## PROBLEM 1: Stationary and Non-Statonary States

Consider a quantum system whose particles are in the following state:

$$\Psi(x,t) = \frac{1}{\sqrt{8}}\psi_1(x)\mathrm{e}^{-iE_1t/\hbar} - i\sqrt{\frac{3}{8}}\psi_3(x)\mathrm{e}^{-iE_3t/\hbar} + \frac{1}{\sqrt{2}}\psi_5(x)\mathrm{e}^{-iE_5t/\hbar}, \tag{1}$$

where $\psi_n(x)$, $n = 1, 2, 3\ldots$ are stationary states of the Hamiltonian governing the system,

$$H\psi_n(x) = E_n\psi(x).$$

Answer the following questions:

a) Do you expect $\langle x \rangle$, $\langle x^2 \rangle$ and $\langle E \rangle$ to be time dependent or time independent? Discuss briefly, but do not calculate. (2 Points)

b) Is the uncertainty $\Delta E$ positive, negative or zero? Is $\Delta E$ time dependent or time independent? Again, discuss briefly but do not calculate. (2 Points)

c) Is $\Psi(t)$ above a solution of the time dependent Schroedinger equation? Demonstrate. (2 Points)

d) If the stationary states $\psi_1(x)$, $\psi_3(x)$ and $\psi_5(x)$ are eigenstates of the harmonic oscillator, will any of your answers to part a) change? Justify. (2 Points)

e) Now assume the particles are in the state

$$\Psi(x,t) = \psi_3(x)\mathrm{e}^{-iE_3t/\hbar}.$$

Answer parts a) and b) for this state. (2 Points)

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# Quantum #1

a) Operators that commute with the Hamiltonian will be time independent. Since in general $[H, x] \neq 0$, $[H, x^2] \neq 0$, and $[H, E] = 0$, we would expect $\langle x \rangle$ and $\langle x^2 \rangle$ to vary with time while $\langle E \rangle$ will not

b) $\Delta E = \sqrt{\langle E^2 \rangle - \langle E \rangle^2}$

* From this equation, we know $\Delta E$ cannot be negative and there is no reason to expect $\langle E^2 \rangle = \langle E \rangle$, thus our answer should not be 0. $\Delta E$ should be time independent as $[E^2, E] = 0$ and thus $[H, E^2] = 0$

c) The time-dependent Schrödinger eqn is:

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = H|\psi\rangle \quad \text{where } H|\psi_n\rangle = E_n|\psi\rangle$$

$$|\psi\rangle = \frac{1}{\sqrt{8}} e^{-iE_1 t/\hbar} |\psi_1\rangle - i\sqrt{3/8}\, e^{-iE_3 t/\hbar} |\psi_3\rangle + \frac{1}{\sqrt{2}} e^{-iE_5 t/\hbar} |\psi_5\rangle$$

$$\Rightarrow H|\psi\rangle = \frac{E_1}{\sqrt{8}} e^{-iE_1 t/\hbar} |\psi_1\rangle - E_3 i\sqrt{3/8}\, e^{-iE_3 t/\hbar} |\psi_3\rangle + \frac{E_5}{\sqrt{2}} e^{-iE_5 t/\hbar} |\psi_5\rangle$$

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = i\hbar \left[ \frac{-iE_1/\hbar}{\sqrt{8}} e^{-iE_1 t/\hbar} |\psi_1\rangle + \frac{i^2 E_3}{\hbar} \sqrt{3/8}\, e^{-iE_3 t/\hbar} |\psi_3\rangle + \frac{-iE_5}{\hbar} \sqrt{\frac{1}{2}}\, e^{-iE_5 t/\hbar} |\psi_5\rangle \right.$$

$$= \frac{E_1}{\sqrt{8}} e^{-iE_1 t/\hbar} |\psi_1\rangle - iE_3 \sqrt{3/8}\, e^{-iE_3 t/\hbar} |\psi_3\rangle + \frac{E_5}{\sqrt{2}} e^{-iE_5 t/\hbar} |\psi_5\rangle$$

$\therefore$ $|\psi\rangle$ is a solution to time dependent Schrödinger Eqn

d) If we now specify that $|\psi_n\rangle$ are the states of the SHO, the only answer that changes from part a is $\langle x \rangle$ should now be time independent since $\langle x \rangle = 0$

e) If $|\psi\rangle = e^{-iE_3 t/\hbar} |\psi_3\rangle$, All our answers in part a will be time independent b/c $|\psi_3\rangle$ is a stationary state and the time dependences will cancel out $(e^{iE_3 t/\hbar} \cdot e^{-iE_3 t/\hbar} = 1)$ Additionally, since we now definitely know $E$ $\langle E \rangle^2 = \langle E^2 \rangle$ which means $\Delta E = 0$