Jan 2008

Problem 1: The Infinite Square Well: (10 Points)

A single particle is in a one dimensional infinitely deep potential well of width L where $V(x)$ is given by:

$$V(x) = \begin{cases} 0, & \text{if } 0 \leq x \leq L \\ \infty, & \text{otherwise} \end{cases}$$

1. Find the allowed energies (E_n) and the normalized eigenfunctions ($\Psi(x)$) to Schrodinger's Equation for this potential. Show all your work. **(2 Points)**

2. Sketch the wave functions for the first three stationary states for this potential. **(2 Points)**

3. Now, if four spin-1/2 identical particles of mass m are placed in this potential, calculate the three lowest values for the total energy of the system of particles. **(3 Points)**

4. Determine the degeneracy for each of the three energy states found in part 3. **(3 Points)**

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Quantum #1

a) $H\psi = E\psi$

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + 0(\psi) = E\psi$$

$$\frac{\partial^2}{\partial x^2}\psi = \frac{2mE}{-\hbar^2}\psi$$

*if $k = \sqrt{\frac{2mE}{\hbar^2}}$

$$\frac{\partial^2}{\partial x^2}\psi = -k^2\psi$$

$\hookrightarrow \psi = A\sin(kx) + B\cos(kx)$

* We know that $\psi(0) = \psi(L) = 0$

$\hookrightarrow 0 = A\cancel{\sin(k\cdot 0)}^{0} + B\cos(k\cdot 0)$

$0 = B$

$\Rightarrow \psi(x) = A\sin(kx)$; for this to be 0 at $x = L$, $kx = n\pi \Rightarrow k_n = \frac{n\pi}{L}$

$\hookrightarrow \psi(x) = A\sin\left(\frac{n\pi x}{L}\right)$

* Normalizing the wave function, we see:

$$1 = \int_{-\infty}^{\infty} \left|A\sin\left(\frac{n\pi x}{L}\right)\right|^2 dx$$

$$1 = A^2\int_0^L \sin^2\left(\frac{n\pi x}{L}\right)dx$$

$$1 = A^2\cdot\frac{L}{2}$$

$\hookrightarrow A = \sqrt{\frac{2}{L}} \quad \Rightarrow \boxed{\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)}$

* Returning to k_n:

$$k_n = \frac{n\pi}{L} = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\frac{n^2\pi^2}{L^2} = \frac{2mE}{\hbar^2}$$

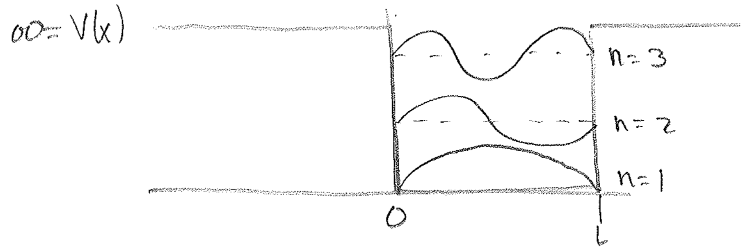
$\hookrightarrow \boxed{E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}}$, $n \in \mathbb{Z}^+$ for non-trivial solutions

#1 (cont.)

b) $\psi_1 = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$

$\psi_2 = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right)$

$\psi_3 = \sqrt{\frac{2}{L}} \sin\left(\frac{3\pi x}{L}\right)$



c) * Since spin-½ particles are fermions, no more than one particle can occupy a single state

$\hookrightarrow \; E_{sys} = \dfrac{(n_1^2 + n_2^2 + n_3^2 + n_4^2)\pi^2\hbar^2}{2mL^2}$

(1)(2)(4)(5)
$1+4+16+25 = 46$

(1)(2)(3)(6)
$1+4+9+36 = 50$

$\Rightarrow$ our lowest energy configurations are:

$n = \{1, 2, 3, 4\}$, $E_{sys} = \dfrac{30\pi^2\hbar^2}{2mL^2}$

$n = \{1, 2, 3, 5\}$, $E_{sys} = \dfrac{39\pi^2\hbar^2}{2mL^2}$

$n = \{1, 2, 4, 5\}$ $E_{sys} = \dfrac{46\pi^2\hbar^2}{2mL^2}$

d) Each state has 4! degeneracies, 24 overall for each state