Centripetal Force

1. Introduction

When an object travels in a circle, even at constant speed, it is undergoing acceleration. In this case the acceleration acts not to increase or decrease the *magnitude* of the velocity vector, but rather to change its *direction*. Newton's second Law tells us that in the absence of any outside Force, an object will travel at constant speed in a *straight* line. Therefore if we observe the direction of motion changing, we know there is a Force acting. The direction of that Force is the direction that the velocity vectors turns toward. In the case of circular motion, we can see that the acceleration in inward since the direction of motion is forever turning inward toward the center.

We can show that in order for an object to continue moving in a circle that is at a constant radius, at constant speed, the Force must exactly match the mass, speed and radius of the circle according to the equation

$$F = m\frac{v^2}{R}$$

As shown in Figure 1, if the Force is too small or the object traveling too fast, then it will move outward from the circular path. If the object travels too slowly or the Force is too large, then it will fall inward toward the center. For a given speed and radius, there is only one magnitude of Force which is exactly right for keeping the object moving in a circle. From the geometry of the situation, we could prove that this magnitude is mv^2/R.

There are many cases of circular motion: orbits of planets and satellites are nearly circular, we can talk about a person on a merry-go-round or a child playing on a rotating platform, we might calculate the motion of electrons or protons in a particle accelerator. The Forces acting in these cases are gravitational forces, friction or electromagnetic forces. Whatever their origin, because they act to keep objects moving in a circle, we shall call them the *Centripetal Force*, F_c and call the acceleration they produce *Centripetal Acceleration*, a_c.
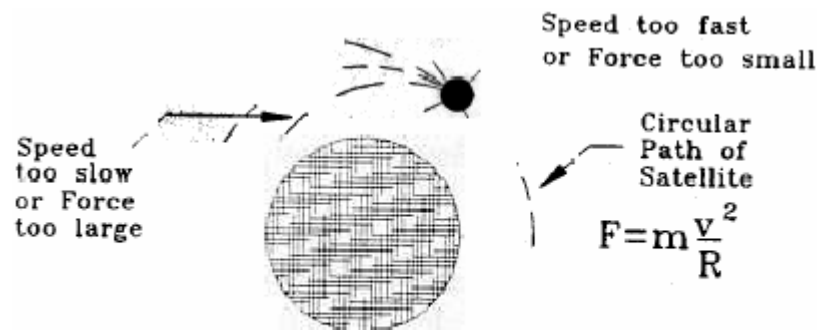

Figure 1: A satellite orbiting the Earth must travel at exactly the right speed and height so that the gravitational Force provides the desired Centripetal acceleration.

The word Centripetal comes from Latin and means "center seeking" because the Forces and accelerations must point exactly to the center of the circle in order for there to be circular motion. *Because we see that these Forces and accelerations produce circular motion, we know their magnitudes must be:*

$$F_c = m\frac{v^2}{R}$$

$$a_c = \frac{v^2}{R}$$

We can also relate the centripetal acceleration and Force to the period of rotation, *T*, since

$$v = \frac{2\pi R}{T}$$

it follows that

$$F_c = ma_c = m\frac{v^2}{R} = m\frac{(2\pi R)^2}{RT^2} = m\frac{4\pi^2 R}{T^2}$$

OUR EXPERIMENT

In this experiment we will measure the force required to keep a mass moving at constant angular velocity in a circle of constant radius. This force will be applied by a spring suspended between the mass and a post at the center of the circle about which the mass rotates. The general outline of the apparatus is shown in Figures 2 and 3. A Bob is suspended from a crossarm such that with no other masses or springs attached to it, it hangs straight down over a Pointer. When the system is at rest and we attach a spring to the Bob, a hanging mass is needed to keep the spring stretched and the Bob directly over the pointer. If we remove this mass, then the spring will pull the Bob inward towards the axis. However if we begin to spin the Bob about the axis, its inertia will try to make it go in a straight line and the Spring will have to pull inward to keep it moving in a circle. Thus *the spring provides the Centripetal Force*. By **Newton's Third Law**, the Bob applies an equal and opposite Force to the spring, and so the Spring will begin to stretch outward. As the rotational speed increases, the spring must apply more and more Force according to the equation F = mv^2/R. The spring therefore stretches until at exactly the right rotational velocity the Bob once again hangs over the Pointer (Figure 4.6).

For this experiment we must rotate the apparatus at constant speed while trying to measure the period of rotation. This is difficult to achieve and often the experimental results have significant uncertainties for this experiment. Try as best you can to measure the frequency of rotation for a constant speed by keeping the bob rotating at a constant radius.
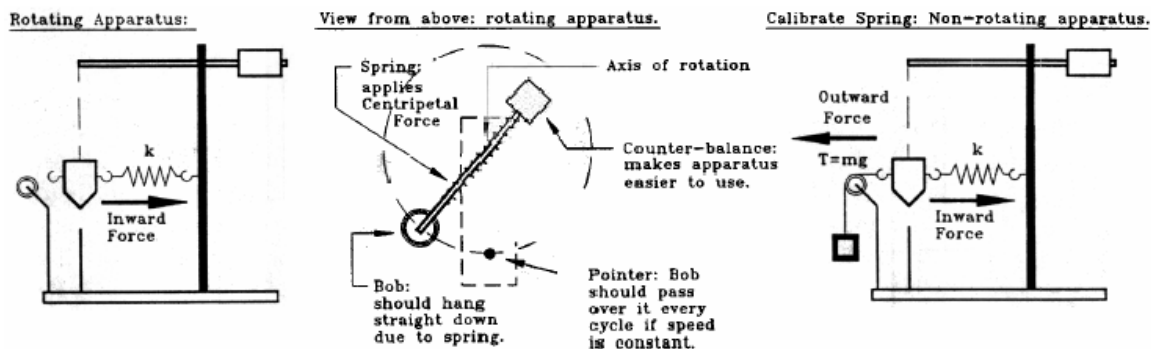
Figure 2: The Centripetal Force Apparatus.

2 Procedure

OPERATION OF THE CENTRIPETAL FORCE APPARATUS

The rotor or axis of the apparatus is rolled by hand between your thumb and forefingers. When the rotational speed is just right, the bob hanging from the cross-arm will hang straight down over the pointer, the Centripetal Force required for uniform circular motion is then supplied by the spring. The purpose of the counterweight is to balance the bob so that the apparatus spins smoothly. When properly balanced, it applies no force.

INITIAL SETUP:
• Remove the bob, weigh it, and record its mass. If additional masses are available to add to the bob, now is the time to weigh them as well. Weigh the mass holder hook and any masses you will use with it as well. Remember to record an estimated uncertainty for these values according to how well you think you can read the scales.

Bob Mass:_____

Mass Holder Hook :_____

CALIBRATION OF THE SPRING

If the spring is an "'ideal" spring, the force it can apply is equal to $F = -k \times \Delta R$, where k is a constant known as the "Spring Constant" or "Elastic Constant" and ΔR is the distance the spring stretches or compresses from its relaxed, equilibrium length. In other words, as you pull on or compress the spring, the force with which it "pushes back" increases linearly. This is known as "Hook's Law", but it is not an actual Law of Physics, but merely a model for the spring which holds fairly well as long as the spring is not compressed or stretched too much or too little.
• Set up the apparatus as shown in Figure 1. Remove the mass holder hook. Move the cross-arm so that the black bob is hanging straight down. Line up the vertical pointer directly under the tip of the bob as a reference for your measurement. Measure the distance from the pointer to the center of the spindle as accurately as you can. This is the equilibrium distance (the unstretched length) of the spring. Record this length as "R_0". This distance R_0 corresponds to a force, F_{spring}, of zero being applied to the spring.
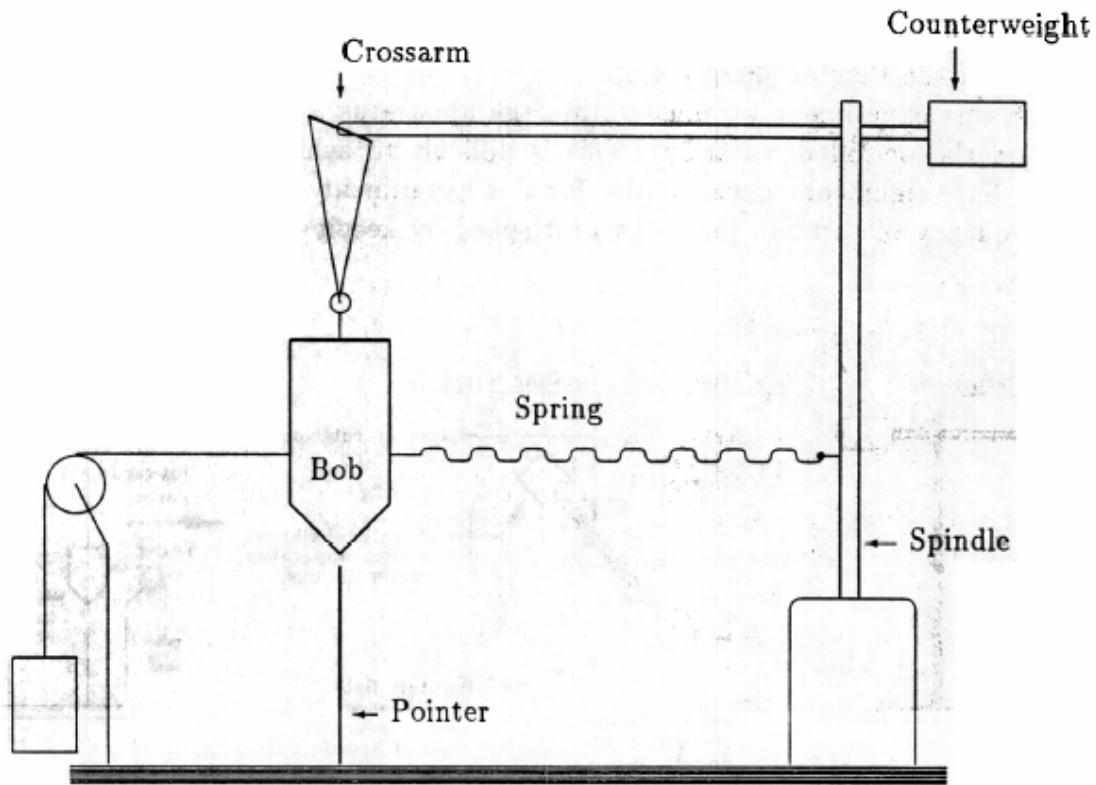
$R_{0:}$_____

Figure 3: Centripetal Force Apparatus

• Now replace the mass holder hook, making sure the string goes back over the pulley smoothly. Hang a slotted mass on the hook. Hang enough mass to stretch the spring about 20%. Record the weight of the hanging mass. This weight (plus the holder!) is equal to the Tension in the string that goes over the pulley, and so this is the Force being applied to the Spring. For the non-rotating apparatus, the bob hangs straight down when the hanging mass' weight is equal to the tension in the spring, so by measuring the weight required to keep the mass vertical at various lengths for the stretched spring, you will be able to calibrate the spring.

Mass	Spring Length

Graph the displacement vs. force so that you can calculate the spring constant from the slope.
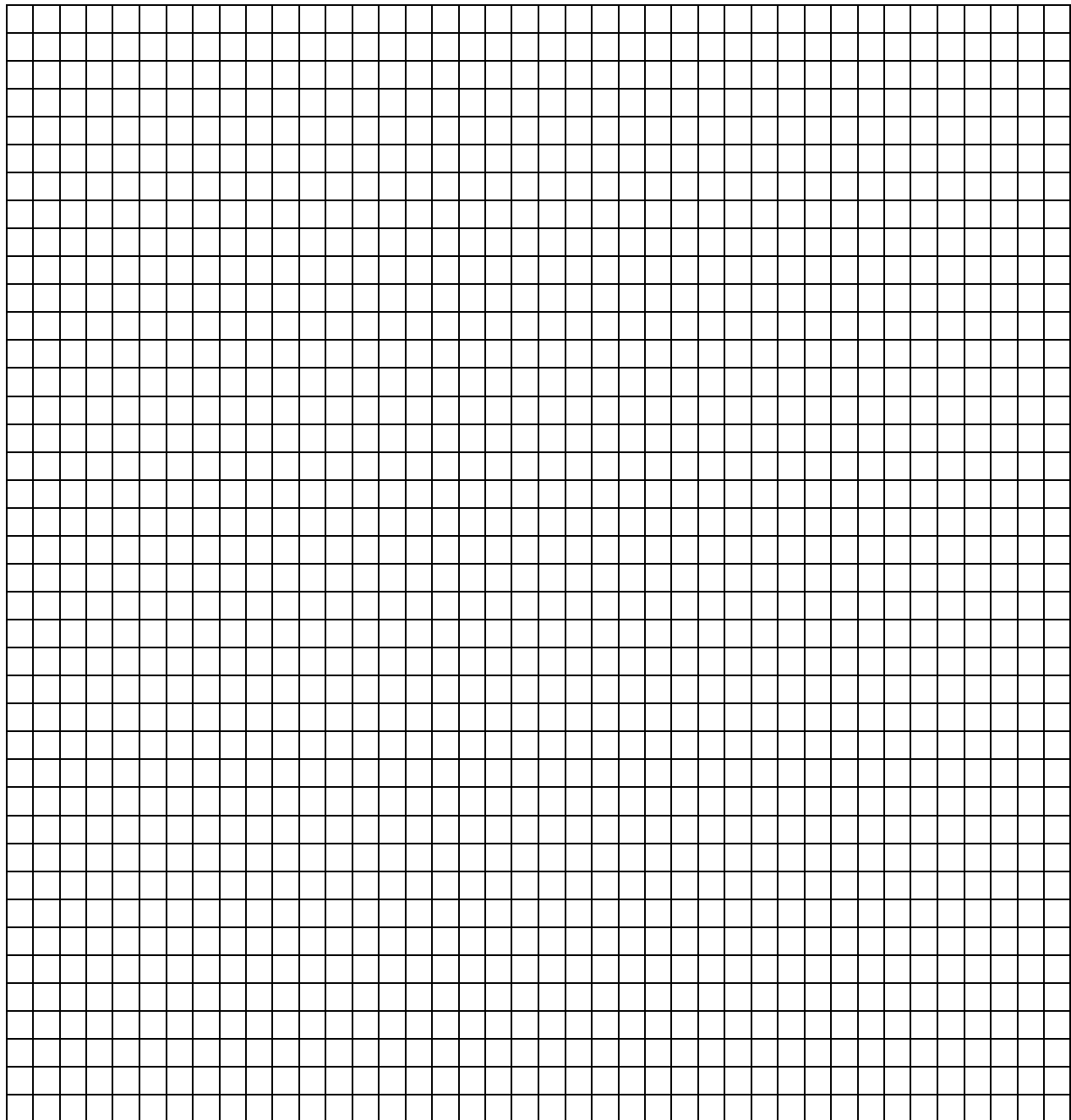
Spring Constant :_____

• Adjust the cross-arm to move the mass outward so that it is again hanging straight down. Lineup the Pointer under the Bob and use it as a reference to measure the distance from the axis to the Bob. Record this new distance, R_1. The Spring stretches until the force with which it pulls on the weight $F_1 = -k (R_1 - R_0) = -k \Delta R$ balances the weight, $M g$. Therefore record R_1, the weights and the value of $F_1 = M g$.

R_1:_____
Mass:_____
F_1 :_____

• Repeat this procedure for several different distances, R_1. Move the pointer to a distance at least 20% greater than the relaxed length of the spring and then find the weight required to balance force of the spring. Mark the various positions of the Pointer on the base platform to refer to these distances again later. BE CAREFUL NOT TO DAMAGE THE SPRING BY STRETCHING IT TOO FAR!!

R_1:_____
Mass:_____
F_1 :_____

R_1:_____
Mass:_____
F_1 :_____

R_1:_____
Mass:_____
F_1 :_____

MEASURING CENTRIPETAL FORCE

1. Hang the Bob straight down. Now detach the weights, hook, and Spring so that the Bob hangs freely down. Move the vertical Pointer to some distance, R_1, for which you have previously measured the force that balances the spring, F_1, Readjust the cross-arm so that the mass again hangs straight down at this distance *when the spring is not attached*. Lock the cross-arm into position. Adjust the position of the counterweight, if necessary, so that when you rotate the axis its motion seems smooth and balanced.

2. Rotate the apparatus and let the spring provide the inward centripetal force. Reconnect the spring. *The stretched spring will pull the bob inward*. Practice rotating the axis until you can achieve a smooth motion that will cause the bob to pass directly over the pointer as the apparatus rotates. By doing this you have determined a speed such that the inward pull of the stretched spring provides exactly enough Centripetal Force to keep the bob moving in a circle at constant radius $R = R_1$ and at constant speed.

3. Note that the *only Forces on the bob* are the upward force of the string, which balances the downward force of the weight of the bob, and the inward force of the stretched spring. The forces are therefore "unbalanced". *There is an inward force on the Bob applied by the spring.* The bob seems to "want" to go outward simply because its own inertia is trying to keep it moving in a straight line, and *an inward force is required to force it to move in a circle*. The necessary force is called the Centripetal Force and the spring is what is providing the Centripetal Force in this case.

4. Spin the bob at a roughly constant speed so that the point of the bob passes over the Pointer at the distance you have chosen. Use your stopwatch to measure the average period of revolution, *T*. To do this accurately, measure the time it takes to complete 10 turns (if possible), then divide the total time by the number of turns. Repeat this process four more times until you get nearly repeatable results. Have one lab partner checking to make sure that the point of the bob is hanging over the vertical marker through all turns counted during each timing. If this can be achieved, it will mean that the bob was spinning with constant velocity.

Trial 1 Time :_____

Trial 2 Time:_____

Trial 3 Time:_____

Trial 4 Time:_____

Trial 5 Time:_____

Average Time:_____

5. Using the stretched distance, R_1, for this trial and results from your previous measurements determine the corresponding value of force, $F_1 = M_{weight}g$.

   F_1:_____

6. You now know the radius of the circle in which the bob is traveling and, from the Period, *T* you have measured, the speed with which it rotates. For this first trial, calculate the Centripetal Force that is needed to keep your bob moving in a circle at the radius and speed you have determined. This should be the Force applied by the Spring.

   Centripetal Force :_____

7. Using propagation of errors, calculate the uncertainty in the centripetal force based on the uncertainties in the radius, period and mass. What uncertainty (radius, period, mass) contributes the most to the uncertainty on the centripetal force?

8. Do the two values of force agree within your uncertainty? Discuss with your lab partners any sources of systematic or random uncertainty.

9. Now qualitatively observe what happens when you rotate the bob at a speed much greater than the speed you measured previously. In your lab notebook discuss what you observe. What is the net force on the bob? (If you say there is a net outward force, discuss this with your lab partners and your TA!) Why does the bob move the way it does?

10. Make two additional measurements of the rotational period for different values of R_1. For each trial remember to first adjust pointer to the distance, and then the crossarm with the spring detached so that the bob hangs straight down at the chosen distance of revolution. If the apparatus starts rocking, reset the position of the counterweight and begin again.

11. Before you make your measurements, predict the new value you will measure for rotational period

R_1 :_____
Period Prediction:_____

R_1 :_____
Period Prediction:_____

R_1:_____

Trial 1 Time :_____

Trial 2 Time:_____

Trial 3 Time:_____

Trial 4 Time:_____

Trial 5 Time:_____

Average Time:_____
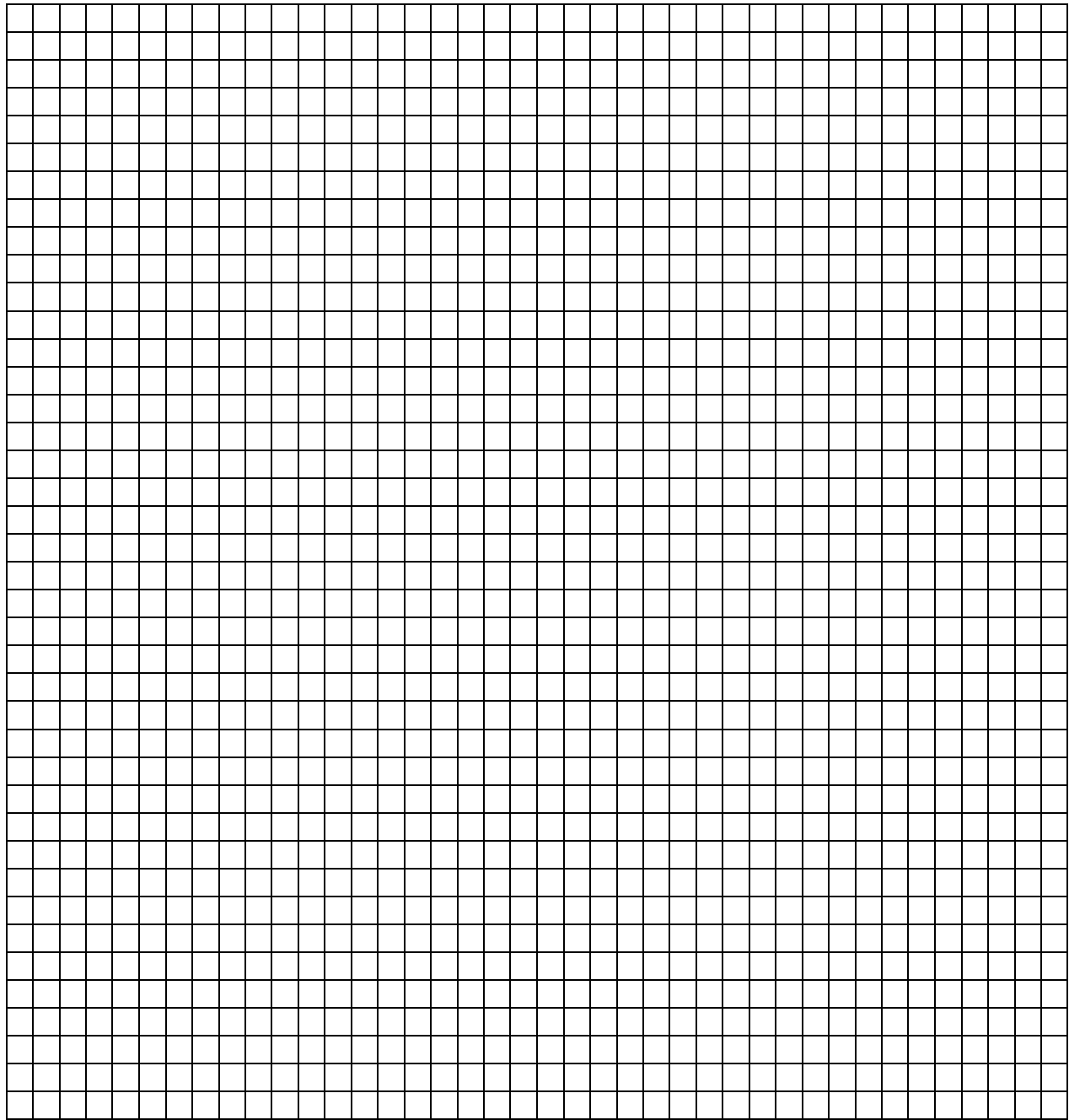

R_1:_____

Trial 1 Time :_____

Trial 2 Time:_____

Trial 3 Time:_____

Trial 4 Time:_____

Trial 5 Time:_____

Average Time:_____

2. Plot the centripetal force (proportional to the stretch of the spring) to $4\pi^2 R/T^2$, where R is the radius of the circular orbit, and T is its period.
The slope the above graph should be the mass m of the "bob". You should compare that to the mass you determine by weighing it.

Mass of Bob from Slope of line: _____

Mass of Bob based on direct weighing :_____

3 Questions:

- How accurate was your ability to measure the mass? What was the largest source of error? Be specific. How could the error (systematic and random) be minimized?

- Do your two methods of determining the mass measure the same *type* of mass: that is gravitational or inertial? Discuss.

- As the radius of the circular orbit in your experiment increased, how did each of the following change: velocity, period, centripetal force?

- Assume you wish to design a centrifuge that will spin samples at a high rate of speed in order to separate them by density. However, you want to make sure your centrifuge does not spin so fast that it breaks up! Can you design (based on this experiment) a switch or governor that will shut off the electric motor when the centrifuge spins too rapidly?