 and 3 tie, then 1
- D) 1, 2, 3

Interactive Question

The diagram shows three circuits with identical batteries, inductors and resistors. Rank these according to the current through the battery after the switch has been closed a long time, least to greatest.



A) 3, 2, 1

B) 2 and 3 tie, then 1

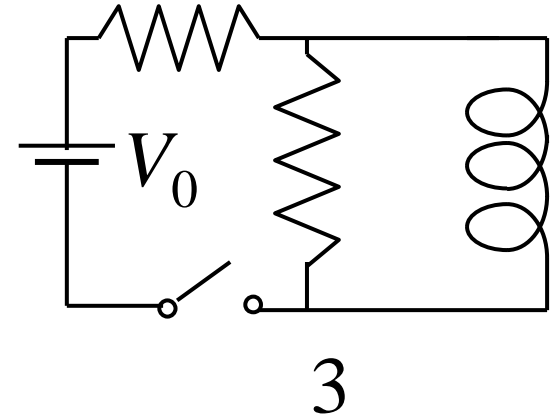
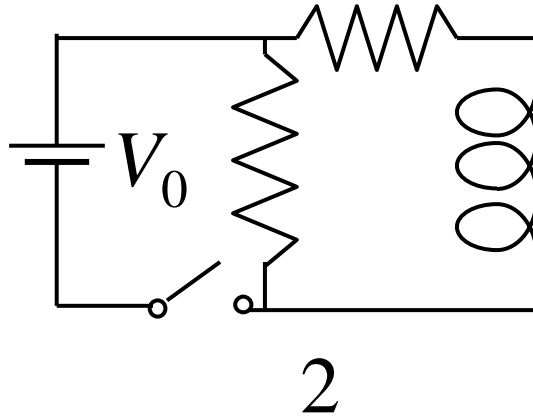
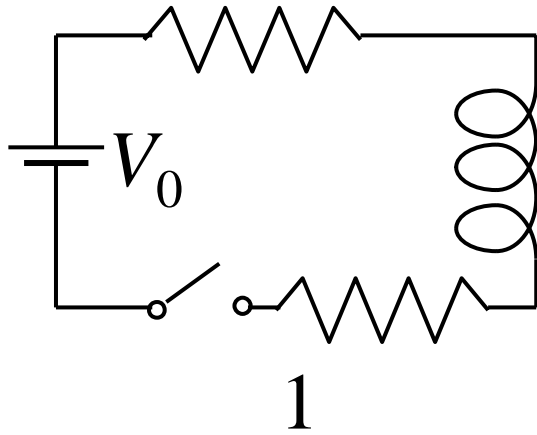
C) 1, 3, 2

D) 1, 2, 3

E) 3, 1, 2

Interactive Question

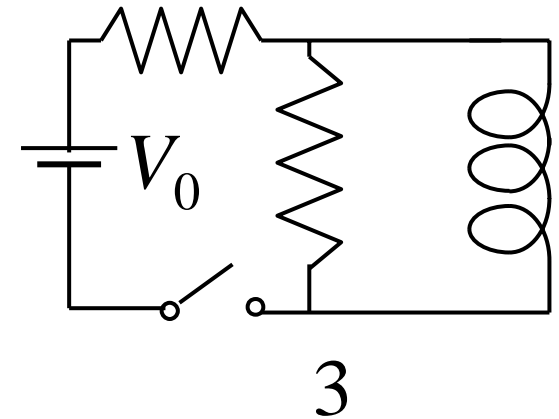
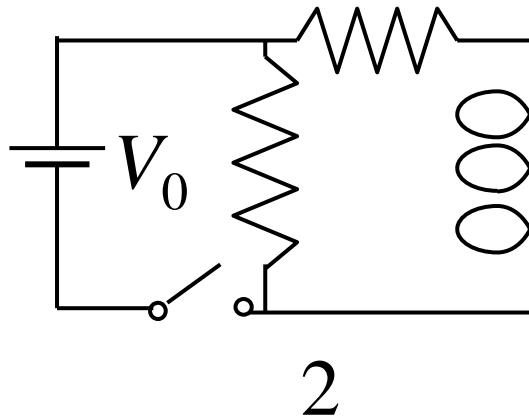
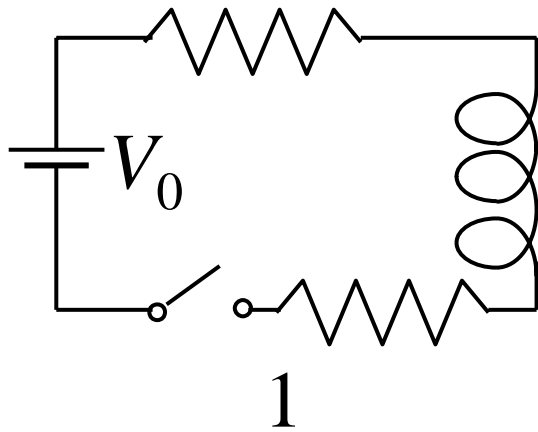
The diagram shows three circuits with identical batteries, inductors and resistors. Which of the following circuits has the greatest potential difference across the inductor just after the switch is closed?



- A) 1
- B) 2
- C) 3
- D) 1 and 2 tie
- E) All are the same

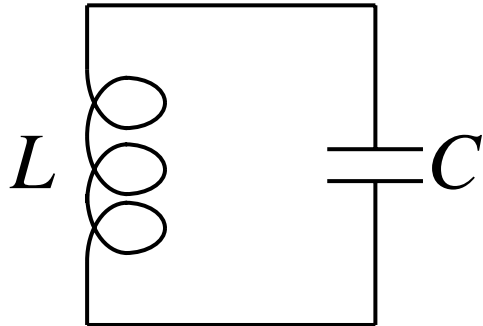
Interactive Question

The diagram shows three circuits with identical batteries, inductors and resistors. Which of the following circuits has the greatest potential difference across the inductor after the switch has been closed a long time?



- A) 1
- B) 2
- C) 3
- D) 1 and 2 tie
- E) All are the same

LC Circuits with No Generator



Use the Loop Rule:

$$-L \, dI/dt - Q/C = 0$$

$$L \, d(dQ/dt)/dt + Q/C = 0$$

$$d^2Q/dt^2 + Q/LC = 0$$

The solution is:

$$Q = Q_0 \cos(\omega t - \delta)$$

$$\text{with } \omega = 1/(LC)^{1/2}$$

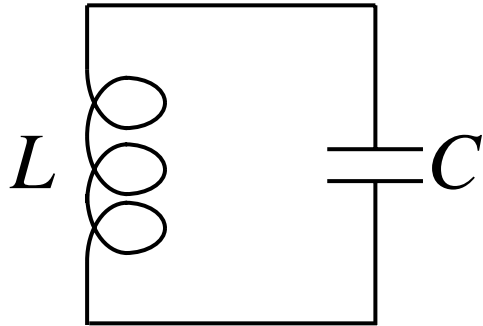
Check the answer:

$$dQ/dt = I = -Q_0 \omega \sin(\omega t - \delta) = -I_0 \sin(\omega t - \delta)$$

$$d^2Q/dt^2 = -Q_0 \omega^2 \cos(\omega t - \delta)$$

$$d^2Q/dt^2 + Q/LC = -Q_0 \omega^2 \cos(\omega t - \delta) + Q_0 \omega^2 \cos(\omega t - \delta) = 0$$

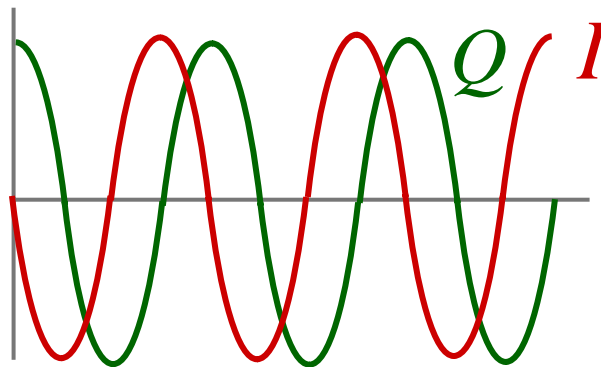
LC Circuits with No Generator



$$Q = Q_0 \cos(\omega t - \delta)$$

$$I = -I_0 \sin(\omega t - \delta)$$

The current in the circuit is 90° out of phase (1/4 period) with the charge on the capacitor.



Interactive Question

A charged capacitor and an inductor are connected in series. At time $t=0$, the current is zero. If T is the period of the resulting oscillations, then the next time, after $t=0$, that the charge on the capacitor is a maximum is:

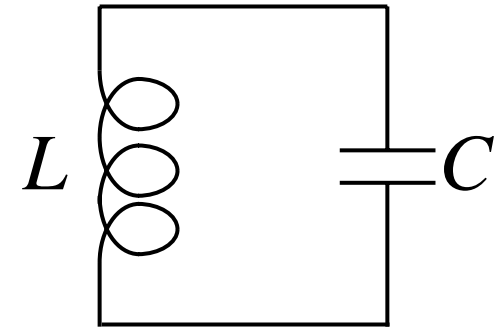
- A) $T/4$
- B) $T/2$
- C) T
- D) $2T$
- E) $4T$

Interactive Question

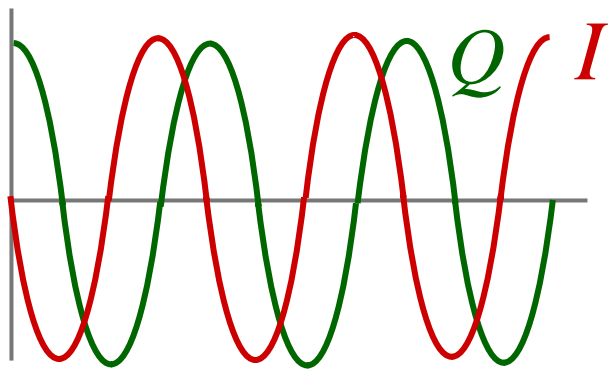
A charged capacitor and an inductor are connected in series. At time $t=0$, the current is zero. If T is the period of the resulting oscillations, then the next time, after $t=0$, that the voltage across the inductor is a maximum is:

- A) $T/4$
- B) $T/2$
- C) T
- D) $3T/2$
- E) $2T$

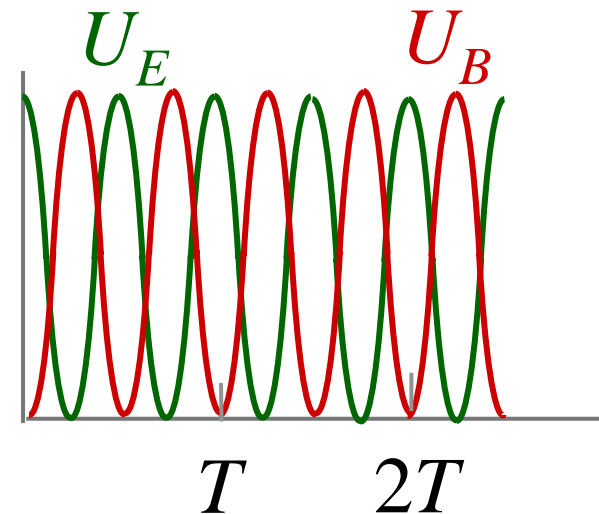
Energy in LC Circuit



$$\begin{aligned}
 U &= U_E + U_B \\
 &= Q^2/(2C) + LI^2/2 \\
 &= \{Q_0^2/(2C)\} \cos^2(\omega t - \delta) + \{LI_0^2/2\} \sin^2(\omega t - \delta) \\
 &= \{Q_0^2/(2C)\} \cos^2(\omega t - \delta) + \{LQ_0^2\omega^2/2\} \sin^2(\omega t - \delta) \\
 &= \{Q_0^2/(2C)\} \cos^2(\omega t - \delta) + \{LQ_0^2/(2LC)\} \sin^2(\omega t - \delta) \\
 &= \{Q_0^2/(2C)\} \{\cos^2(\omega t - \delta) + \sin^2(\omega t - \delta)\} \\
 U &= Q_0^2/(2C)
 \end{aligned}$$



$$\begin{aligned}
 U_E &= Q^2/(2C) \\
 U_B &= LI^2/2
 \end{aligned}$$



Interactive Question

A charged capacitor and an inductor are connected in series. At time $t=0$, the current is zero. If T is the period of the resulting oscillations, then the next time, after $t=0$, that the energy stored in the magnetic field is a maximum is:

- A) $T/4$
- B) $T/2$
- C) T
- D) $2T$
- E) $4T$

- Problem: A 300 V dc power supply is used to charge a 25 μF capacitor. After the capacitor is fully charged, it is disconnected from the power supply and connected across a 10 mH inductor. The resistance is negligible.
- (a) Find the capacitor charge 1.2 ms after the connection is made.
 - (b) Find the magnetic and electric energy at the same time.

Interactive Question

In an oscillating LC circuit, the total stored energy is U . When $Q = Q_0/2$, what is the energy stored in the magnetic field, U_B ?

- A) $U/4$
- B) $U/2$
- C) U
- D) $U/\sqrt{2}$
- E) $3U/4$