S-2010



## PROBLEM 5: Two Level System

Consider a quantum system that can be accurately approximated as having two energy levels $|+\rangle$ and $|-\rangle$ such that

$$H_0|\pm\rangle = \pm\epsilon|\pm\rangle\,,$$

where $\epsilon$ is energy.

When placed in an external field, the eigenstates of $H_0$ are mixed by another term in the total Hamiltonian

$$V|\pm\rangle = \delta|\mp\rangle\,.$$

For simplicity, we choose $\epsilon$ to be real.

(a) [1 points] Using the states $|+\rangle$ and $|-\rangle$ as your basis states, write down the matrix representations for the operators $H_0$ and $V$.

(b) [3 points] What will be the possible results if a measurement is made of the energy for the full Hamiltonian $H = H_0 + V$?

(c) [2 points] Experiments are performed that measure the transition energies between eigenstates. Without the external field ($\delta = 0$) it is found that the transition energy is 4 eV and with the external field ($\delta \neq 0$) the transition energy is 6 eV. What is the coupling between the states $|\pm\rangle$, $\delta$, in this case?

(d) [2 points] We can write the eigenstates of the total Hamiltonian in terms of two energy levels $|\pm\rangle$ as

$$\begin{aligned} |1\rangle &= \cos(\theta_1)|+\rangle + \sin(\theta_1)|-\rangle \\ |2\rangle &= \cos(\theta_2)|+\rangle + \sin(\theta_2)|-\rangle\,. \end{aligned}$$

Letting $\delta/\epsilon = C$, solve for the angles $\theta_1$ and $\theta_2$ in terms of $C$.

(e) [2 points] Consider an experiment where the two-level system starts in the eigenstate of $H_0$ with eigenvalue $-\epsilon$. A very weak field is turned on so that $C \ll 1$. To the lowest order in $C$, what is the probability of measuring a positive energy for the system when $\delta \neq 0$?

S-2010

5 A

$$\boxed{H_0 = \begin{pmatrix} +\epsilon & 0 \\ 0 & -\epsilon \end{pmatrix} \qquad V = \begin{pmatrix} 0 & \delta \\ \delta & 0 \end{pmatrix}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \delta \end{pmatrix} = \checkmark$$

B

$$H = \begin{pmatrix} \epsilon & \delta \\ \delta & -\epsilon \end{pmatrix} \rightarrow \begin{vmatrix} \epsilon-\lambda & \delta \\ \delta & -\epsilon-\lambda \end{vmatrix} = \cancel{\phantom{xx}}$$

$$(\epsilon-\lambda)(-\epsilon-\lambda) - \delta^2 = \phi$$

$$-\epsilon^2 \cancel{-\epsilon\lambda + \lambda\epsilon} + \lambda^2 - \delta^2 = \phi$$

$$\lambda^2 = \delta^2 + \epsilon^2$$

$$\rightarrow \boxed{\lambda = \pm\sqrt{\delta^2+\epsilon^2}}$$

C

(c) don't know what $|\pm\rangle$ are

Assume of the form $\begin{pmatrix}A\\B\end{pmatrix}\begin{pmatrix}C\\D\end{pmatrix}$

$$\langle +|H_0|-\rangle = (A\ B)\begin{pmatrix}\epsilon & 0\\ 0 & -\epsilon\end{pmatrix}\begin{pmatrix}C\\D\end{pmatrix}$$

$$= (A\ B)\begin{pmatrix}C\epsilon\\ -\epsilon D\end{pmatrix} = AC\epsilon - BD\epsilon = 4$$

$$\rightarrow \langle +|H|-\rangle = (A\ B)\begin{pmatrix}\epsilon & \delta\\ \delta & -\epsilon\end{pmatrix}\begin{pmatrix}C\\D\end{pmatrix}$$

$$= (A\ B)\begin{pmatrix}\epsilon C+\delta D\\ \delta C-\epsilon D\end{pmatrix} = A\epsilon C + A\delta D + B\delta C - B\epsilon D = 6$$

$$= 4 + A\delta D + B\delta C = 6$$

Or

$\langle +|H_0|-\rangle = 4$

$\langle +|H_0|-\rangle + \langle +|V|-\rangle$

$\langle +|H_0|-\rangle = 6$

$\langle +|+\rangle$

$\boxed{2 = \delta}$

$\boxed{?\ 2\ ?}$

(D) $|\lambda=1\rangle = \begin{pmatrix} \epsilon-\sqrt{\delta^2+\epsilon^2} & \delta \\ \delta & -\epsilon-\sqrt{\delta^2+\epsilon^2}\end{pmatrix}\begin{pmatrix}A\\ B\end{pmatrix} = \phi$

$$= \left(\epsilon - \sqrt{\delta^2+\epsilon^2}\right)A + B\delta = \phi$$

$$\Rightarrow \quad B = \left[-\frac{1}{c} + \sqrt{1+\frac{1}{c}}\right]A$$

$$\cancel{\Rightarrow 1 = A^2 + \left(-\frac{1}{c} + \sqrt{1+\frac{1}{c}}\right)^2 A^2}$$

$$\cancel{A^2 + \left(\frac{1}{c^2} + 1 + \frac{1}{c} - \frac{2}{c}\sqrt{1+\frac{1}{c}}\right)A^2}$$

$$\cancel{A^2\left(2 + \frac{1}{c} + \frac{1}{c^2} - \frac{2}{c}\sqrt{1+\frac{1}{c}}\right) = 1}$$

$$\frac{B}{A} = \boxed{\tan\theta_1 = \sqrt{1+\frac{1}{c}} - \frac{1}{c}}$$

$\boxed{\text{Same For } \theta_2}$

(D) + energy is $|1\rangle$

$$\Rightarrow \quad P = |\langle 1|-\rangle|^2 = \sin^2\theta_1$$

$c \ll 1$

$\tan\theta_1 \approx -\infty$

which means $\theta_1 \to \pi/2$

$\boxed{P=1} \longleftarrow$

S-2009



# Problem 2: A two-state system (10 points)

We can approximate the ammonia molecule $NH_3$ by a simple two-state system. The three $H$ nuclei are in a plane, and the $N$ nucleus is at a fixed distance either above or below the plane of the $H$'s. Each is approximately a stationary state with some energy $E_0$. But there is a small amplitude for transition from up to down. Thus the total Hamiltonian is $H = H_0 + H_1$, where

$$H_0 = \begin{pmatrix} E_0 & 0 \\ 0 & E_0 \end{pmatrix} \text{ and } H_1 = \begin{pmatrix} 0 & -A \\ -A & 0 \end{pmatrix}$$

with $|A| \ll |E_0|$.

(a) Find the exact eigenvalues of $H$. (*1 points*)

(b) Now suppose the molecule is in an electric field that distinguishes the two states. The new Hamiltonian is $H = H_0 + H_1 + H_2$, where

$$H_2 = \begin{pmatrix} \epsilon_1 & 0 \\ 0 & \epsilon_2 \end{pmatrix}$$

Find the new exact energy levels. (*1 points*)

(c) Apply perturbation theory and find the energy levels to the lowest non-vanishing order for $\epsilon_i \ll |A|$. Compare the results to the exact answer in (b). (*4 points*)

(d) Apply perturbation theory and find the energy levels to the lowest non-vanishing order for $\epsilon_i \gg |A|$. Compare the results to the exact answer in (b). (*4 points*)

2 $H_0 = \begin{pmatrix} E_0 & 0 \\ 0 & E_0 \end{pmatrix}$ $\quad H_1 = \begin{pmatrix} 0 & -A \\ -A & 0 \end{pmatrix}$

A $\quad H = \begin{pmatrix} E_0 & -A \\ -A & E_0 \end{pmatrix}$

$$\rightarrow \begin{vmatrix} E_0 - E & -A \\ -A & E_0 - E \end{vmatrix} = (E_0 - E)^2 - A^2 = \phi$$

$$E_0^2 + E^2 - 2E_0 E - A^2 = \phi$$

$$E^2 - 2E_0 E + (E_0^2 - A^2) = \phi$$

$$E = \frac{2E_0 \pm \sqrt{4E_0^2 - 4(E_0^2 - A^2)}}{2}$$

$$= \frac{2E_0 \pm 2A}{2} = \boxed{E_0 \pm A = E_\pm}$$

B $\quad H = \begin{pmatrix} (E_0 - \epsilon_1) & -A \\ -A & (E_0 - \epsilon_2) \end{pmatrix}$

$$\rightarrow \begin{vmatrix} E_0 - \epsilon_1 - E & -A \\ -A & E_0 - \epsilon_2 - E \end{vmatrix} = (E_0 - \epsilon_1 - E)(E_0 - \epsilon_2 - E) - A^2 = \phi$$

$$\cancel{E_0^2} - \cancel{E_0\epsilon_2} - \cancel{E_0 E} - \cancel{E_0\epsilon_1}$$

$$+ \cancel{\epsilon_1\epsilon_2} + \cancel{E\epsilon_1} - \cancel{EE_0} + \cancel{E\epsilon_2}$$

$$\cancel{+E^2} - \cancel{A^2} = \phi$$

$$\rightarrow E_0^2 - A^2 + E^2 + \epsilon_1\epsilon_2 - 2E_0 E - E_0 \epsilon_2 - E_0 \epsilon_1$$

$$+ E\epsilon_1 + E\epsilon_2$$

B Continued

$$E^2 + E(\epsilon_1+\epsilon_2-2E_0) + (\epsilon_1\epsilon_2 - E_0\epsilon_2 - E_0\epsilon_1 - A^2 + E_0^2) = \phi$$

$$E = \frac{2E_0-\epsilon_1-\epsilon_2 \pm \sqrt{(\epsilon_1+\epsilon_2-2E_0)^2 - 4(\epsilon_1\epsilon_2 - E_0\epsilon_2 - E_0\epsilon_1 - A^2 + E_0^2)}}{2}$$

$$\to \quad = (\epsilon_1+\epsilon_2-2E_0)(\epsilon_1+\epsilon_2-2E_0)$$

$$= \epsilon_1^2 + \epsilon_1\epsilon_2 - 2E_0\epsilon_1$$
$$+ \epsilon_2\epsilon_1 + \epsilon_2^2 - 2E_0\epsilon_2$$
$$- 2E_0\epsilon_1 - 2E_0\epsilon_2 + 4E_0^2$$

$$= \cancel{\epsilon_1^2} \cancel{+\epsilon_2^2} + 2\epsilon_2\epsilon_1 - 4\cancel{E_0}\epsilon_1 - 4\cancel{E_0}\epsilon_2$$
$$\cancel{+4E_0^2}$$

$$-4\cancel{\epsilon_1}\epsilon_2 + 4\cancel{E_0}\epsilon_2 + 4\cancel{E_0}\epsilon_1 + 4A^2 \cancel{-4E_0^2}$$

$$= -2\epsilon_1\epsilon_2 + 4A^2 + \epsilon_1^2 + \epsilon_2^2$$

$$= (\epsilon_1-\epsilon_2)^2 + 4A^2$$

$$\to \boxed{E_{\pm} = \frac{2E_0-\epsilon_1-\epsilon_2 \pm \sqrt{(\epsilon_1-\epsilon_2)^2+4A^2}}{2}}$$

(c) $\epsilon_1 \ll |A|$

Take $H = \begin{pmatrix} E_0 & -A \\ -A & E_0 \end{pmatrix}$

& $H' = \begin{pmatrix} \epsilon_1 & 0 \\ 0 & \epsilon_1 \end{pmatrix}$ where $H'$ is perturbation

$\rightarrow E_0^{(1)} = \langle n | H' | n \rangle$ don't know n

$\rightarrow \begin{vmatrix} E_0 - \lambda & -A \\ -A & E_0 - \lambda \end{vmatrix} \rightarrow E_\pm = E_0 \pm A$

$\underline{E_+} \rightarrow \begin{pmatrix} A & -A \\ -A & A \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \emptyset$

$\rightarrow a = b$

$|E_+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\underline{E_-} \rightarrow \begin{pmatrix} -A & -A \\ -A & -A \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \rightarrow |E_-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

(c) Continued

$$\underline{E_+} \rightarrow \quad E_+^{(1)} = \frac{1}{2}\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}\epsilon_1 & 0 \\ 0 & \epsilon_2\end{pmatrix}\begin{pmatrix}1 \\ 1\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}\epsilon_1 \\ \epsilon_2\end{pmatrix} = \boxed{\frac{1}{2}(\epsilon_1 + \epsilon_2)}$$

$$E_-^{(1)} = \frac{1}{2}\begin{pmatrix}1 & -1\end{pmatrix}\begin{pmatrix}\epsilon_1 & 0 \\ 0 & \epsilon_2\end{pmatrix}\begin{pmatrix}1 \\ -1\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1 & -1\end{pmatrix}\begin{pmatrix}\epsilon_1 \\ -\epsilon_2\end{pmatrix} = \boxed{\frac{1}{2}(\epsilon_1 + \epsilon_2)}$$

$$B \rightarrow \quad 2E_0 - \epsilon_1 - \epsilon_2 \pm \sqrt{(\epsilon_1 - \epsilon_2)^2 + 4A^2}$$

$$A \gg \epsilon_i$$

$$\cancel{\rightarrow E_0 - \frac{\epsilon_1}{2} - \frac{\epsilon_2}{2} \pm \frac{\epsilon_1 - \epsilon_2}{2}\left[1 + \frac{2A^2}{(\epsilon_1 - \epsilon_2)^2}\right]}$$

$$\cancel{\rightarrow E_0 - \frac{\epsilon_1}{2} - \frac{\epsilon_2}{2} \pm \frac{\epsilon_1}{2} - \frac{\epsilon_2}{2} + \frac{A^2}{\epsilon_1 - \epsilon_2}}$$

$$\rightarrow E_0 - \frac{\epsilon_1}{2} - \frac{\epsilon_2}{2} \pm A\left[1 + \frac{(\epsilon_1 - \epsilon_2)^2}{8A^2}\right]$$

They agree but I think there are prime

D $A \angle\angle \epsilon_i$

$$\cancel{H = \begin{pmatrix} E_0 - \epsilon_1 & 0 \\ 0 & E_0 - \epsilon_2 \end{pmatrix}}$$

$$H = \begin{vmatrix} E_0 + \epsilon_1 - \lambda & 0 \\ 0 & E_0 + \epsilon_2 - \lambda \end{vmatrix} = (E_0 + \epsilon_1 - \lambda)(E_0 + \epsilon_2 - \lambda) = \phi$$

$$\rightarrow E_0^2 + \epsilon_2 E_0 - \lambda E_0 + \epsilon_1 E_0 + \epsilon_1 \epsilon_2 - \lambda \epsilon_1 - \lambda E_0 - \lambda \epsilon_2 + \lambda^2 = \phi$$

$$\rightarrow E_0^2 + \epsilon_2 E_0 - 2\lambda E_0 + \epsilon_1 E_0 + \epsilon_1 \epsilon_2 + \lambda^2 - \lambda \epsilon_2 - \lambda \epsilon_1$$

$$\rightarrow \lambda^2 + \lambda(-\epsilon_2 - \epsilon_1 - 2E_0) + (E_0^2 + \epsilon_2 E_0 + \epsilon_1 E_0 + \epsilon_1 \epsilon_2) = \phi$$

$$\rightarrow \lambda = \frac{(2E_0 + \epsilon_2 + \epsilon_1) \pm \sqrt{(\epsilon_2 + \epsilon_1 + 2E_0)^2 - 4(E_0^2 + \epsilon_2 E_0 + \epsilon_1 E_0 + \epsilon_1 \epsilon_2)}}{2}$$

$$\Gamma = (\epsilon_2 + \epsilon_1 + 2E_0)(\epsilon_2 + \epsilon_1 + 2E_0) = \epsilon_2^2 + \epsilon_1 \epsilon_2 + 2E_0 \epsilon_2 + \epsilon_1 \epsilon_2 + \epsilon_1^2 + 2E_0 \epsilon_1 + 2E_0 \epsilon_2 + 2E_0 \epsilon_1 + 4E_0^2$$

$$= 4E_0^2 + 4E_0 \epsilon_2 + 4E_0 \epsilon_1 + 2\epsilon_1 \epsilon_2 + \epsilon_2^2 + \epsilon_1^2$$

1 Continued

$$\Gamma \rightarrow -2\epsilon_1\epsilon_2 + \epsilon_1^2 + \epsilon_2^2$$

$$= (\epsilon_1 - \epsilon_2)^2$$

$$\rightarrow \lambda = \frac{2E_0 + \epsilon_2 + \epsilon_1 \pm \epsilon_1 - \epsilon_2}{2}$$

$$\rightarrow \lambda = E_0 + \epsilon_1$$

$$= E_0 + \epsilon_2$$

$$\rightarrow \quad E_0 + E_1 \rightarrow \begin{pmatrix} 0 & 0 \\ 0 & E_0 + \epsilon_2 - \lambda \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix}$$

$$A = 1$$

$$|\lambda_1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|\lambda_2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

7

$\lambda_1 \rightarrow$ ~~$(1\ 0)\begin{pmatrix} E_0+\epsilon_1 & 0 \\ 0 & E_0+\epsilon_2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$~~

~~$= (1\ 0)\begin{pmatrix} E_0+\epsilon_1 \\ 0 \end{pmatrix} = E_0+\epsilon_1$~~

$$(1\ 0)\begin{pmatrix} 0 & -A \\ -A & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= (1\ 0)\begin{pmatrix} 0 \\ -A \end{pmatrix} = \phi$$

$\lambda_2 = \phi$

Second order

$$E^{(2)}_{0} = \sum_{k \neq n} \frac{\langle n | H' | k \rangle^2}{E_n - E_k}$$

$$= \frac{\left| (1\ 0)\begin{pmatrix} 0 & -A \\ -A & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|^2}{E_0+\epsilon_1-E_0-\epsilon_2} = \frac{\left| (1\ 0)\begin{pmatrix} -A \\ -A \end{pmatrix} \right|^2}{\epsilon_1-\epsilon_2}$$

$$= \boxed{\frac{+A^2}{\epsilon_1-\epsilon_2}}$$

other is $\boxed{\dfrac{+A^2}{\epsilon_2-\epsilon_1}}$

1 Continued

$$E_0 + \frac{\epsilon_1 + \epsilon_2}{2} \pm \frac{(\epsilon_1 - \epsilon_2)}{2}\left[1 + \frac{2A^2}{(\epsilon_1 - \epsilon_2)^2}\right]$$

$$= \boxed{E_0 + \frac{\epsilon_1}{2} + \frac{\epsilon_2}{2} \pm \frac{\epsilon_1}{2} - \frac{\epsilon_2}{2} + \frac{A^2}{\epsilon_1 - \epsilon_2}}$$

agrees

F-2013

## Problem 5: Two Level Systems

Consider the Hamiltonian for a two-state system:

$$H = \begin{pmatrix} \epsilon & \lambda\Delta \\ \lambda\Delta & -\epsilon \end{pmatrix} \tag{1}$$

where $\lambda$ (a unitless parameter) determines the strength of the perturbation on the two-level system and $\epsilon$ and $\Delta$ are constants with the unit of energy.

The energy eigenvectors for the unperturbed Hamiltonian ($\lambda = 0$) are

$$\psi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \psi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2}$$

(a) [2 pt] Solve for the energy eigenvalues $E_1$ and $E_2$ for the full Hamiltonian (for any $\lambda$). What is the functional form of the eigenenergies in the limits $\lambda \to 0$ and $\lambda \to \infty$?

(b) [2 pt] For the case that $\lambda|\Delta| \ll \epsilon$, solve for the energy eigenvalues to first order and second order in $\lambda$.

Compare these results with the exact results obtained in part (a) and show that they are in agreement.

(c) [1 pt] For the case that $\lambda|\Delta| \ll \epsilon$, what is the change in the unperturbed eigenstate $\psi_+$ to first order in $\lambda$?

(d) [2 pt] For the case that the unperturbed Hamiltonian is nearly degenerate, $\epsilon \ll \lambda|\Delta|$ show that the exact results obtained in part (a) agree with the results of applying first order degenerate perturbation theory with $\epsilon = 0$.

(e) [3 pts] For the case that $\epsilon \ll \lambda|\Delta|$, it would advantageous to use a different set of basis states to describe the system. Using basis states that are approximately eigenstates of the Hamiltonian for small $\epsilon$, determine the Hamiltonian matrix in this new basis. Show that the exact solutions for the eigenenergies are the same as in part (a) in this basis.

F-2013

5

$$H=\begin{pmatrix}\epsilon & \lambda\Delta\\ \lambda\Delta & -\epsilon\end{pmatrix}$$

A $\begin{vmatrix}\epsilon-E & \lambda\Delta\\ \lambda\Delta & -\epsilon-E\end{vmatrix}=0 \rightarrow (\epsilon-E)(-\epsilon-E)-\lambda^2\Delta^2=0$

$$-\epsilon^2-\cancel{E\epsilon+E\epsilon}+E^2-\lambda^2\Delta^2=0$$

$$\rightarrow E^2=\epsilon^2+\lambda^2\Delta^2$$

$$\rightarrow \boxed{E=\pm\sqrt{\epsilon^2+\lambda^2\Delta^2}}$$

$\rightarrow \underline{\lambda\rightarrow 0} \rightarrow \cancel{E=\pm\epsilon}$

$$\epsilon\sqrt{1+\frac{\lambda^2\Delta^2}{\epsilon^2}}$$

$$\boxed{=\pm\left(\epsilon+\frac{\lambda^2\Delta^2}{\epsilon}\right)}$$

$\rightarrow \underline{\lambda\rightarrow\infty} \rightarrow \cancel{E=\pm\lambda\Delta\left(1+\frac{1}{2}\frac{\epsilon^2}{\lambda^2\Delta^2}\right)}$

$$\downarrow$$

$$\boxed{E=\pm\lambda\Delta} = \cancel{\pm\left(\lambda\Delta+\frac{\epsilon^2}{2\lambda\Delta}\right)}$$

B $\langle H'\rangle=E_0^{(1)}$

$$\rightarrow \psi_+ \rightarrow \psi_+ H'\psi_+ = (1\ 0)\underbrace{\begin{pmatrix}0 & \lambda\Delta\\ \lambda\Delta & 0\end{pmatrix}}_{H'}\begin{pmatrix}1\\0\end{pmatrix}$$

$$=\boxed{0}$$

$$\rightarrow \psi_- \rightarrow \psi_- H'\psi_- \rightarrow (0\ 1)\begin{pmatrix}0 & \lambda\Delta\\ \lambda\Delta & 0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$$

$$=\boxed{0}$$

B Contined

$$E_0^{(2)} = \sum_{k \neq n} \frac{|\langle n|H'|k\rangle|^2}{E_n - E_k}$$

$n = \psi_+ \quad k = \psi_-$

$$\rightarrow E_+ = \begin{pmatrix} \epsilon & 0 \\ 0 & -\epsilon \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \epsilon \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\rightarrow E_- = \begin{pmatrix} \epsilon & 0 \\ 0 & -\epsilon \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = -\epsilon \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\rightarrow E_0^{(2)} = \frac{\left[ (1\ 0) \begin{pmatrix} 0 & \lambda\Delta \\ \lambda\Delta & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \right]^2}{2\epsilon}$$

$$= \frac{\left[ (1\ 0) \begin{pmatrix} \lambda\Delta \\ 0 \end{pmatrix} \right]^2}{2\epsilon} = \boxed{\frac{\lambda^2 \Delta^2}{2\epsilon}}$$

Should be The Same for The other one

C $$|\psi_+\rangle^1 = \sum_{k \neq n} \frac{\langle k|H'|n\rangle}{E_n - E_k} |k\rangle$$

$$= (0\ 1)\begin{pmatrix} 0 & \Delta\lambda \\ \Delta\lambda & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = (0\ 1)\begin{pmatrix} 0 \\ \lambda\Delta \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \boxed{\frac{\lambda\Delta}{2\epsilon}\begin{pmatrix} 0 \\ 1 \end{pmatrix}}$$

D $H' = \begin{pmatrix} 0 & \lambda\Delta \\ \lambda\Delta & 0 \end{pmatrix}$

$$\rightarrow H' \doteq \begin{array}{c} \\ + \\ - \end{array}\begin{array}{c} \begin{array}{cc} + & - \end{array} \\ \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \end{array}$$

$1 = 4 = \emptyset$

$\rightarrow 2 = \lambda^2\Delta^2 = 3$

$$\rightarrow H' \doteq \begin{pmatrix} 0 & \lambda^2\Delta^2 \\ \lambda^2\Delta^2 & 0 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} -E & \lambda^2\Delta^2 \\ \lambda^2\Delta^2 & -E \end{pmatrix} = \boxed{E = \pm\lambda^2\Delta^2}$$

E $H_{New} = U H_{old} U^{\dagger}$

$$U = \begin{array}{c} \\ |+old\rangle \\ |-old\rangle \end{array}\begin{array}{c} \begin{array}{cc} |+New\rangle & |-New\rangle \end{array} \\ \begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix} \end{array}$$

$$\rightarrow H = \begin{pmatrix} 0-E & \lambda\Delta \\ \lambda\Delta & 0-E \end{pmatrix} \rightarrow E = \pm\lambda\Delta$$

$$\rightarrow |\lambda\Delta\rangle = \begin{pmatrix} -\lambda\Delta & \lambda\Delta \\ \lambda\Delta & -\lambda\Delta \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = \emptyset \rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

E Continued

$$|-\lambda\Delta\rangle = \quad \rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

$$U = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix}$$

$$\rightarrow \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}0 & \lambda\Delta\\ \lambda\Delta & 0\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix}$$

$$\begin{matrix}1 & 1\\ 1 & -1\end{matrix}$$

$$= \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}\lambda\Delta & -\lambda\Delta\\ \lambda\Delta & \lambda\Delta\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}2\lambda\Delta & 0\\ 0 & -2\lambda\Delta\end{pmatrix} = \begin{pmatrix}\lambda\Delta & 0\\ 0 & \lambda\Delta\end{pmatrix} = H_{new}$$

$$\rightarrow \boxed{\pm\lambda\Delta}$$