Visualizing 2D Materials Using Multi-Layered Structures

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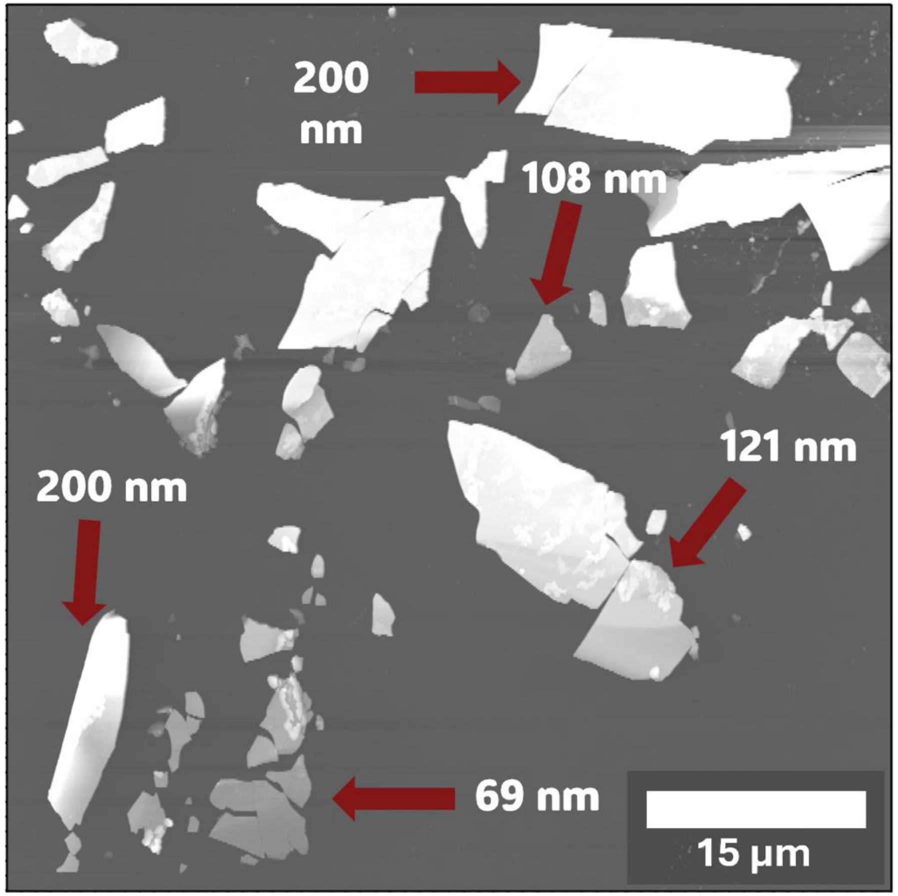
Motivation Behind Project



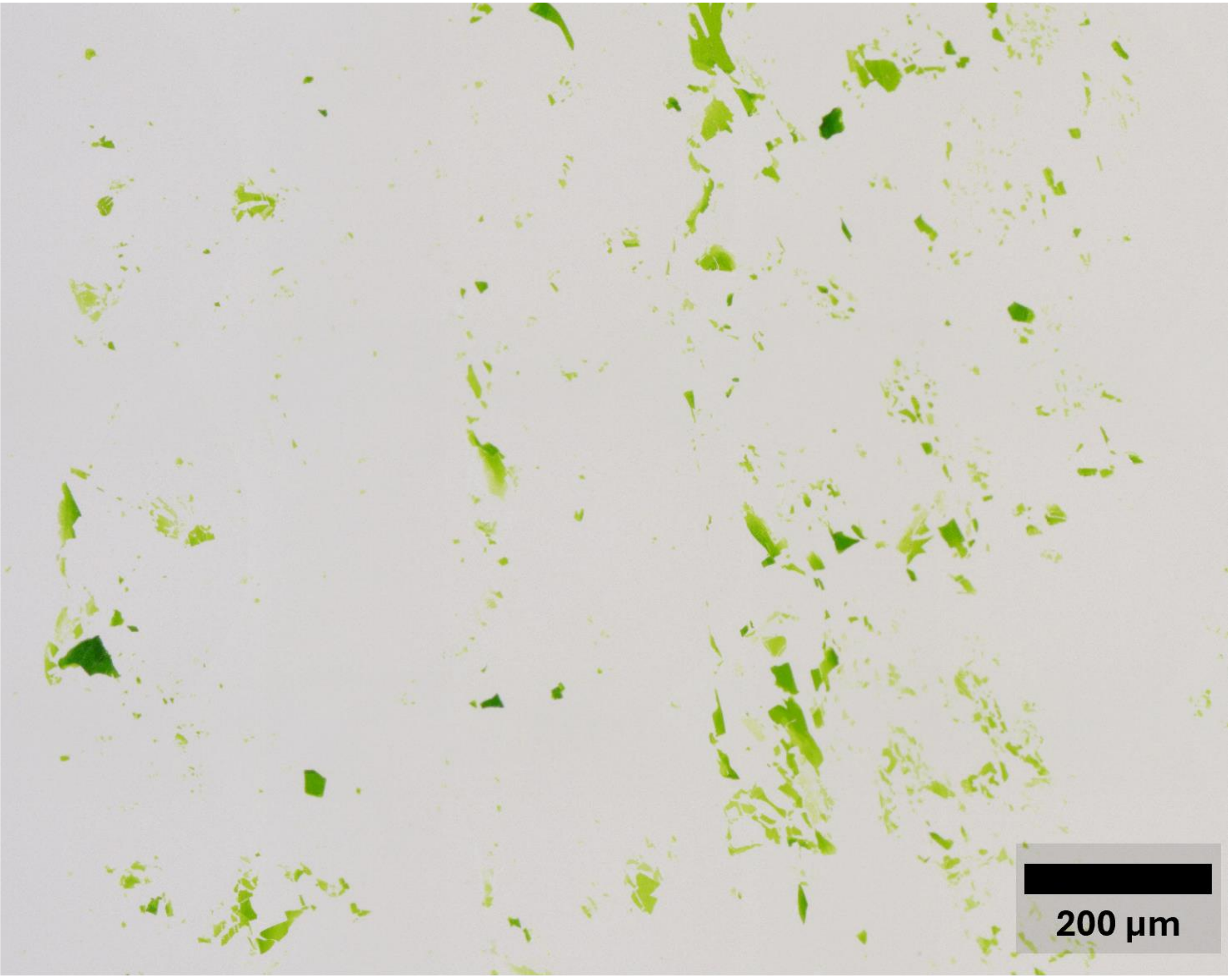
AFM Image (Atomic Force Microscopy)

- Wanted to make 2D, hard-to-see ultra-thin organic materials easier to see through an optical microscope.
- Need to exfoliate material into thin, transparent flakes on top of a substrate.
- Thinner flakes are harder to see.
- AFM image takes a long time; the optical microscope is a lot quicker.

Contrast

- Harder to see thin flakes with a microscope
- Easier to see means we want a higher contrast
- Contrast is the color/density difference between the flake and the substrate
- Enhance the thinner, less visible flakes via an optical path

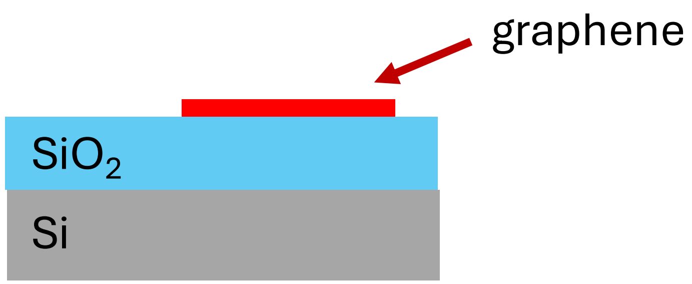
$$\frac{no\ flake\ - flake}{no\ flake}$$

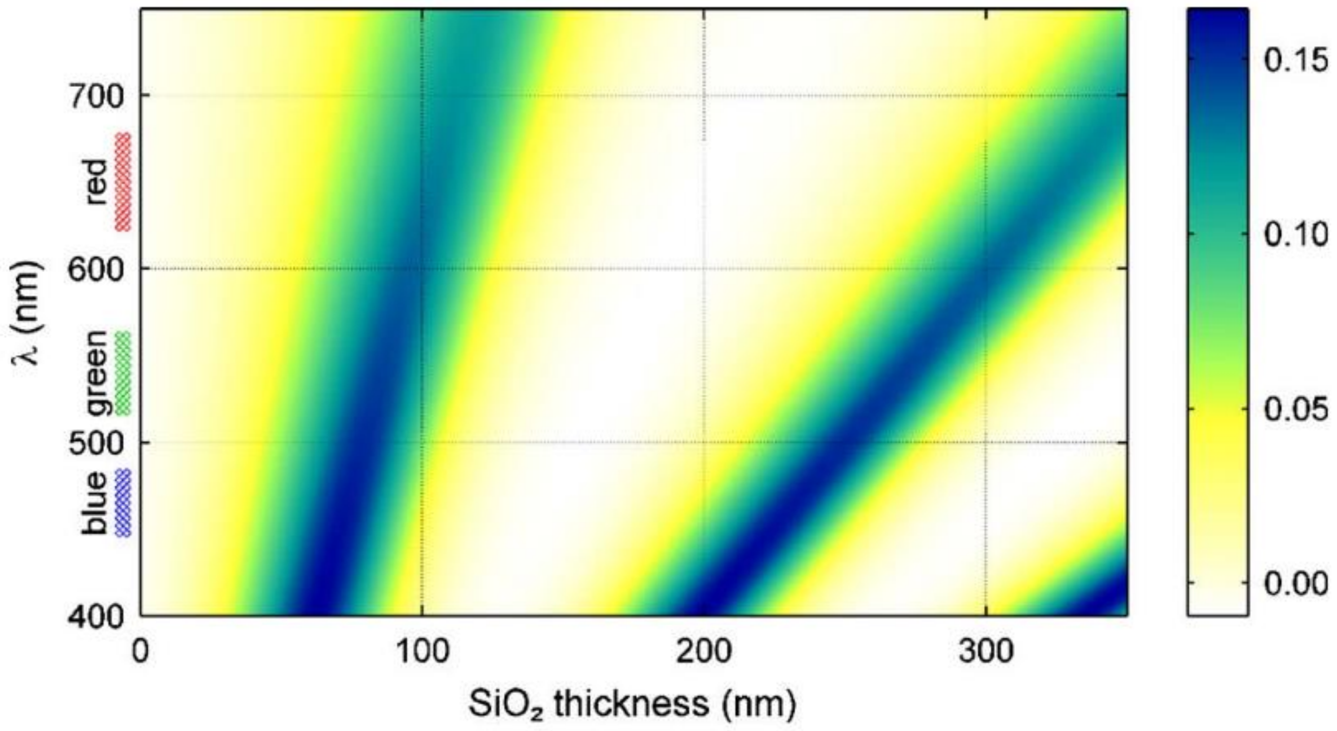


Optical Microscope

Goal

- Blake et al. show interference enhances contrast for a single layer of graphene
- Certain thicknesses of SiO_2 lead to enhanced graphene contrast/visibility: e.g. 90 nm
- Central question: Can this idea be applied to ultra-thin organic materials?
- Make a Python script to simulate a multi-layer structure containing air, graphene, SiO_2, and Si to calculate the contrast of graphene with varying parameters
  - Python is an open-source language that is easily accessible
- Graphene index, SiO_2 thickness, and SiO_2 index are functions of wavelength

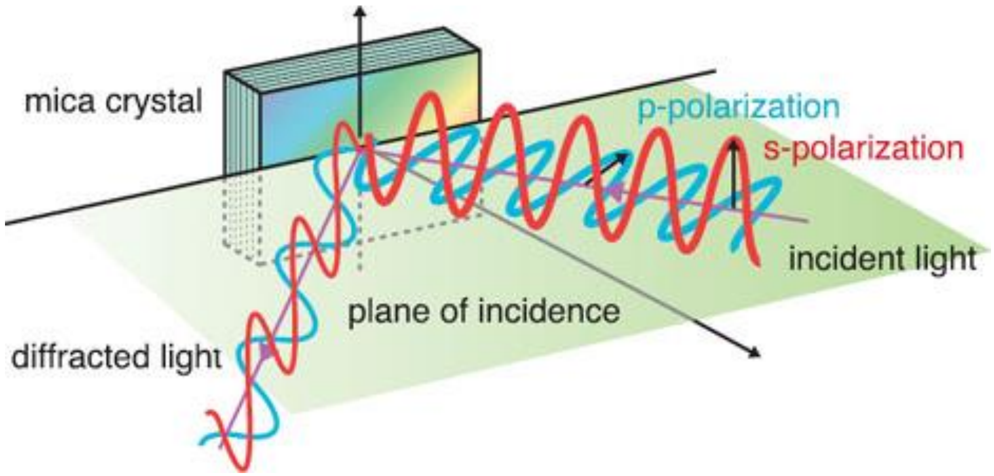




P. Blake, E. W. Hill, A. H. Castro Neto, K. S. Novoselov, D. Jiang, R. Yang, T. J. Booth, A. K. Geim; Making Graphene Visible. *Appl. Phys. Lett*. 6 August 2007; 91 (6): 063124. https://doi.org/10.1063/1.2768624

Reflection and Transmission Coefficients
Fresnel Equations

- Explain how the reflection and transmission of light are affected when changing mediums
- Amplitude coefficients r and t: magnitude/phase
- Power coefficients R and T: how light moves through medium
- In terms of the wave relative vector



$$r_s = \frac{\tilde{k}_{n\perp} - \tilde{k}_{n+1\perp}}{\tilde{k}_{n\perp} + \tilde{k}_{n+1\perp}}$$

$$r_p = \frac{\frac{\tilde{k}_{n+1}}{\tilde{k}_n}\tilde{k}_{n\perp} - \frac{\tilde{k}_n}{\tilde{k}_{n+1}}\tilde{k}_{n+1\perp}}{\frac{\tilde{k}_{n+1}}{\tilde{k}_n}\tilde{k}_{n\perp} + \frac{\tilde{k}_n}{\tilde{k}_{n+1}}\tilde{k}_{n+1\perp}}$$

$$R = \left|r^2\right|$$

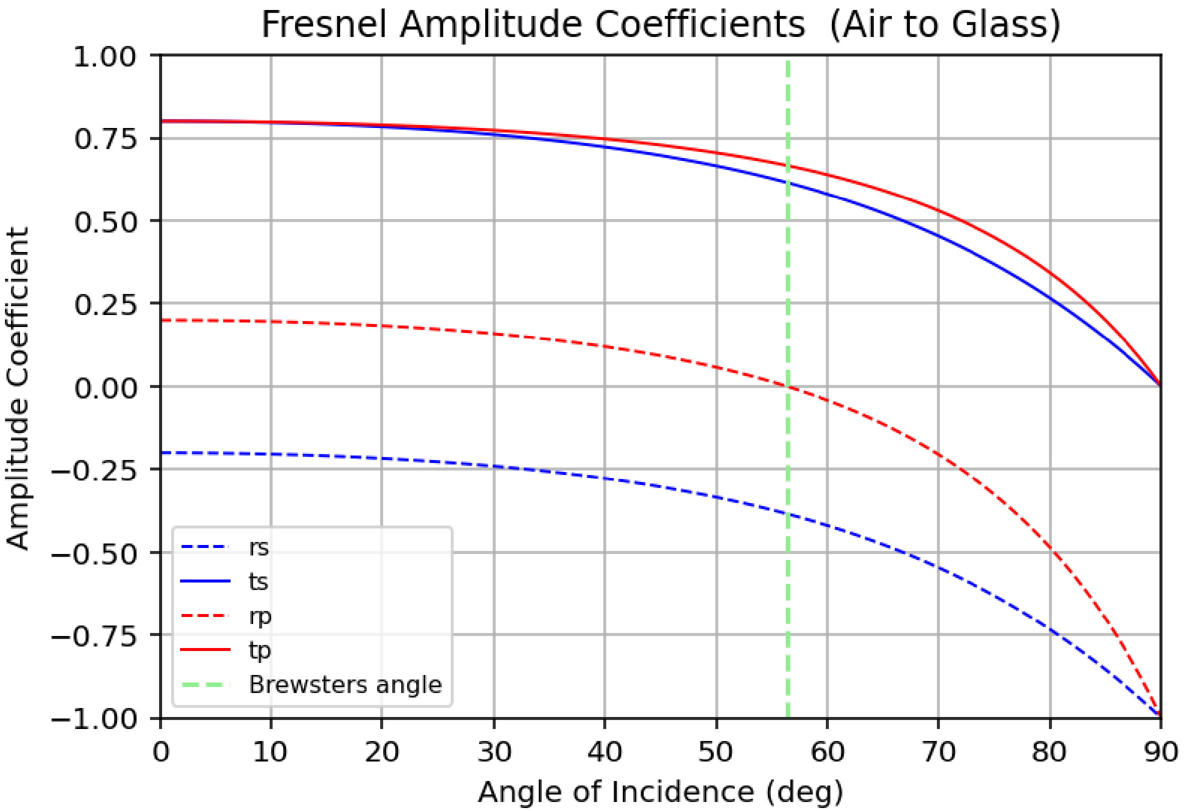
$$T_s = \mathrm{Re}\left(\frac{\tilde{k}^*_{n+1z}}{\tilde{k}^*_{nz}}\left|t_s\right|^2\right)$$

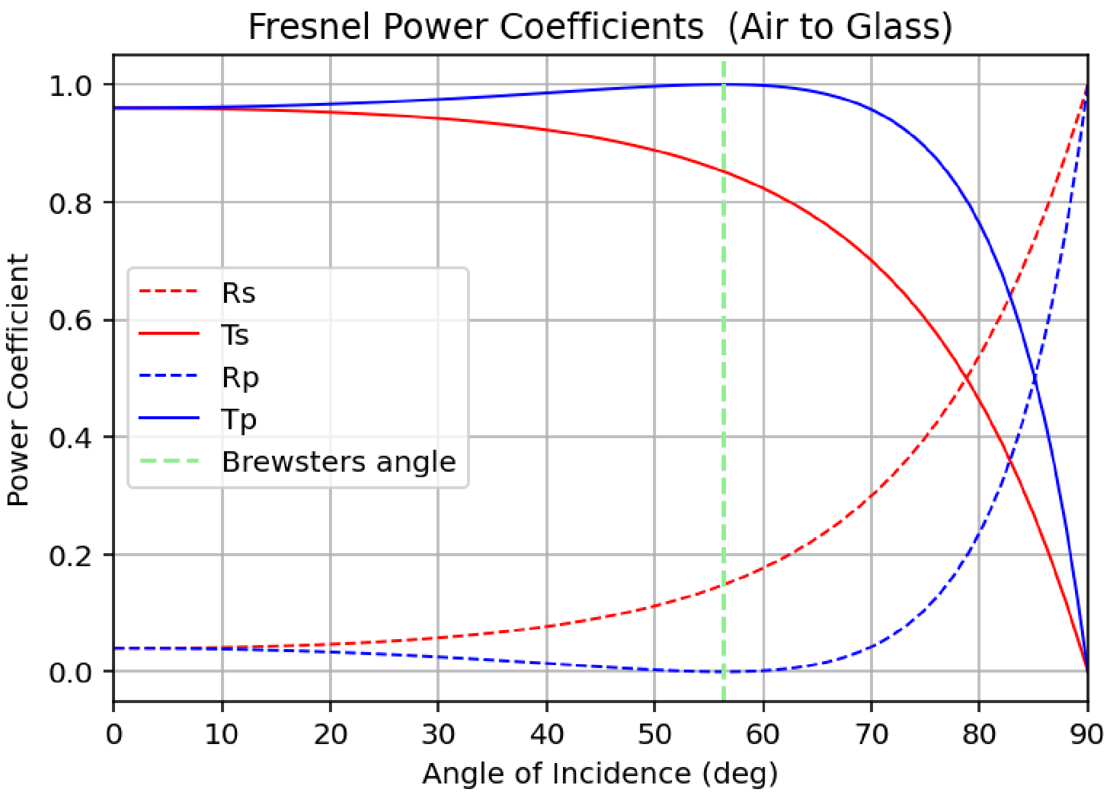
$$t_s = \frac{2\tilde{k}_{n\perp}}{\tilde{k}_{n\perp} + \tilde{k}_{n+1\perp}}$$

$$t_p = \frac{2k_{n\perp}}{\frac{\tilde{k}_{n+1}}{\tilde{k}_n}\tilde{k}_{n\perp} + \frac{\tilde{k}_n}{\tilde{k}_{n+1}}\tilde{k}_{n+1\perp}}$$

$$T_p = \mathrm{Re}\left(\frac{\tilde{k}_n^2}{\left|\tilde{k}_n\right|^2}\frac{\tilde{k}^{*2}_{n+1}}{\left|\tilde{k}_{n+1}\right|^2}\frac{\tilde{k}_{n+1z}}{\tilde{k}_{nz}}\left|t_p\right|^2\right)$$

Testing Fresnel Equations/Coefficients

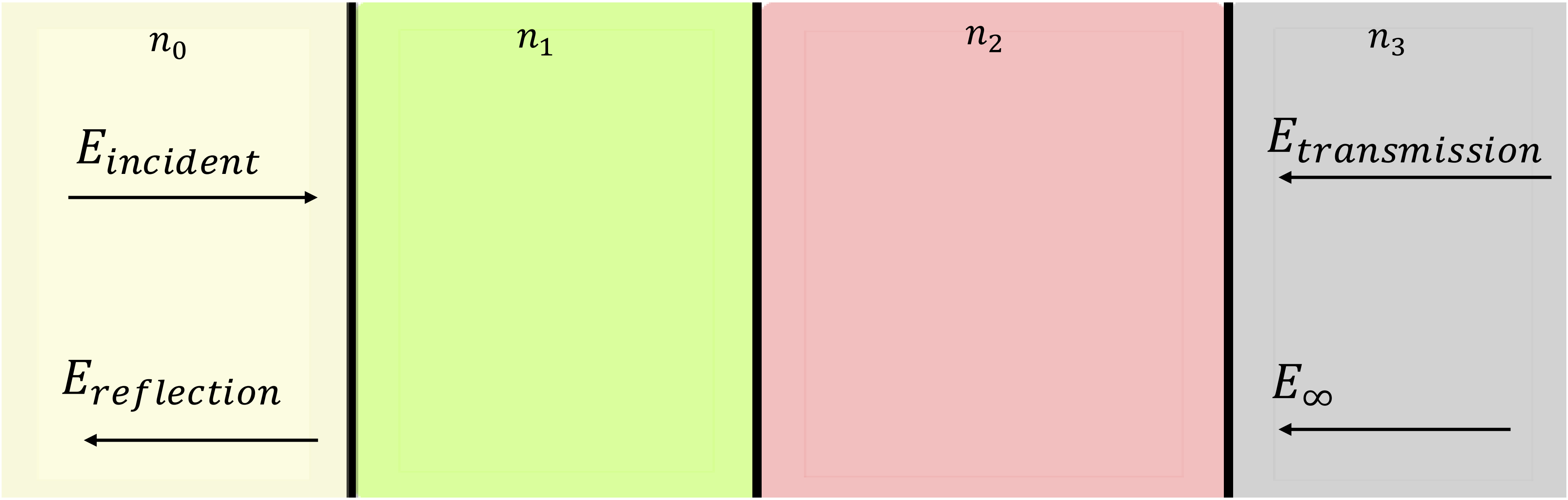




Graphs made by me!

- Started with only two layers: air → glass
- Calculates the reflection and transmission Coefficients at each interface

Multi-Layered Structures

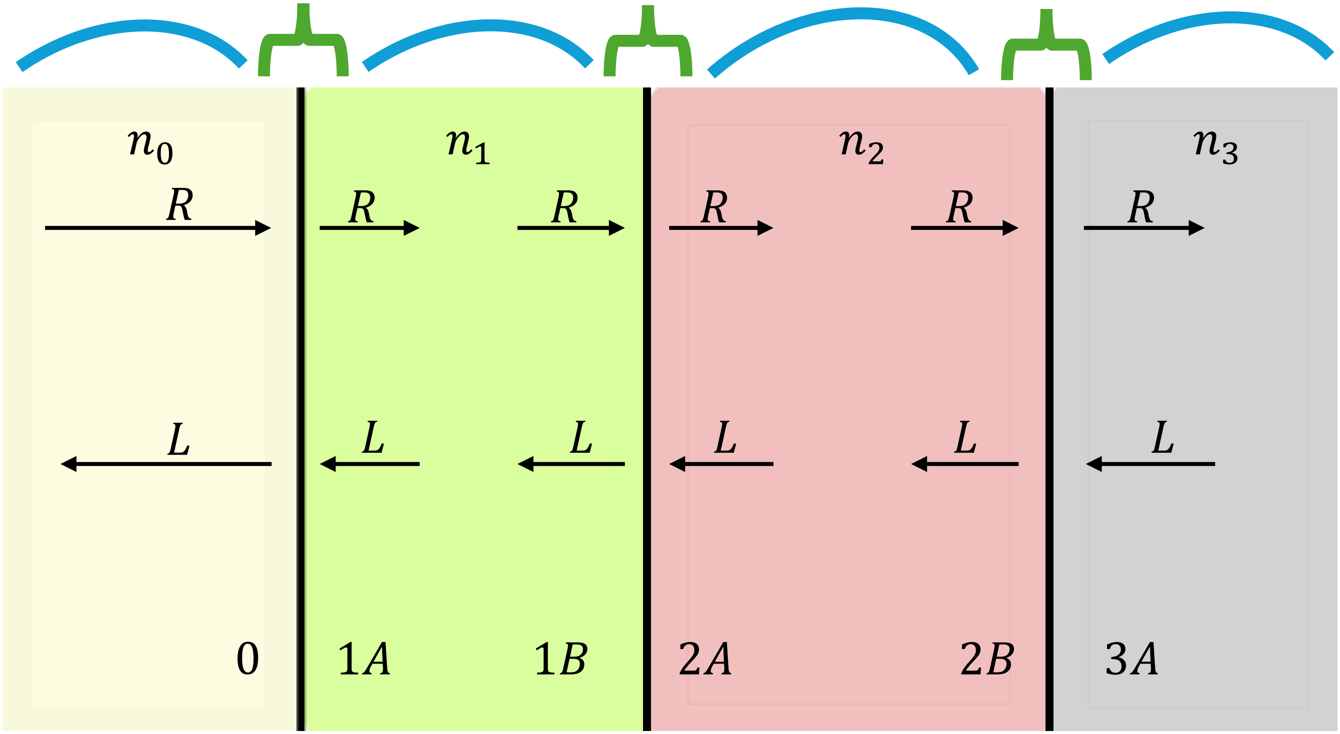


- Multiple layers of mediums with different refractive indexes.
- Must account for all incidence, transmission, and interference.

Transfer Matrix

- COULD calculate this out with each individual incidence, transmission, and interference of light, but isn't scalable.

- Easier way to compute all the components in a scalable way.



Transmission matrix: $\boldsymbol{D} = \frac{1}{t_{01}}\begin{bmatrix} 1 & r_{01} \\ r_{01} & 1 \end{bmatrix}$

$$M = D_{23}P_2D_{12}P_1D_{01}$$

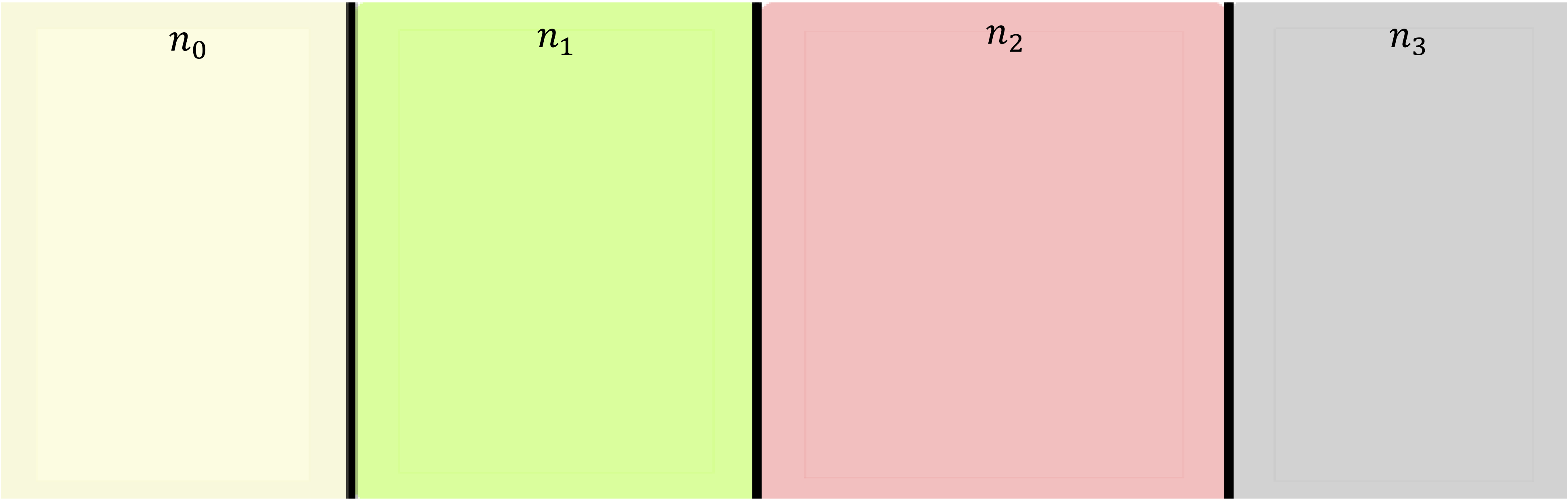
Propagation matrix: $\boldsymbol{P} = \begin{bmatrix} e^{+jk_1L_1} & 0 \\ 0 & e^{-jk_1L_1} \end{bmatrix}$

Transmission Matrix

- The in-between of mediums
- What the Fresnel equations calculate
- Earlier graphs calculate one transmission matrix

Propagation Matrix

- The individual medium
- Dependent on thickness of medium and angle of incidence
- Thickness of first and last medium is semi-finite



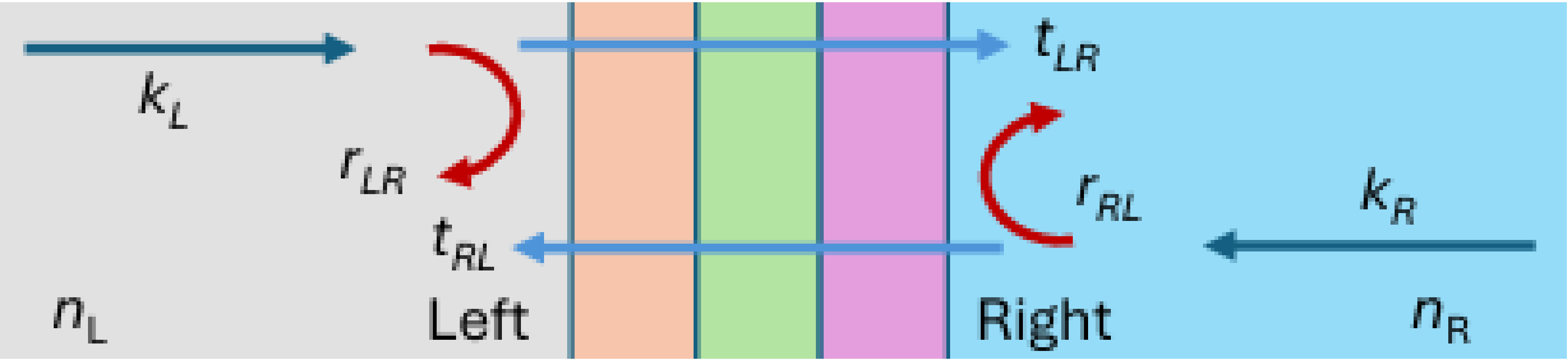
Scattering Matrix

- Accounts for multiple reflection, transmission, and interference.
- Can calculate the reflection and transmission coefficients from the system from the resulting matrix
- Specifically want the reflection coefficient (R) at normal incidence.

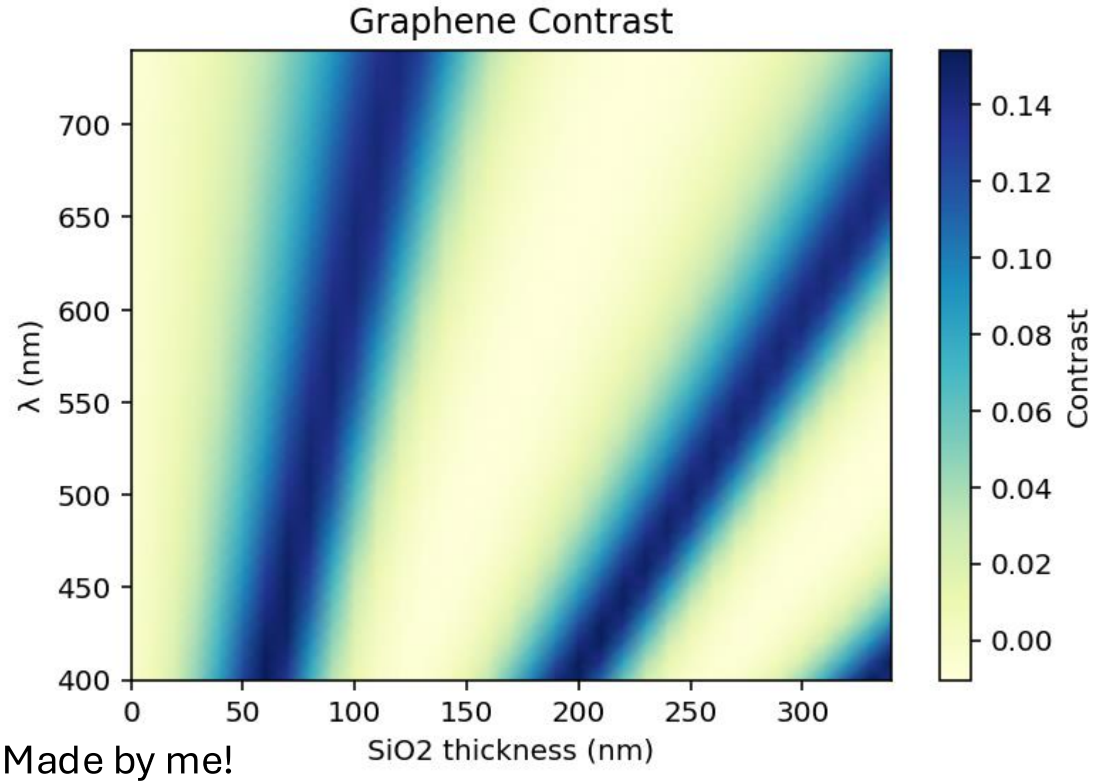
$$r_{LR} = \frac{M_{21}}{M_{11}}; \quad t_{LR} = \frac{1}{M_{11}}$$

$$r_{RL} = -\frac{M_{21}}{M_{11}}; \quad t_{RL} = \frac{\det(\mathbf{M})}{M_{11}}$$

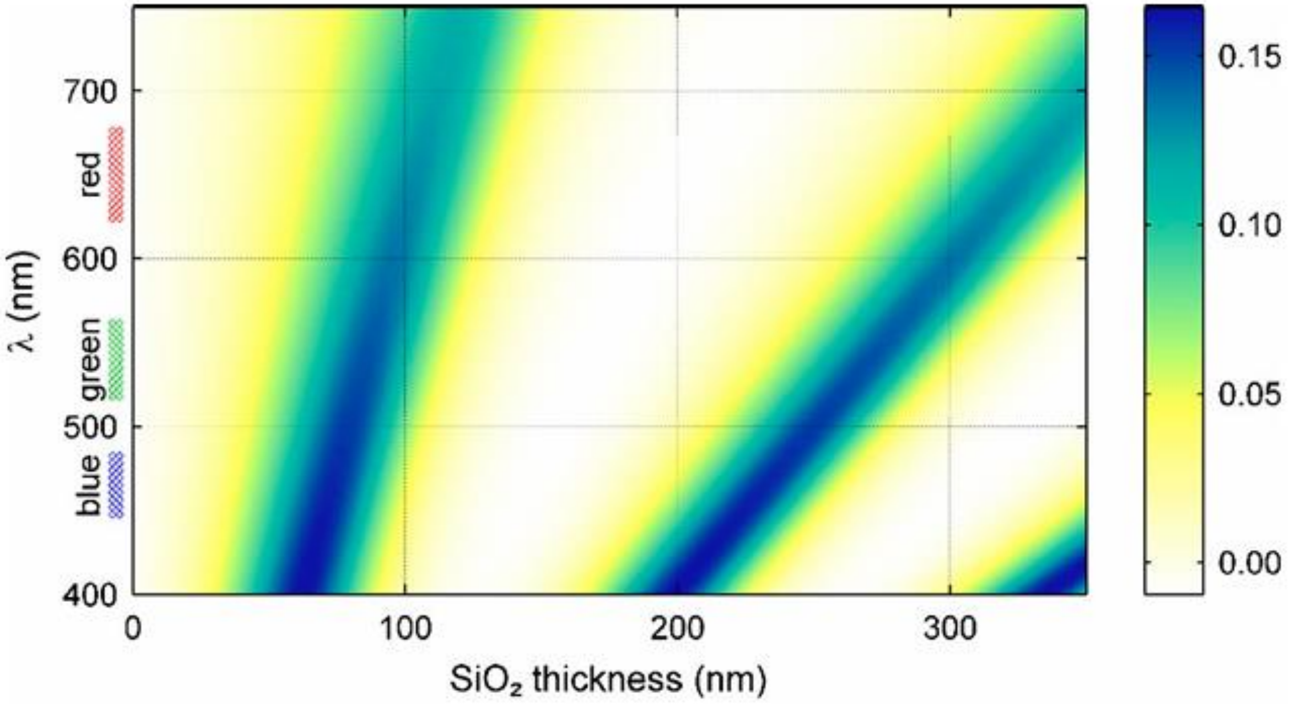
$$R = \left|r^2\right|$$



Result: Replication of Graphene Contrast

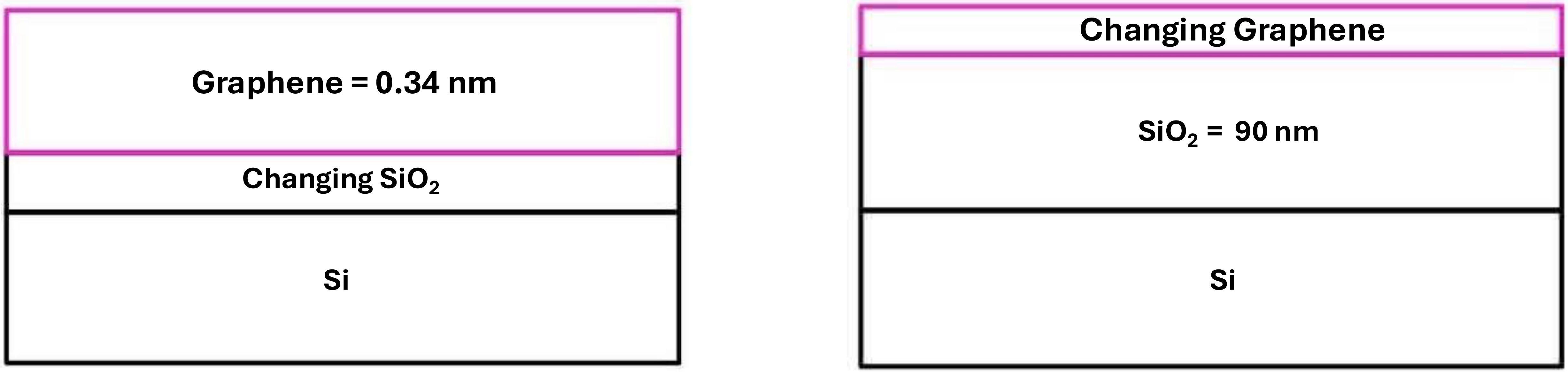


Made by me!



- Successfully replicated the graph from the paper! Yay!
- Used the graphene index and SiO_2 thickness both as functions of wavelength
- SiO_2 at 90 nm is optimal for graphene contrast

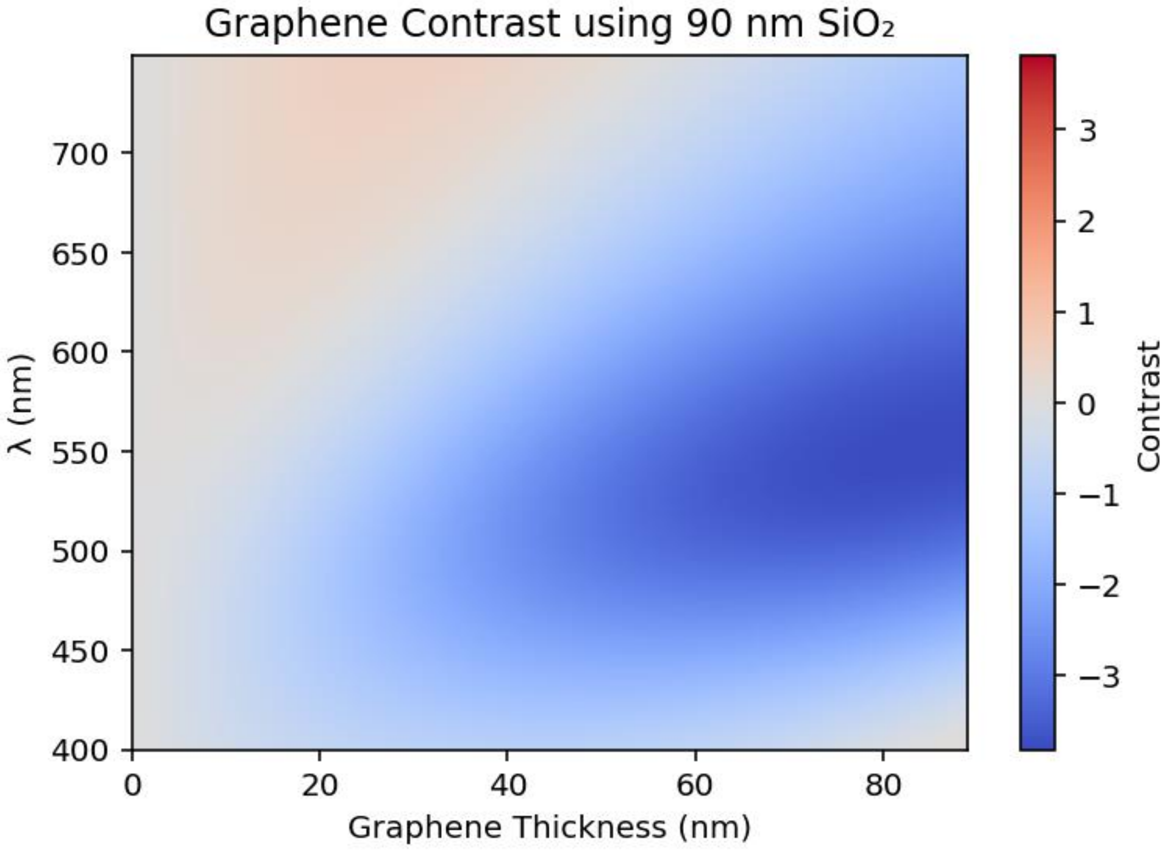
Extend Blake et al. to Thicker Graphene Layers



Made by me!

- Know that 90 nm SiO_2 is where the graphene contrast is at a peak
- Can make SiO_2 thickness constant to see how changing graphene's thickness affects the contrast.

Effect of Graphene Thickness



Made by me!

- Contrast dependent on graphene thickness and wavelength, now with a constant SiO_2

- Demonstrates that graphene is best seen with green light at thicknesses of 35 nm to 70 nm

$$\frac{no\ graphene\ - graphene}{no\ graphene}$$

- Negative contrast = graphene is more reflective than the substrate

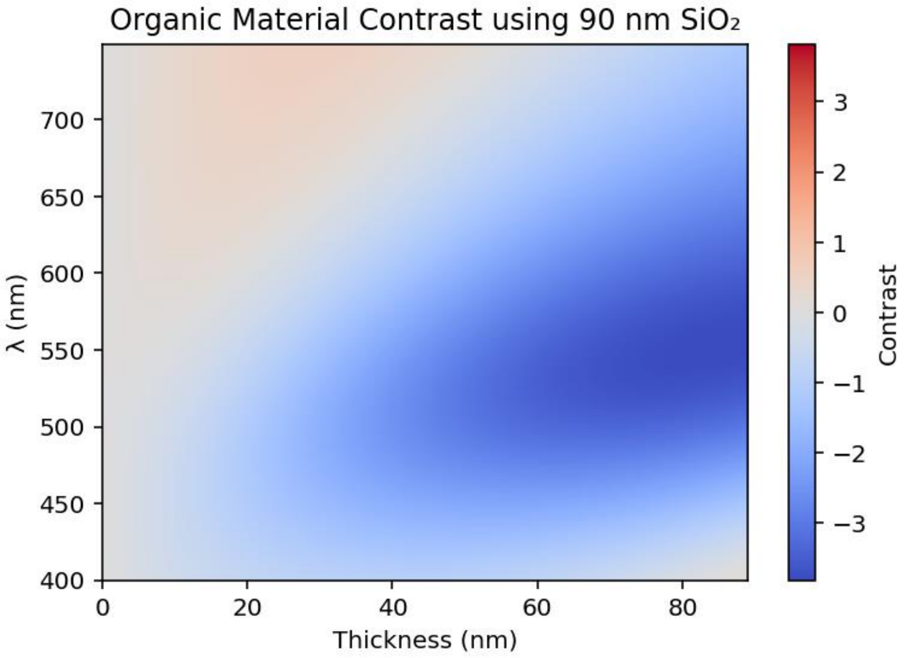
- Now can translate this to organic materials!

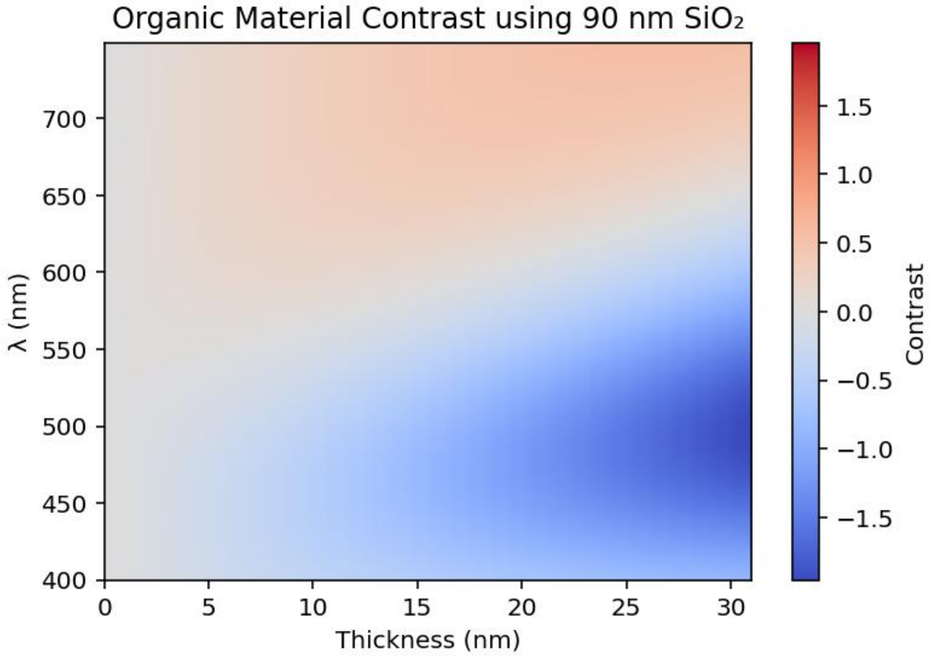
Organic Materials

- Assume a non-absorbing organic material with n = 1.7 + 0i

- Certain thicknesses paired with specific wavelengths are most optimal for contrast.

- Green/yellow light paired with 90nm SiO_2 and 40-100 nm organic thickness





Graphs made by me!

What I’ve Learn This Summer!

- Fresnel Equations (both with and without the wave vector)
- I’m now a master at Python
- How to ask for help
- Better social skills
- No one expects me to know everything!

Acknowledgements

Dr. Lloyd Bumm (best mentor ever)

Mikey Walkup

Steven Raybould

References

P. Blake, E. W. Hill, A. H. Castro Neto, K. S. Novoselov, D. Jiang, R. Yang, T. J. Booth, A. K. Geim; Making Graphene Visible. *Appl. Phys. Lett.* 6 August 2007; 91 (6): 063124. https://doi.org/10.1063/1.2768624

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Questions?

Thank You All So Much! Best Summer Ever! <3