PROBLEM 1: Postulates of Quantum Mechanics

A physical system consists of three distinct physical states. For this system, an operator Λ has eigenvalues λ_1, λ_2 and λ_3.

(a) Write down the completeness relation for this system. [2 points]

(b) Apply the completeness relation, then write down the expansion of a general state $|\psi\rangle$ in terms of eigenvectors of Λ [1 point]

(c) What is the probability that a measurement Λ of the state $|\psi\rangle$ yields the value λ_1? [2 points]

(d) A measurement of Λ on the state $|\psi\rangle$ is found to give a value λ_2. What is the state of the system immediately after the measurement? [1 point]

(e) A second measurement of Λ on the system is immediately performed. What is the probability of finding $\langle\Lambda\rangle = \lambda_1$? What is the probability of finding $\langle\Lambda\rangle = \lambda_2$? [2 points]

(f) Let us assume that the Hamiltonian H is time independent. Write down an equation that determines the time evolution of the state $|\psi(t)\rangle$ in the Schrödinger picture. Write down an equation that determines the time evolution of $\Lambda(t)$ in the Heisenberg picture. [2 points]

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Quantum #1

a) The completeness relation for any system is such that

$$1 = \sum_i |\lambda_i\rangle\langle\lambda_i|$$

$\Rightarrow$ For this system where we define the kets as $|\lambda_1\rangle, |\lambda_2\rangle, |\lambda_3\rangle$ (in Λ-basis)

$$1 = |\lambda_1\rangle\langle\lambda_1| + |\lambda_2\rangle\langle\lambda_2| + |\lambda_3\rangle\langle\lambda_3|$$

b) To expand the state, we use the projection operator $\tilde{P} = 1$

$\hookrightarrow$ $P|\psi\rangle = \langle\lambda_1|\psi\rangle|\lambda_1\rangle + \langle\lambda_2|\psi\rangle|\lambda_2\rangle + \langle\lambda_3|\psi\rangle|\lambda_3\rangle$

c) $|\langle\lambda_1|1|\psi\rangle|^2$

d) Assuming the system is defined in the Λ eigenbasis, we would expect to find $|\psi\rangle = |\lambda_2\rangle$ immediately after measurement

e) λ_1: $|\langle\lambda_1|\Lambda|\lambda_2\rangle| = 0$

λ_2 $|\langle\lambda_2|\Lambda|\lambda_2\rangle| = |\lambda_2|^2$

f) $|\psi(t)\rangle = \exp[iHt/\hbar]|\psi\rangle$

$$\Lambda(t) = e^{+iHt/\hbar}\,\Lambda\,e^{-iHt/\hbar}$$