S-2016

Problem 6: Electron in a Finite Square Well (10 pts)

Consider an electron of energy E incident from x=$-\infty$ on a symmetric one-dimensional square well of depth V_0 and width L.

$$V(x) = \begin{cases} 0, & x < -L/2 \\ -V_0, & -L/2 < x < L/2 \\ 0, & x > L/2 \end{cases}$$

a) Write down the solutions to the time-independent Schrodinger Equation for this situation. There should be five integration constants (2 points)

b) Apply boundary conditions to find the probability that the electron is transmitted past the finite well (4 points)

c) For what values of E is there a 100% probability for transmission past the well? (2 points)

d) Consider a potential well with V_0 large enough for there to be two bound states. For this well, what is the smallest electron energy (E > 0) for which there is a 100% probability for transmission? Your answer will depend on V_0 and other parameters in the problem. (2 points)

S-2016

(b) Bound → $E < \emptyset$ → $V_0 < E < \emptyset$ → $|V_0| > |E|$

→ $-V_0 < -E$

Scattering → $E > \emptyset$ → $|E| > |V_0|$

IF bond in well

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} - V_0\psi = E\psi \rightarrow$$

$|V_0| > |E|$ so This is a posive #

$$\frac{d^2\psi}{dx^2} = -\frac{2m\,(E+V_0)}{\hbar^2}$$

oscillatory

(A) but we will do Scattering

$\psi_I \; (x < -\frac{L}{2}) \rightarrow \frac{d^2\psi}{dx^2} = \frac{-2mE}{\hbar^2}\psi$ ← k_1^2

→ $\boxed{\psi_I = Ae^{ik_1x} + Be^{-ik_1x}}$

$\psi_{III} \; (x > \frac{L}{2}) \rightarrow \boxed{\psi_{III} = Ee^{ik_1x}} + Fe^{-ik_1x}$

assuming nothing goes us

$\psi_{II} \; (-\frac{L}{2} < x \leq \frac{L}{2}) \rightarrow$ $\boxed{\psi_{II} = C\sin k_2 x + D\cos k_2 x}$

$$\frac{d^2\psi}{dx^2} = -\frac{2m\,(E+V_0)}{\hbar^2}$$

B Want $\frac{E}{A}$

$$\boxed{Ae^{-ik_1 L/2} + Be^{+ik_1 L/2} = C \sin k_2 \tfrac{-L}{2} + D\cos k_2 \tfrac{-L}{2} = -C\sin k_2 \tfrac{L}{2} + D\cos k_2 \tfrac{L}{2}}$$

$$\boxed{ik_1 Ae^{-ik_1 L/2} - ik_1 Be^{ik_1 L/2} = C\cos k_2 \tfrac{L}{2} + D\sin k_2 \tfrac{L}{2}}$$

$$\boxed{Ee^{ik_1 L/2} = C\sin k_2 \tfrac{L}{2} + D\cos k_2 L/2}$$

$$\boxed{ik_1 Ee^{ik_1 L/2} = C\cos k_2 \tfrac{L}{2} - D\sin k_2 \tfrac{L}{2}}$$

$$\to Ee^{ik_1 L/2} + Ae^{-ik_1 L/2} + Be^{ik_1 L/2} = 2D\cos k_2 \tfrac{L}{2}$$

$$\to ik_1 Ae^{-ik_1 L/2} - ik_1 \cancel{B}e^{ik_1 L/2} - ik_1 \cancel{E}e^{ik_1 L/2} = 2D\sin k_2 \tfrac{L}{2}$$

$$ik_1 \cancel{E}e^{ik_1 L/2} + ik_1 Ae^{-ik_1 L/2} + ik_1 \cancel{B}e^{ik_1 L/2} = 2ik_1 D\cos k_2 L/2$$

$$\to \underline{ik_1 Ae^{-ik_1 L/2} = D(\sin k_2 L/2 + ik_1 \cos k_2 L/2)}$$

$$\to \cos k_2 \tfrac{L}{2}\, Ee^{ik_1 L/2} = C\cos k_2 \tfrac{L}{2} \sin k_2 \tfrac{L}{2} + D\cos^2 k_2 L/2$$

$$-ik_1 \sin k_2 L/2\, Ee^{ik_1 L/2} = -C\cos k_2 \tfrac{L}{2} \sin k_2 \tfrac{L}{2} + D\sin^2 k_2 L/2$$

$$\to D = \cos k_2 L/2\, Ee^{ik_1 L/2} - ik_1 \sin k_2 \tfrac{L}{2} Ee^{ik_1 L/2}$$

$$\to \frac{ik_1 Ae^{-ik_1 L/2}}{\sin k_2 L/2 + ik_1 \cos k_2 L/2} = Ee^{ik_1 L/2}(\cos k_2 \tfrac{L}{2} - ik_1 \sin k_2 \tfrac{L}{2})$$

B Continued

$$\left(\cos k_2 \tfrac{L}{2} - i k_1 \sin k_2 \tfrac{L}{2}\right)\left(\sin k_2 \tfrac{L}{2} + i k_1 \cos k_2 \tfrac{L}{2}\right)$$

$$= \cos k_2 \tfrac{L}{2} \sin k_2 \tfrac{L}{2} + i k_1 \cos^2 k_2 \tfrac{L}{2} - i k_1 \sin^2 k_2 \tfrac{L}{2} + k_1^2 \cos k_2 \tfrac{L}{2} \sin k_2 \tfrac{L}{2}$$

$$= \frac{(k_1^2+1)}{2} \sin k_2 L + i k_1 \cos k_2 L$$

$$\to \frac{E}{A} = \frac{i k_1 e^{-ik_1}}{\frac{(k_1^2+1)}{2} \sin k_2 L + i k_1 \cos k_2 L}$$

$$\to \boxed{\frac{|E|^2}{|A|^2} = \frac{k_1^2}{\frac{(k_1^2+1)^2}{4} \sin^2 k_2 L + k_1^2 \cos k_2 L} = \frac{4k_1^2}{(k_1^2+1)^2 \sin^2 k_2 L + 4k_1^2 \cos^2 k_2 L}}$$

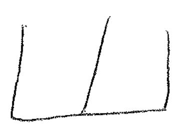
C when $k_2 L = n\pi$

$$\sqrt{\frac{2m(E+V_0)}{\hbar^2}}\, L = n\pi$$

$$\to (E+V_0) = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

Infinite square well

D Solve for bound state



in well $A\sin + B\cos$

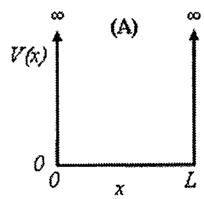
solve for even

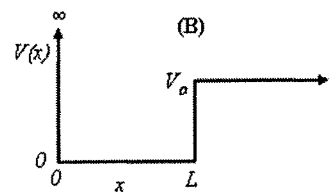
at $\frac{L}{2}$ boundary

apply minimum $V_0 L$ the thing

S-2007

PROBLEM 2





a) [2 pts] Calculate the energy eigenvalues for a particle of mass m in the one-dimensional infinite well shown in Figure A.

b) [4 pts] For the time-independent Schrödinger Equation corresponding to potential (B), find a transcendental equation in E giving the eigenenergies in terms of $V_o, L, m,$ and $\hbar$

c) [4 pts] For the time-independent Schrödinger Equation corresponding to potential (B), what is the smallest value of V_o that gives one bound state? What is the smallest value of V_o that gives two bound states?

S-2007

2 A $\boxed{\psi = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}}$

$$\boxed{E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}}$$

B For $x > L$

$$\rightarrow \frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V_0\psi = E\psi$$

$$\rightarrow \frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}(E - V_0)\psi$$

assume bound state

$$= \underbrace{\frac{2m(V_0 - E)}{\hbar^2}}_{k_2^2}\psi$$

$$\rightarrow \psi = Ae^{k_2 x} + Be^{-k_2 x}$$

at $x \rightarrow \infty$, $\psi = \phi$

$$\psi_2 = Be^{-k_2 x}$$

For $x < L$ $\rightarrow$ $A \sin k_1 x$

at $x = L$ $\rightarrow$ $A \sin k_1 L = Be^{-k_2 L}$

$$k_1 A \cos k_1 L = -k_2 Be^{-k_2 L}$$

$k_1 = \frac{\sqrt{2mE}}{\hbar}$

$k_2 = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$

$$\rightarrow -\frac{k_1}{k_2}\cos k_1 L = \sin k_1 L$$

$$\rightarrow \boxed{-\frac{k_1}{k_2} = \tan k_1 L}$$

$$\boxed{-\sqrt{\frac{E}{V_0 - E}} = \tan\left(\frac{\sqrt{2mE}}{\hbar} L\right)}$$

(c) Make a variable $y = k_1 L$ (argument of trig function)

$\Rightarrow -k_1 \cot y = k_2$

↑ want this to be y too

$\Rightarrow -y \cot y = k_2 L$

$$= \sqrt{\frac{2mL^2 V_0}{\hbar^2} - \frac{2mL^2 E}{\hbar^2}}$$

$R^2 = x^2 + y^2$

Circle of radius $R = \sqrt{\frac{2mL^2 V_0}{\hbar^2}}$

For one bound state $0 < R < \frac{\pi}{2}$

$\Rightarrow 2mL^2 V_0 < \frac{\pi^2}{4} \hbar^2$

$$\Rightarrow \boxed{V_0 < \frac{\pi^2 \hbar^2}{8mL^2}}$$

For two $\rightarrow$ $0 < R < \pi$

F-2004

Problem 5: Quantum statistics (10 points)

1. Write down the energy eigenvalues and wave functions for a particle of mass m in an infinite square well, with $V = 0$ for $-L/2 < x < L/2$ and $V = \infty$ for $|x| > L/2$. (2 pts)

2. What is the ground state energy and wave-function if 2 identical non-interacting bosons are in the well? (4 pts)

3. What is the ground state energy and wave-function if 2 identical non-interacting spin-up fermions are in the well? (4 pts)

F-2004

5 $V = \phi \qquad -L/2 < x < L/2$

$V = \infty \qquad |x| > L/2$

A For Symmetric well $\quad 0 \to L$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \to \frac{d^2\psi}{dx^2} = \underbrace{-\frac{2mE}{\hbar}}_{k^2}\psi$$

$$\psi = A\sin kx + \cancel{B\cos kx}$$

at $x = \phi \to \psi = \phi$

$$= A\sin kx$$

at $L = x \to \psi = \phi$

$$kx = n\pi \to k = \frac{n\pi}{L}$$

$$\to \psi = A\sin\frac{n\pi}{L}x$$

$$\to 1 = A^2\int_0^L \sin^2\frac{n\pi}{L}x\,dx = A^2\left[\frac{x}{2} - \frac{\sin\frac{2n\pi x}{L}}{\frac{4n\pi}{L}}\Big|_0^L\right]$$

$$= A^2\frac{L}{2} = 1 \to A = \sqrt{\frac{2}{L}}$$

$$\to \boxed{\psi = \sqrt{\frac{2}{L}}\sin\frac{n\pi}{L}x} \to \frac{2mE}{\hbar} = \frac{n^2\pi^2}{L^2}$$

$$\to \boxed{E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}}$$

(A) Continued

Now shift $L \to \frac{L}{2}$

$$\psi = \sqrt{\frac{2}{L}} \sin\left[\frac{n\pi}{L}\left(x - \frac{L}{2}\right)\right] = \sin\frac{n\pi x}{L}\cos\frac{n\pi}{2} + \cos\frac{n\pi x}{L}\sin\frac{n\pi}{2}$$

If

$$\boxed{\begin{array}{l} n = \text{even} \rightarrow \psi = \sin\frac{n\pi x}{L} \\ n = \text{odd} \rightarrow \psi = \cos\frac{n\pi x}{L} \end{array}}$$

(B) Bosons $\rightarrow$ Symmetric

spin 0

$$\psi = \frac{1}{\sqrt{2}}\left(\psi_1(x_1)\psi_2(x_2) + \psi_1(x_2)\psi_2(x_1)\right)$$

$$E = \frac{\pi^2\hbar^2}{2mL^2}\left(n_1^2 + n_2^2\right) = \boxed{\frac{\pi^2\hbar^2}{mL^2} = E}$$

If both are in same state, no need to add, just take distinguishable

$$\boxed{\psi = \frac{2}{L}\left[\cos\frac{\pi x_1}{L}\cos\frac{\pi x_2}{L}\right]}$$

C) Fermions

Antisymmetric

$$\psi(x)_A \ \phi_S \quad \text{or} \quad \psi(x)_S \ \phi_A$$

spin

symmetric is $\uparrow\uparrow$

antisymmetric is $\uparrow\downarrow$

both are spin up so,

$$\boxed{\psi = \frac{1}{\sqrt{2}}\left[\cos\frac{\pi x_1}{L}\sin\frac{2\pi x_2}{L} - \cos\frac{\pi x_2}{L}\sin\frac{2\pi x_1}{2}\right]\chi_\uparrow \chi_\uparrow}$$

$\times \frac{2}{L}$

If $\uparrow\downarrow$ you can have $(\psi_1\psi_2 - \psi_1\psi_2)(\uparrow\downarrow + \uparrow\downarrow)$

or $(\psi_1\psi_2 + \psi_1\psi_2)(\downarrow\uparrow - \uparrow\downarrow)$

$$E = m\,2^2 + 1^2 = \boxed{\frac{5}{2}\frac{\pi\hbar^2}{2mL^2} = E}$$

S-2014

PROBLEM 2: Particle in a Box

A particle of mass m is in the ground state of a one dimension box of length L. At $t = 0$, the box suddenly expands *symmetrically* to *three* times its size, leaving the wavefunction of the particle undisturbed. Assume the particle was in the ground state before the expansion.

a) Solve the Schrodinger equation and calculate the eigenenergies and eigenfunctions in the box before and *after* the expansion (show all your work). (3 Points)

b) What is the probability of finding the particle in the ground state immediately after the expansion? (4 Points)

c) Compute the wave function of the particle $\psi(x,t)$ for $t \geq 0$. Hint: express your answer as a superposition of eigenstates. (3 Points)

Hint: $\int_{-\pi/2}^{\pi/2} d\theta \, \cos\theta \cos(q\theta) = \frac{2}{1-q^2} \cos\left(q\frac{\pi}{2}\right)$,

$\int_{-\pi/2}^{\pi/2} d\theta \, \cos\theta \sin(q\theta) = 0.$

S-2014

2 L

A Before

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \rightarrow \frac{d^2\psi}{dx^2} = -k^2\psi$$

$$\psi = A\sin kx + B\cancelto{0}{\cos kx}$$

$$kL = n\pi$$

$$\boxed{\psi = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}}$$

$$\boxed{E = \frac{n^2\pi^2\hbar^2}{2mL^2}}$$

After

$$\boxed{\psi = \sqrt{\frac{2}{3L}}\sin\frac{n\pi x}{3L} \qquad E = \frac{n^2\pi^2\hbar^2}{18mL^2}}$$

B $P = C_n^2$

$$\psi_{\text{expanded}} = C_1 \psi_1 \rightarrow C_1 = \int_0^L \sqrt{\frac{2}{3L}} \sin\frac{\pi x}{3L} \sqrt{\frac{2}{L}} \sin\frac{\pi x}{L}\, dx$$

$$= \frac{1}{\sqrt{3}} \frac{2}{L} \int_0^L \sin\left[\frac{\pi x}{3L}\right] \sin\left[\frac{\pi x}{L}\right] dx$$

$\rho \qquad x$

$$= \frac{1}{\sqrt{3}} \frac{2}{L} \left[\frac{\sin\left(\frac{2\pi}{3L}\right)x}{2\left(\frac{2\pi}{3L}\right)} - \frac{\sin\left(\frac{4\pi}{3L}\right)x}{2\left(\frac{4\pi}{3L}\right)} \Bigg|_0^L \right]$$

$$= \frac{1}{\sqrt{3}} \frac{2}{L} \left[\underbrace{\frac{\sin\frac{2\pi}{3}(3L)}{\frac{4\pi}{}}}_{} - \underbrace{\frac{\sin\frac{4\pi}{3}(3L)}{\frac{8\pi}{}}}_{} \right]$$

$$\frac{(2)(3L)\frac{1}{2}\sqrt{3}}{8\pi} + \frac{\frac{3L}{2}\sqrt{3}}{8\pi}$$

$$= \frac{6}{2}\sqrt{3}L + \frac{3}{2}\sqrt{3}L$$

$$= \frac{9}{\cancel{2}}\cancel{\sqrt{3}L}\left(\frac{\cancel{2}}{\cancel{\sqrt{3}L}\, 8\pi}\right)$$

$$= \frac{9}{8\pi}$$

$$\rightarrow \boxed{P = \frac{9^2}{(8\pi)^2}}$$

(c)

at $T = 0 \rightarrow \psi_{expanded} = \psi_1$

$$= \sum_n |\phi_n\rangle \underbrace{\langle \phi_n | \psi_1 \rangle}_{c_n}$$

$$= c_n \phi_n$$

$$\phi_n = \sqrt{\frac{2}{3L}} \sin \frac{n\pi x}{3L}$$

$$\psi_{expanded}(t) = \sum_n c_n \exp\left(-\frac{itH}{\hbar}\right) \phi_n$$

$$= \sum_n c_n \phi_n e^{\frac{-itE_n}{\hbar}}$$

$$E_n = \frac{n^2 \pi^2 \hbar^2}{18 m L^2}$$

F-2014

PROBLEM 4: Two Particles in a 1D Box

Consider two noninteracting particles of mass m inside a 1D box,

$$V(x) = \begin{cases} 0 & , 0 < |x| < a \\ \infty & , \text{otherwise} \end{cases} .$$

Make sure to consider the spin part of the wavefunction in this problem.

a) Let n_1 and n_2 be the quantum numbers of particle 1 and 2 respectively. What are the wavefunctions of the single particle states for the each particle in the box? What are the single particle energies? (2 Points)

b) If the particles are distinguishable what is the two-particle wavefunction that describes the state? What is the energy? Write out explicitly the state (or states) and energies for the ground state and first excited states of the system. (2 Points)

c) If the two particles are identical spin 0 bosons what are the ground state and first excited state wavefunctions and energies? (2 Points)

d) If the two particles are identical spin 1/2 fermions what are the ground state and first excited state wavefunctions and energies? (2 Points)

e) Write down the Hamiltonian for the two particles in the box and show that when the particles are identical H commutes with the exchange operator. (2 Points)

F-2014

4 $V(x) = \begin{cases} 0 & ~~0 < x < a~~ \quad 0 < |x| < a \\ \infty & \text{otherwise} \end{cases}$

A $$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \rightarrow \frac{d^2\psi}{dx^2} = -k^2\psi$$

$$\rightarrow \psi = A\sin kx + \cancel{B\cos kx}$$

0 at $x = \phi$

$$ka = n\pi$$

$$\rightarrow k = \frac{n\pi}{a}$$

$$\rightarrow \boxed{\begin{aligned} \psi_1 &= \sqrt{\frac{2}{a}}\,\sin\left(\frac{n_1\pi}{a}x_1\right) \\ \psi_2 &= \sqrt{\frac{2}{a}}\,\sin\left(\frac{n_2\pi}{a}x_2\right) \end{aligned}}$$

$$k^2 = \frac{n^2\pi^2}{a^2} = \frac{2m}{\hbar^2}E \rightarrow \boxed{\begin{aligned} E_1 &= \frac{n_1^2\pi^2\hbar^2}{2ma^2} \\ E_2 &= \frac{n_2^2\pi^2\hbar^2}{2ma^2} \end{aligned}}$$

B Distinguishable $\rightarrow \boxed{\psi_{n_1,n_2}(x_1,x_2) = \frac{2}{a}\sin\left(\frac{n_1\pi x}{a}\right)\sin\left(\frac{n_2\pi x}{a}\right)}$

$$\rightarrow E = \frac{\pi^2\hbar^2}{2ma^2}(n_1^2 + n_2^2)$$

$gs = E_{11}$

$1s = E_{12} = E_{21}$

(C) Bosons

Need symmetric

$$\Rightarrow \psi = \frac{1}{\sqrt{2}}\left[\frac{2}{a}\sin\frac{n_1\pi x}{a}\sin\frac{n_2\pi x}{a} + \frac{2}{a}\sin\frac{n_1\pi x}{a}\sin\frac{n_2\pi x}{a}\right] \times \chi_s$$

symmetric spin function

antisymmetric

or $\left(\sim - \sim \right)\chi_a$

minus

antisymmetric

$$GS = \psi = \frac{2}{a}\sin\frac{\pi x_1}{a}\sin\frac{\pi x_2}{a}$$

First ~~state~~ excited easy, but realize it is NOT degenerate (you don't know which is which)

(D) Fermions

Need antisymmetric

$\psi_s\chi_a$ or $\psi_a\chi_s$

singlet

triplet

E $-\frac{\hbar^2}{2m}\frac{d^2}{dx_1^2} - \frac{\hbar^2}{2m}\frac{d^2}{dx_2^2} = E\psi$

identical $\Rightarrow \psi = \psi_1 \psi_2$

$\boxed{P\psi_{n_1}(x_1)\psi_{n_2}(x_2) = \psi_{n_1}(x_2)\psi_{n_2}(x_1)}$

↑ exchange operator

key

$[P, H]\psi = PE\psi - HP\psi = \emptyset$

$\rightarrow HP\psi = -\frac{\hbar^2}{2m}\frac{d^2}{dx_1}\psi_{n_1}(x_2)\overset{\psi_{n_2}(x_1)}{} - \frac{\hbar^2}{2m}\frac{d^2}{dx_2}\psi_{n_1}(x_2)\psi_{n_2}(x_1)$

$= -\frac{\hbar^2}{2m}\Big[-\frac{2}{a}\frac{\pi^2 n_2^2}{a^2}$ sin

do this out &

you'll get the

answer

S-2013

Problem 6: Spherical Square Well

Consider a spin 0 particle of mass m moving in a 3D square well, given by the potential

$$V(\vec{r}) = -V_0 \quad 0 \leq |\vec{r}| \leq a_0\ , \qquad V(\vec{r}) = 0 \quad |\vec{r}| > a_0 \qquad (V_0 > 0). \tag{1}$$

In this problem we will only consider the bound states of this well, so that $-V_0 < E < 0$.

(a) [1 pt] Explain why we can write the eigenstates of this potential as

$$\Psi_{k,l,m} = f_{k,l}(r) Y_l{}^m(\theta, \phi). \tag{2}$$

(b) [2 pts] Defining the function $u_{k,l}(r) = r f_{k,l}(r)$, write the radial Schrödinger equation for $u_{k,l}(r)$.

(c) [2 pts] For $l = 0$, write the form for the function $u_{k,0}(r)$ in the regions $0 \leq r \leq a_0$ and $r \geq a_0$. Define any constants that you use.

(d) [3 pts] Using the boundary conditions on the function $u_{k,0}(r)$, derive an equation that gives the bound state energies for the $l = 0$ states. Hint: Considering that $f(r) = u(r)/r$, what is the boundary condition on u as $r \to 0$?

(e) [2 pts] For a fixed radius for the potential, a_0, calculate the minimum depth, $V_0 = V_{min}$, for the potential to have a bound state.

S-2013

6 $V(\vec{r}) = -V_0, \quad \emptyset \leq |\vec{r}| \leq a_0, \qquad V(\vec{r}) = \emptyset \quad |\vec{r}| > a_0$

$-V_0 < E < \emptyset$

A 3D Shrodinger equation

$$\left[\frac{p^2}{2m} + V \right] \psi = E\psi$$

$$p = -i\hbar\vec{\nabla} \Rightarrow p^2 = -\hbar^2\nabla^2$$

$$\Rightarrow \left[-\frac{\hbar^2}{2m}\nabla^2 + V \right] \psi = E\psi$$

if ψ is seperable, i.e. $\psi = R(r)\Phi(\phi)\Theta(\theta)$

~~Then~~ ~~Then~~ The potential only depends. on V,

So angular terms can be written as one function $Y_l^m(\theta,\phi)$

(B) $\frac{1}{r}\frac{\partial^2}{\partial r^2}r + \frac{1}{r^2}\underbrace{\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]}_{-\frac{L^2}{\hbar^2}}$ (unitless) ②

$\rightarrow -\frac{L^2}{\hbar^2}\psi = l(l+1)\psi$

$\rightarrow \left[-\frac{\hbar^2}{2m}\frac{1}{r}\frac{\partial^2}{\partial r^2}r + \frac{L^2}{2mr^2}\right]\psi = E\psi \rightarrow \left[-\frac{\hbar^2}{2m}\frac{1}{r}\frac{\partial^2}{\partial r^2}r + \frac{\hbar^2 l(l+1)}{2mr^2}\right]$

$+V\psi = E\psi$

$\psi = fY$ ← divide $\rightarrow \psi \rightarrow f$

$\rightarrow U = rf \quad \rightarrow f = \frac{U}{r}$

$$-\frac{\hbar^2}{2m}\frac{1}{r}\frac{\partial^2}{\partial r^2}r\psi = \boxed{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2}U + \frac{\hbar^2 l(l+1)U}{2mr^2} + UV = EU}$$

(c) $\ell = \phi \rightarrow -\frac{\hbar^2}{2m}\frac{d^2}{dr^2}u + uV = Eu$

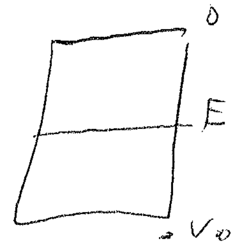
$\underline{0 \le r \le a_0}$

$$\rightarrow -\frac{\hbar^2}{2m}\frac{d^2 u}{dr^2} - V_0 u = Eu$$

E is negative, but $|V_0| > |E|$

$$\rightarrow \frac{d^2 u}{dr^2} = -\frac{2m(E+V_0)}{\hbar^2} u$$

positive #



$$= -\underbrace{\frac{2m(E+V_0)}{\hbar^2}}_{= k^2} u$$

$$\rightarrow u = A\sin kr + B\cos kr$$

$\psi = \phi$ at $r = \phi$

$\rightarrow B = \phi$

$$\rightarrow \boxed{u = A\sin kr}$$

E is $-$

$\underline{r \ge a_0} \rightarrow -\frac{\hbar^2}{2m}\frac{d^2}{dr^2}u = Eu \rightarrow \frac{d^2}{dr^2}u = -\frac{2mE}{\hbar}u$

$= K_2^2 u$

$$u = Ae^{kr} + Be^{-kr}$$

at $r \rightarrow \infty$ $A \rightarrow \phi$

$$\boxed{u = Be^{-kr}}$$

D $u(r) = r f(r)$ as $r \to 0$, $u \to 0$

which is True

$$U_1 = U_2 \text{ at } a_0$$

$$\to A \sin ka_0 = B e^{-k_2 a_0}$$

$$\frac{du_1}{dr} = \frac{du_2}{dr}\Big|_{a_0}$$

$$\to kA\cos ka_0 = -k_2 B e^{-k_2 a_0} \to -\frac{kA\cos ka_0}{k_2} = e^{-k_2 a_0} B$$

$$\to \cancel{A} \sin ka_0 = -\left(\cancel{A}\cos ka_0\right)\frac{k}{k_2}$$

$$\to \boxed{k_2 \tan ka_0 = -k}$$

Can Solve For energies

E This equation needs to hold For bound States

~~at ka_0 = 0 → Tan = 0~~

~~$k = \sqrt{\frac{2m(E+V_0)}{\hbar^2}} = 0$ No~~

~~Find For $ka \to \frac{\pi}{2} n$ →~~

E Continued

$$\tan k a_0 = -\frac{\sqrt{E}}{\sqrt{E+V_0}} = -\sqrt{1+\frac{E}{V_0}}$$

$$\cancel{k a_0 = \sqrt{\frac{2m(E+V_0)}{\hbar^2}}} \qquad \cancel{k_2 a_0 = \sqrt{\frac{2mV_0}{\hbar^2}}}$$

$$\tan k a_0 = -\sqrt{1+\frac{E}{V_0}}$$

~~Finite ... not approach ...~~

$$k a_0 > \frac{\pi}{2} \qquad \boxed{V_0 = \frac{\pi^2 \hbar^2}{8 a_0^2 m}} - E$$

F-2015

Problem 4: Square Well Expansion

Consider a 1D quantum particle of mass m in a square well of width a:

$$\begin{aligned} V(x) &= 0, \quad |x| \leq \frac{a}{2} \\ V(x) &= \infty, \quad |x| > \frac{a}{2} \end{aligned} \tag{1}$$

(a) [1 pt] Write down the energy eigenvalues, E_n, and energy eigenstates, $\psi_n(x)$ for this well. You do not need to derive the states in all detail.

You might want to write the solutions for even and odd values of n separately.

(b) [2 pts] The well expands very suddenly to a new width $L > a$. The expansion is uniform about $x = 0$ so that for the new well, $V(x) = 0$ for $x \leq \frac{L}{2}$.

Assuming the particle is in the state n initially, for the well of width a, write an expression for the probability for the particle to be in the state n' after the expansion, for the well of width L. You don't have to solve for this probability yet, but write this expression in as much detail as you can. Explain why, for half of the possible values of n' this probability is zero.

(c) [2 pts] Consider the case where the particle is initially in the ground state of the well of width a. Show that the probability that the particle will end up in the ground state of the expanded well, of width L is

$$P_{11}\left(\frac{a}{L}\right) = \frac{16}{\pi^2}\frac{a}{L}\frac{\cos^2\left(\frac{\pi}{2}\frac{a}{L}\right)}{\left(1-\left(\frac{a}{L}\right)^2\right)^2} \tag{2}$$

(d) [3 pts] Calculate the limiting functional form for $P_{11}(a/L)$ from part (c) for $L >> a$, $\frac{a}{L} \to 0$. (Calculate the lowest order non-constant term in $\frac{a}{L}$.)

Calculate the limiting functional form for $P_{11}(a/L)$ from part (c) for $\frac{a}{L} \to 1$. It might be helpful to define $\frac{a}{L} = 1 - \delta$. (Calculate the lowest order non-constant term in δ.)

Explain physically why you would predict the two limiting values of the probability.

(e) [2 pts] Consider the case where the particle is initially in the ground state of the well and the potential well is completely removed suddenly ($V(x) = 0$ for all x).

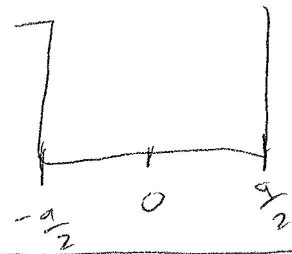
Write down an expression that can be solved for the probability density of the particle having a momentum p after the well disappears. Just as in part (b), provide as much detail as you can, without actually solving for the probability.

Show that this will be very similar to the result in (b) so that calculating this probability would be a simple modification of the results in part (c).

Hint: The fact that $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$ and $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$ might be useful.

(4) $V(x) = \emptyset \ , \quad |x| \le \frac{a}{2}$

$V(x) = \infty \ , \quad |x| > \frac{a}{2}$



(A)
$$\boxed{\begin{aligned} E_n &= \frac{n^2 \hbar^2 \pi^2}{2ma^2} \\ \psi_n &= \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \quad \text{For } n = \text{even} \\ &= \sqrt{\frac{2}{a}} \cos\left(\frac{n\pi x}{a}\right) \quad \text{For } n = \text{odd} \end{aligned}}$$

(B) $\psi_L = \sum_n c_n \psi_a \rightarrow |c_n|^2 = \left| \int_{-a/2}^{a/2} \frac{2}{\sqrt{aL}} \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{n'\pi x}{L}\right) dx \right|^2$

$= P$

IF n = odd, & n' = even

$P = \emptyset$

IF n = even & n' = odd

$P = \emptyset$

↳ $\int \cos \sin = \emptyset$

(c) $n=1$ For ground state

$$C = \int_{-a/2}^{a/2} \frac{2}{\sqrt{aL}} \cos\left(\frac{\pi x}{a}\right) \cos\left(\frac{n\pi}{L}\right)$$

$$= \frac{2}{\sqrt{aL}} \frac{\sin\left(\left[\frac{\pi}{a} - \frac{\pi}{L}\right]x\right)}{2\left(\frac{\pi}{a}-\frac{\pi}{L}\right)} + \frac{\sin\left(\frac{\pi}{a}+\frac{\pi}{L}\right)x}{2\left(\frac{\pi}{a}+\frac{\pi}{L}\right)}\Bigg|_{-a/2}^{a/2}$$

$$= \frac{2}{\sqrt{aL}}\left[\frac{\sin\left(\frac{\pi}{2} - \frac{a\pi}{2L}\right)}{2\left(\frac{\pi}{a}-\frac{\pi}{L}\right)} + \frac{\sin\left(\frac{\pi}{2} + \frac{a\pi}{2L}\right)}{2\left(\frac{\pi}{a}+\frac{\pi}{L}\right)}\right.$$

$$\left. - \frac{\sin\left(\frac{a\pi}{2L} - \frac{\pi}{2}\right)}{2\left(\frac{\pi}{a}-\frac{\pi}{L}\right)} - \frac{\sin\left(+\frac{a\pi}{2L} + \frac{\pi}{2}\right)}{2\left(\frac{\pi}{a}+\frac{\pi}{L}\right)}\right]$$

$$= \frac{2}{\sqrt{aL}}\left[\frac{1}{2\pi\left(\frac{1}{a}-\frac{1}{L}\right)}\left(\sin A\cos B - \cancel{\cos A \sin B} + \sin A \cos B + \cancel{\cos A \sin B}\right) - \left(\cancel{\sin B \cos A} - \cos B \sin A - \cancel{\sin B \cos A} - \cos B \sin A\right)\frac{1}{2\pi\left(\frac{1}{a}+\frac{1}{L}\right)}\right.$$

$$= \frac{2}{\sqrt{aL}}\left[\frac{2\sin A\cos B}{2\pi\left(\frac{1}{a}-\frac{1}{L}\right)} + \frac{2\cos B \sin A}{2\pi\left(\frac{1}{a}+\frac{1}{L}\right)}\right]$$

$$= \frac{2}{\pi\sqrt{aL}}$$

Doesn't work

$\alpha = \pi \qquad B = \frac{\pi a}{L}$

(3)

(c) Continued

$$= \frac{2}{\sqrt{aL}}\left[\frac{\sin\left(\frac{\pi}{2}-\frac{a\pi}{2L}\right)}{\underbrace{\frac{2\pi}{a}-\frac{2\pi}{L}}_{A}} - \frac{\sin\left(\frac{a\pi}{2L}-\frac{\pi}{2}\right)}{\underbrace{\frac{2\pi}{a}+\frac{2\pi}{L}}_{B}}\right]$$

$$= \frac{2}{\sqrt{aL}}\left[\frac{\sin\frac{\pi}{2}\cos\frac{a\pi}{2L} - \cancel{\cos\frac{\pi}{2}}}{A} - \cancel{\sin\frac{a\pi}{2L}\cos\frac{\pi}{2}} + \frac{\sin\frac{\pi}{2}\cos\frac{a\pi}{2L}}{B}\right]$$

$AB =$

$\frac{\pi^2}{a^2} + \frac{\pi^2}{L^2}$

$\pi^2\left[\frac{1}{a^2} - \frac{1}{L^2}\right]$

$$= \frac{2}{\sqrt{aL}}\left[\frac{\cos\frac{a\pi}{2L}}{A} + \frac{\cos\left(\frac{a\pi}{2L}\right)}{B}\right]$$

$$= \frac{2}{\sqrt{aL}}\left[\cos\frac{a\pi}{2L}\left[\frac{\frac{2\pi}{a}+\cancel{\frac{\pi}{L}}+\frac{2\pi}{a}-\cancel{\frac{\pi}{L}}}{AB}\right]\right]$$

$$= \frac{4\pi}{a}\frac{2}{\sqrt{aL}}\;\cos\frac{a\pi}{2L}$$

$$= \frac{4}{\pi a}\frac{}{\sqrt{aL}}\;\frac{\cos\frac{a\pi}{2L}}{\left(\frac{1}{a^2}-\frac{1}{L^2}\right)} \Rightarrow \frac{4a}{\pi\sqrt{aL}} \times \frac{\cos\frac{a\pi}{2L}}{\left(1-\frac{a^2}{L^2}\right)}$$

$$\boxed{P = \frac{16a}{\pi^2 L}\frac{\cos^2\left(\frac{\pi}{2}\frac{a}{L}\right)}{\left(1-\left(\frac{a}{L}\right)^2\right)^2}}$$

D $\dfrac{a}{L} \to 0$

$$P_{11} = \frac{2}{\pi}\frac{16}{\pi^2}\, x\, \frac{\cos^2(x)}{1} \quad \text{where} \quad x = \frac{\pi a}{2L}$$

expand about $x = \phi$.

$$P_{11}\left(\frac{a}{L}\right) = \cancel{\phi} + \frac{2}{\pi}\frac{16}{\pi^2}\left[\underbrace{\cos^2(x)}_{=1} - 2x\cos(x)\sin(x)\Big|_{x=\phi}\right]$$

$=$

$\dfrac{a}{L} \to 1$

$$P_{11} = \frac{16}{\pi^2}\cos^2\left(\frac{\pi}{2}\right)$$

Not really sure here

E ?

F-2008

Problem 4: The Infinite Square Well: (10 Points)

A single particle is in a one dimensional infinite well whose potential $V(x)$ is given by:

$$V(x) = \begin{cases} 0, & \text{if -L} \leq \text{x} \leq \text{L} \\ \infty, & \text{otherwise} \end{cases}$$

a. Find the allowed energies (E_n) and the normalized eigenfunctions ($\Phi_n(x)$) to Schrodinger's Equation for this potential. Show all your work. **(2 Points)**

Assume the particle is in the ground state and a position measurement of the particle is made. Since any measuring apparatus has a finite resolution, the exact location of the particle cannot be detemined. We therefore only know the location of the particle within some resolution ϵ. After making the position measurement the wave function $\Psi(x)$ is:

$$\Psi(x) = \frac{1}{\sqrt{\epsilon}} \quad |x| < \frac{\epsilon}{2}$$

$$\Psi(x) = 0 \quad |x| > \frac{\epsilon}{2}$$

b. What is the probability that the particle has energy E_n? **(2 Points)**

c. If $\epsilon = 2L$, we know that the particle is somewhere in the box. What is the probability that the particle is in the ground state? **(1 Point)**

d. Before the position measurement we knew the particle was in the box and in the ground state. If after the measurement and $\epsilon = 2L$ we know that the particle is in the box, why is probability that the particle is in the ground state not 1? **(1 Point)**

For parts e), f) and g) now assume that the particle is in the potential $V(x)$

$$V(x) = \begin{cases} 0, & \text{if -L} \leq \text{x} \leq \text{L} \\ \infty, & \text{otherwise} \end{cases}$$

and in the ground state. The position of the walls are quickly increased to

$$V(x) = \begin{cases} 0, & \text{if -L'} \leq \text{x} \leq \text{L'} \\ \infty, & \text{otherwise} \end{cases}$$

where $|L'| > |L|$

e. After the expansion, what is the probability that the particle has energy E_n? You do not need to solve the integral. **(2 Points)**

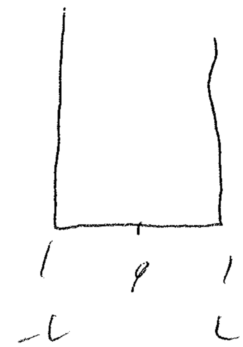
f. Before the walls of the potential are increased, does $|\Psi(x,t)|^2$ (where $\Psi(x,t)$ is a solution to Schrodinger's equation before the expansion) have any time dependance? Explain **(1 Point)**

g. After the position of the walls are increased to L', does $|\Psi(x,t)|^2$ (where $\Psi(x,t)$ is a solution to Schrodinger's equation after the expansion) have any time dependance? Explain. **(1 Point)**

F-2008

1

4 $V = \begin{cases} 0 & -L \le x \le L \\ \infty & \text{otherwise} \end{cases}$



A $\dfrac{-\hbar^2}{2m}\dfrac{d^2\psi}{dx} = E\psi$

$$\rightarrow \frac{d^2}{dx^2}\psi = \underbrace{\frac{-2mE}{\hbar^2}}_{-k^2}\psi$$

$$\psi = A\sin kx + B\cos kx$$

$\psi = \varnothing$ at $\pm L$ $\rightarrow$ shift the well to $0 \rightarrow 2L$

$\rightarrow \psi = \varnothing$ at $\varnothing$ & $2L$ $\rightarrow B = \varnothing \rightarrow \psi = A\sin kx$

$kx = n\pi$

$\rightarrow 2kL = n\pi \rightarrow k = \dfrac{n\pi}{2L}$

$\rightarrow \psi = A\sin\left(\dfrac{n\pi x}{2L}\right)$

$$\rightarrow 1 = A^2\int_0^{2L} \sin^2\left(\frac{n\pi x}{2L}\right)dx$$

$$= A^2\left[\frac{x}{2} - \frac{\sin\frac{n\pi x}{L}}{4\left(\frac{n\pi}{2L}\right)}\right]\Bigg|_{\varnothing}^{2L} \rightarrow A^2\left[(L - \varnothing) - (0 - \varnothing)\right]$$

$\rightarrow A = \dfrac{1}{\sqrt{L}}$

A Continued

$$\psi = \sqrt{\frac{1}{L}} \sin\left(\frac{n\pi x}{2L}\right)$$

Shift back $\rightarrow$ $\psi = \sqrt{\frac{1}{L}} \sin\left(\frac{n\pi}{2L}(x-L)\right)$

$$\sin\left(\frac{n\pi x}{2L} - \frac{n\pi}{2}\right) = \sin\left(\frac{n\pi x}{2L}\right)\cos\left(\frac{n\pi}{2}\right) - \cos\left(\frac{n\pi x}{2L}\right)\sin\left(\frac{n\pi}{2}\right)$$

$$\boxed{\begin{aligned} &\text{If } n = \text{even} \rightarrow \psi = \sqrt{\frac{1}{L}} \sin\left(\frac{n\pi x}{2L}\right) \\ &\text{If } n = \text{odd} \rightarrow \psi = \sqrt{\frac{1}{L}} \cos\left(\frac{n\pi x}{2L}\right) \end{aligned}}$$

$$k = \frac{n\pi}{2L} = \sqrt{\frac{2mE}{\hbar^2}} \rightarrow \frac{n^2\pi^2}{4L^2} = \frac{2mE}{\hbar^2}$$

$$\rightarrow \boxed{E = \frac{n^2\pi^2\hbar^2}{8mL^2}}$$

B $P = \left[\int_{-\epsilon/2}^{\epsilon/2} \sqrt{\frac{1}{L}} \sin\left(\frac{n\pi x}{2L}\right)\frac{1}{\sqrt{\epsilon}}\,dx\right]^2$

$$= \frac{1}{L\epsilon}\left[\int_{-\epsilon/2}^{\epsilon/2} \sin\left(\frac{n\pi x}{2L}\right)\right]^2 = \frac{1}{L\epsilon}\left[\left[-\frac{\cos\left(\frac{n\pi x}{2L}\right)}{n\pi/2L}\right]\Bigg|_{-\epsilon/2}^{\epsilon/2}\right]^2$$

$$= \frac{1}{L\epsilon}\left[\frac{n\pi}{2L}\left[-\cos\left(\frac{n\pi\epsilon}{4L}\right) + \cos\left(\frac{-n\pi\epsilon}{4L}\right)\right]\right]^2$$

$\cos(-x) = \cos x$

$$= \boxed{\emptyset \text{ if } n = \text{even}}$$

$$P = \frac{1}{L\epsilon}\left[\int_{-\epsilon/2}^{\epsilon/2} \cos\left(\frac{n\pi t}{2L}\right)\right]^2$$

$$= \frac{1}{L\epsilon}\left[\frac{\sin\left(\frac{n\pi x}{2L}\right)}{n\pi/2L}\right]^2\Bigg|_{-\epsilon/2}^{\epsilon/2}$$

$$= \frac{4L}{n^2\pi^2}\left[\sin\left(\frac{n\pi t}{2L}\right) - \sin\left(-\frac{n\pi t}{2L}\right)\right]^2$$

$$= \boxed{\frac{16L}{n^2\pi^2}\sin^2\left(\frac{n\pi t}{4L}\right) \quad \text{IF } n = \text{odd}}$$

C

$$P = \left[\int_{-L}^{L} \frac{1}{\sqrt{L}}\frac{1}{\sqrt{2L}} \cos\left(\frac{\pi x}{2L}\right) dx\right]^2$$

$$= \frac{1}{2L^2}\left[\frac{\sin \pi x/2L}{\pi/2L}\right]^2\Bigg|_{-L}^{L} = \frac{1}{2L^2}\frac{4L^2}{\pi^2}\left[\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)\right]^2$$

$$= \frac{16}{2\pi^2} = \boxed{\frac{8}{\pi^2} = P}$$

(D) Because $\psi = \sum c_n \psi_n$?

(E) $\pm L \rightarrow \quad \psi = \frac{1}{\sqrt{L}} \cos\left(\frac{\pi x}{2L}\right)$

$\pm L' \rightarrow \quad \psi = \frac{1}{\sqrt{L'}} \cos\left(\frac{n\pi x}{2L'}\right)$

or $= \frac{1}{\sqrt{L'}} \sin\left(\frac{n\pi x}{2L'}\right)$

$$\rightarrow \quad P = \left[\int_{-L}^{L} \frac{1}{\sqrt{L'}} \frac{1}{\sqrt{L}} \cos\left(\frac{\pi x}{2L}\right) \cos\left(\frac{n\pi x}{2L'}\right) dx \right]^2$$

$$= \frac{1}{L'L} \left[\int_{-L}^{L} \cos\frac{\pi x}{2L} \cos\left(\frac{n\pi x}{2L'}\right) dx \right]^2$$

Same for Sin, one will be $\emptyset$

(F) No, Stationary state

(E) yes, Not Stationary State

F-2008

Problem 1: A 3-D Spherical Well(10 Points)

For this problem, consider a particle of mass m in a three-dimensional spherical potential well, $V(r)$, given as,

$$V = 0 \quad r \leq a/2$$
$$V = W \quad r > a/2.$$

with $W > 0$.

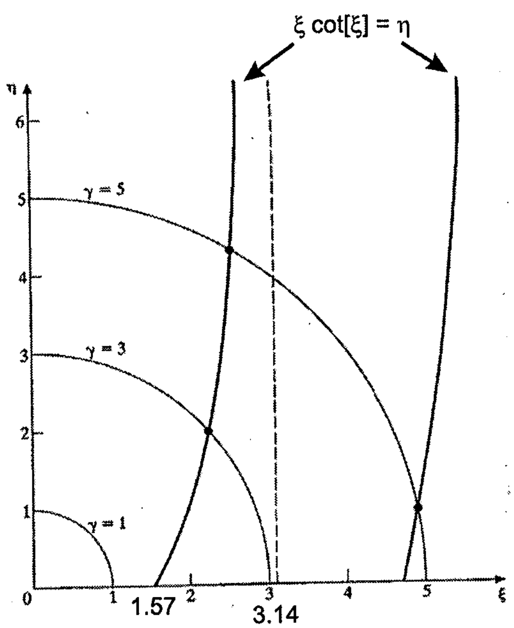
All of the following questions refer to the *zero angular momentum states* of the potential.

a. Find the form of the wave functions (i.e without matching boundary conditions), ψ(r), for this potential for an energy, E, less than the well depth, W.**(3 Points)**

b. The wave function for the one-dimensional symmetric square well has both a cosine and sine solution. Is this true for the three-dimensional spherical well potential? Explain. **(1 Point)**

c. If the potential well was infinitely deep, W$\rightarrow\infty$, what are the energies? Derive the expression using the wave functions you calculated in (a).**(2 Points)**

d. Derive the transcendental equation that determines the energies for the finite spherical well. **(2 Points)**



e. Is there always a bound state in the finite three-dimensional potential? Justify your answer to receive any credit. How does this compare to the one-dimensional finite square well? Use the figure. $\gamma^2 = \eta^2 + \xi^2$, where $\xi = \sqrt{2mE}a/2\hbar$ and $\eta = \sqrt{2m(W-E)}a/2\hbar$.**(2 Points)**

F-2008

1. $V = \phi \quad r \le a/2$

$V = W \quad r > a/2$

(A) $$\nabla^2 \psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) + \underbrace{\frac{1}{r^2}\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \psi}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 \psi}{\partial \phi^2}}_{\frac{L^2}{r^2 \hbar^2}}$$

in order to make dimensionless

$$\rightarrow -\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) - \underbrace{\frac{L^2}{2mr^2}\psi}_{-\frac{\ell(\ell+1)\hbar^2}{2mr^2}\psi} + V\psi = E\psi$$

$\ell = \phi$

$$\rightarrow -\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) + V\psi = E\psi$$

$\rightarrow \psi = R(r)\Theta(\theta)\Phi(\phi)$

No θ, ϕ dependence

Θ & Φ will cancel

$$\rightarrow -\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial R}{\partial r}\right) + VR = ER$$

$\rightarrow u = rR \rightarrow R = \frac{u}{r}$

1 Continued

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\left[\frac{u}{r}\right]\right)$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r} r^2\left[-\frac{u}{r^2}+\frac{1}{r}\frac{du}{dr}\right] = \frac{1}{r^2}\frac{\partial}{\partial r}\left[-u + r\frac{du}{dr}\right]$$

$$= \frac{1}{r^2}\left[\cancel{-\frac{du}{dr}+\frac{du}{dr}} + \frac{r\,d^2u}{dr^2}\right]$$

$$= \frac{1}{r}\frac{d^2u}{dr^2}$$

$$\to -\frac{\hbar^2}{2m}\frac{1}{r}\frac{d^2u}{dr^2} + \frac{Vu}{r} = \frac{Eu}{r} \to -\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + Vu = Eu$$

$$\to \frac{d^2u}{dr^2} - \frac{2mW}{\hbar^2}u = -\frac{2mE}{\hbar^2}u$$

$$\cancel{\to \frac{d^2u}{dr^2} = \frac{2m}{\hbar^2}(W-E)u \to u = Ae^{kr} + Be^{-kr}}$$

$$\cancel{W > E}$$

$$\cancel{\text{where } k^2 = \frac{2m(W-E)}{\hbar^2}}$$

$$\cancel{\to R = \frac{A}{r}e^{kr} + \frac{B}{r}e^{-kr}}$$

$$\cancel{\text{as } r \to \infty \to A \text{ must be } 0}$$

$$\cancel{\to \boxed{R = \frac{B}{r}e^{-kr}}}$$

A Continued

$r \le a/2 \Rightarrow \frac{d^2U}{dr^2} = \underbrace{-\frac{2mE}{\hbar^2}}_{-k^2} U$

$R = \frac{A}{r}\sin kr + \frac{B}{r}\cos kr \Rightarrow \frac{d^2U}{dr^2} = -k^2 U$

$\Rightarrow U = Ae^{ikr} + Be^{-ikr}$

$\Rightarrow \boxed{R = \frac{A}{r}e^{ikr} + \frac{B}{r}e^{-ikr}}$

$r \to 0 \Rightarrow \cos$ term $\to \infty$

$\Rightarrow \boxed{R = \frac{A}{r}\sin kr}$

This is solution

$r > a/2$

$W > E$

$\frac{d^2U}{dr^2} = \underbrace{\frac{2m(W-E)}{\hbar^2}}_{p^2} U$

$\rightarrow U = Ae^{kp} + Be^{-kp}$

$\Rightarrow R = \frac{A}{r}e^{kp} + \frac{B}{r}e^{-kp}$

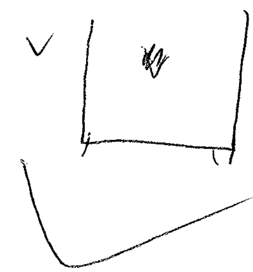
$r \to \infty, \quad A = \phi$

$\boxed{R = \frac{B}{r}e^{-pr}}$

B only depends on sin
→ in symmetric well, odd & ~~[illegible]~~ Even
solutions centered on $\emptyset$.
→ here, treating r in 1D is an asymmetric
well that must go to $\emptyset$ at boundaries

C $R_{r \le a/2} = \frac{A}{r} \sin kr$

$R_{r > a/2} = \frac{B}{r} e^{-\kappa r}$

Think about infinite well, V  V

$\psi = \emptyset$ at side well

→ $R_{r > a/2} = \emptyset$

→ at boundary $r = \frac{a}{2}$ → $\frac{2A}{a} \sin kr = \emptyset$

$\frac{ka}{2} = n\pi$

→ $k = \frac{2n\pi}{a}$

→ $k = \sqrt{\frac{2mE}{\hbar^2}} = \frac{2n\pi}{a}$

→ $\frac{2mE}{\hbar^2} = \frac{4n^2\pi^2}{a^2}$

$$\boxed{\frac{2n^2\pi^2\hbar^2}{ma^2} = E}$$

D $R_{r<\frac{a}{2}} = \frac{A}{r}\sin kr$

$R_{r>\frac{a}{2}} = \frac{B}{r}e^{-pr}$

$\rightarrow \frac{A}{\cancel{r}}\sin kr = \frac{B}{\cancel{r}}e^{-pr} \rightarrow A\sin kr = Be^{-pr}\Big|_{\frac{a}{2}=r}$

$\frac{d}{dr} \quad -\frac{A}{r^2}\sin kr + \frac{Ak}{r}\cos kr = -\frac{B}{r^2}e^{-pr} - \frac{Bp}{r}e^{-pr}\Big|_{\frac{a}{2}=r}$

$\rightarrow A\sin\frac{ka}{2} = Be^{-pa/2}$

$\frac{d}{dr} \quad -\frac{4A}{a^2}\sin\frac{ak}{2} + \frac{2Ak}{a}\cos\frac{ak}{2} = -\frac{4B}{a^2}e^{-pa/2} - \frac{2Bp}{a}e^{-pa/2}$

$\rightarrow A = \frac{Be^{-pa/2}}{\sin ka/2}$

$\rightarrow \cancel{-\frac{4B}{a^2}} + \frac{2Bk}{a}\cot\frac{ak}{2} = \cancel{-\frac{4B}{a^2}} - \frac{2Bp}{a}$

$\rightarrow \frac{2\cancel{B}k}{a}\cot\frac{ak}{2} = -\frac{2\cancel{B}p}{a}$

$\rightarrow \frac{2k}{\cancel{a}}\cot\frac{ak}{2} = \frac{2p}{\cancel{a}}$

$\rightarrow \boxed{\cot\frac{ak}{2} = \frac{-2p}{2k}}$

E Finish

F-2013

Problem 1: 1D Square Wells

(a) [1 pt] Consider an electron confined to an infinitely deep 1D well with walls at $x = 0$ and $x = L$. In the ground state, the electron has an energy of 2.5 eV (the bottom of the well is defined as $V = 0$). What is the width of the well?

(b) [1 pt] A proton is confined to an infinite 1D square well of width 10 fm. What is the wavelength (or frequency) of a photon emitted when the proton undergoes a transition from the first excited state to the ground state of the well?

(c) [2 pt] Sketch the probability density as a function of x for the first 3 energy eigenstates for an electron in an infinite well of width L. Describe qualitatively (or draw) how the probability densities for these states will differ (from the infinite well case) for a square well with an infinite potential barrier at $x = 0$ and a finite potential barrier at $x = L$.

(d) [2 pt] Consider an electron in the nth energy eigenstate of an infinitely deep well with walls at $x = 0$ and $x = L$. Calculate the probability that the electron will be measured between $x = 0$ and $x = \epsilon\ L$, with $0 < \epsilon < 1$. Your answer should be a function of both n and ϵ.

Give a physical explanation for your solution as $n \to \infty$.

(e) [2 pt] The electron is in the ground state of the infinite well when the wall at $x = L$ is very suddenly moved to $x = 2L$. What is the probability that the electron will be found in the ground state of the expanded box?

(f) [1 pt] What energy eigenstate in the expanded box will have the highest probability of being occupied by the electron? What is this probability? Hint: You should be able to determine this result without doing an integral, but you should explain your answer.

(g) [1 pt] Suppose the electron is in the ground state of the infinitely deep well when the walls are suddenly removed completely. Write down an expression for the probability distribution for the momentum of the freed electron. Setup but do not solve the integral.

F-2013

1

1 A $-\frac{\hbar^2}{2m}$ E

$k = \sqrt{\frac{E\,2m}{\hbar^2}}$

$k = \frac{n\pi}{L} \rightarrow \frac{n^2\pi^2\hbar^2}{2L^2 m} = E$

$= 2.5\,eV$

$\rightarrow m_e = 0.511\,MeV \left| \frac{10^6\,eV}{1\,MeV} \right|$

$= 511000\,eV$

natural units, $\hbar \rightarrow 1$

$\rightarrow \boxed{L = \sqrt{\frac{\pi^2\hbar^2}{2mE}} \quad \text{plug in}}$

B $L = 10\,fm$

$\Delta E = \hbar\omega$

$\Delta E = \frac{\pi^2\hbar^2}{2L^2 m}(n_f^2 - n_i^2)$

$= \frac{\pi^2\hbar^2}{2L^2 m}(4-1) = \frac{3\pi^2\hbar^2}{2L^2 m\hbar} = \omega$

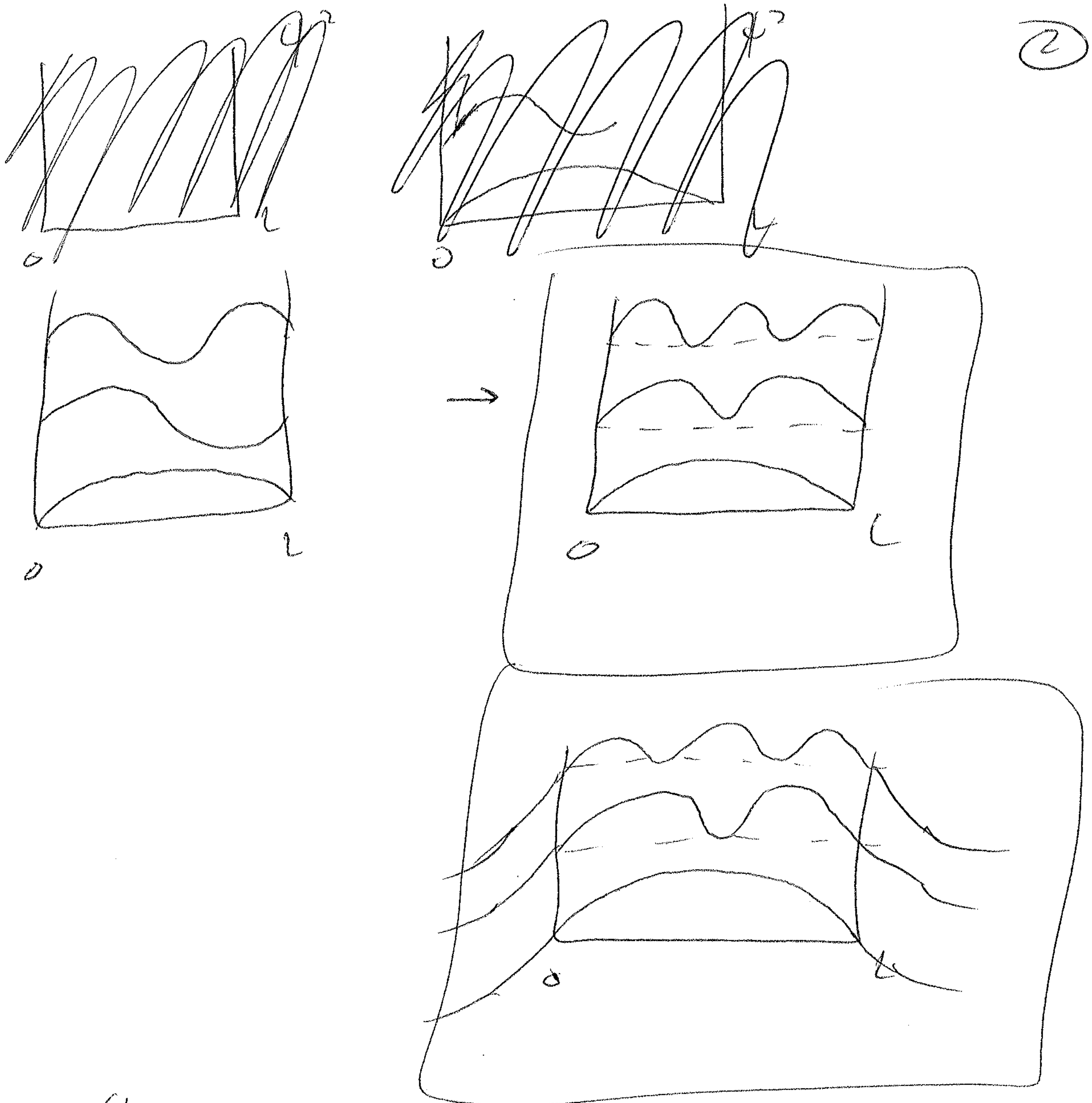
$\rightarrow \omega = 2\pi f \rightarrow c = f\lambda$

$\rightarrow \frac{3\pi^2\hbar}{2L^2 m} = \frac{2\pi c}{\lambda}$

$\rightarrow$ natural units, $c \rightarrow 1$, $\hbar \rightarrow 1$

(C)

(2)



(D) $P = \int_0^{\epsilon L} \psi^2(x)\,dx \quad \rightarrow \quad \psi(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$

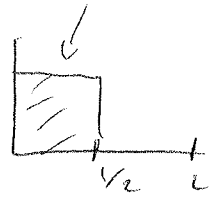
$$= \int_0^{\epsilon L} \frac{2}{L} \sin^2 \frac{n\pi x}{L}\,dx = \left(\frac{x}{2} - \frac{\sin\left(\frac{2n\pi x}{L}\right)}{\frac{4n\pi x}{L}} \right) \frac{2}{L} \Bigg|_0^{\epsilon L}$$

$$= \left(\frac{\epsilon L}{2} - \frac{\sin(2n\pi\epsilon)}{4n\pi\epsilon} \right) \frac{2}{L}$$

Probability $= \boxed{\epsilon - \frac{\sin(2n\pi\epsilon)}{2n\pi\epsilon L} = P}$

$n \to \infty \Rightarrow P = \epsilon$

If $\epsilon = \frac{1}{2}$, $P = \frac{1}{2}$



E $\psi_1 = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$ ← width L

$\phi_1 = \sqrt{\frac{1}{L}} \sin\left(\frac{\pi x}{2L}\right)$ ← width $2L$

$\phi = \sum C_n \psi_n \rightarrow \phi = C_1\psi_1 \rightarrow \int_0^L \phi\psi\,dx = C_1 \underbrace{\int_0^L \psi_1^2\,dx}_{\substack{P \text{ that} \\ \text{it is} \\ \text{anywhere} \\ =1}}$

↑ This can be a linear combination of any complete set, ψ_n is a complete set $\psi_1, \psi_2 \ldots$

$\rightarrow C_1 = \int_0^L \phi\psi_1\,dx$

$= \frac{\sqrt{2}}{L}\int_0^L \sin\left(\frac{\pi x}{L}\right)\left(\sin\frac{\pi x}{2L}\right) dx$

$p = \frac{2\pi}{2L} \quad q = \frac{\pi}{2L}$

$= \left.\frac{\sin \frac{\pi}{2L}x}{\frac{\pi}{L}} - \frac{\sin \frac{3\pi x}{2L}}{\frac{3\pi}{L}}\right|_0^L$

$= \cancel{\frac{2L}{3\pi}} + \frac{L}{3\pi} = \frac{4\cancel{L}}{3\pi}\frac{\sqrt{2}}{\cancel{L}}$

$C_n = \frac{4\sqrt{2}}{3\pi}$

$\rightarrow P = \boxed{C_\phi^2 = \frac{(16)(2)}{9\pi^2}}$

$\boxed{n=2}$ $\sin\frac{2\pi x}{L}$

F $C_n = \frac{\sqrt{2}}{2}\int_0^L \sin\frac{\pi x}{L} \sin\frac{n\pi x}{2L}\,dx$

G Need to do ~~more~~ more work on free particles

Problem 3: 2-d potential (10 points)

A particle of mass m is confined by two impenetrable parallel walls at $x = \pm a$ to move on a two-dimensional strip defined by

$$-a < x < a$$
$$-\infty < y < \infty$$

The wave function for this system can be expressed as the product of two functions: one that depends only on the spatial co-ordinates (x and y), and one that depends only on time t.

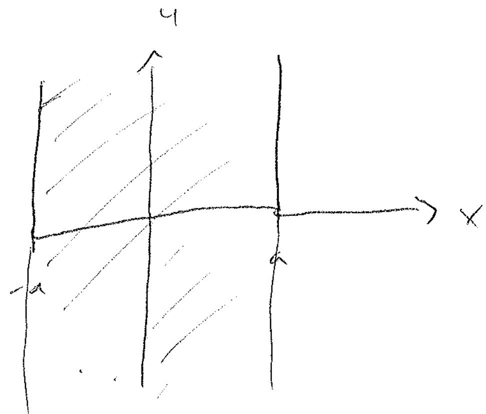
a) Use the separation of variables technique to find the time dependent function. (2 points)

b) The part of the wave function that depends only on spatial co-ordinates can be expressed as the product of two functions: one that depends only on x and one that depends only on y. Use the separation of variables technique to find these two functions. (3 points)

c) What is the minimum energy of the particle that measurement can yield? (2 points)

d) Suppose that two additional walls are inserted at $y = \pm a$. Can a measurement of the particle's energy yield the value $3\pi^2\hbar^2/8ma^2$ Explain your answer. (3 points)

③



(A) $\tilde{H}\psi(x,y,t) = i\hbar \frac{\partial \psi}{\partial t}$

$\rightarrow \left[-\frac{\hbar^2}{2m}\nabla^2 + V(x,y)\right]\psi(x,y,t) = i\hbar\frac{\partial\psi}{\partial t}$

$\rightarrow \psi(x,y,t) = \phi(x,y)\,f(t)$

$\rightarrow \left[-\frac{\hbar^2}{2m}\nabla^2 + V(x,y)\right]\frac{\phi(x,y)f(t)}{\phi(x,y)f(t)} = \frac{1}{f(t)}\frac{i\hbar\,\partial f(t)}{\partial t}$

$\rightarrow \underbrace{-\frac{\hbar^2}{2m}\frac{1}{\phi(x,y)}\nabla^2\phi(x,y) + V(x,y)}_{=E} = \underbrace{\frac{1}{f(t)}\, i\hbar\frac{\partial f}{\partial t}}_{=E}$

$\rightarrow E$ is a Constant

$\rightarrow i\hbar\frac{\partial f}{\partial t} = Ef \rightarrow \frac{\partial f}{\partial t} = \frac{E}{i\hbar}f$

$\cancel{f = \exp\left(-\frac{iEt}{\hbar}\right)}$

$\rightarrow \int\frac{df}{f} = \int_{t_0}^{t}\frac{E}{i\hbar}\,dt$

$\rightarrow \ln f = \frac{E}{i\hbar}(t-t_0)$

$\rightarrow \boxed{f = \exp\left(\frac{-iE(t-T_0)}{\hbar}\right)}$

(3) $\phi(x,y) = X(x)\, Y(y)$ $\qquad E = E_x + E_y$ $\qquad$ (2)

$V(x,y) = V(x) + V(y)$

$$\rightarrow \frac{-\hbar^2}{2m} \frac{1}{X Y}\left[\frac{\partial^2}{\partial x^2} Y X + \frac{\partial^2}{\partial y^2} X Y\right] + V(x) + V(y) = E_x + E_y$$

$$\rightarrow \left[-\frac{\hbar^2}{2m}\frac{1}{X}\frac{\partial^2}{\partial x^2} X + V(x)\right] + \left[-\frac{\hbar^2}{2m}\frac{1}{Y}\frac{\partial^2}{\partial y^2} Y + V(y)\right] = E$$

$\rightarrow$ Since $E = E_x + E_y$ $\qquad = E_x$

$= E_y$

$$\rightarrow -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} X + \cancel{V(x)}^{\to 0} X = E_x X$$

No actual potential

$$\rightarrow \frac{\partial^2}{\partial x^2} X = -\underbrace{\frac{2m}{\hbar^2} E_x}_{k_x^2} X$$

$$X = A e^{-ikx} + B e^{ikx}$$

$$= A \sin k_x x + \cancel{B \cos k_x x}^{\to 0}$$

$\rightarrow$ For a symmetric potential

$$X = A \sin k_x x$$

$$X(0) = 0 = A \sin k_x x = X(2a)$$

$$k_x(2a) = n\pi \rightarrow k_x = \frac{n\pi}{2a}$$

$$\rightarrow X(x) = \sqrt{\frac{1}{a}} \sin\left(\frac{n\pi}{2a} x\right)$$

B Continued

$$X(x) = \begin{cases} \sqrt{\frac{1}{a}} \sin \frac{n\pi}{2a} x & \text{For } n = \text{even} \\ \sqrt{\frac{1}{a}} \cos \frac{n\pi}{2a} x & \text{For } n = \text{odd} \end{cases}$$

$$\rightarrow \frac{\partial^2 Y}{\partial y^2} = \underbrace{-\frac{2m}{\hbar^2} E_y}_{k_y} Y$$

$$\rightarrow \boxed{Y(y) = A e^{-ik_y y} + B e^{ik_y y}}$$

$\boxed{?}$

Can just oscillate in y direction?

C $k_x = \frac{n\pi}{2a} = \sqrt{\frac{2mE}{\hbar^2}} \rightarrow \frac{n^2\pi^2}{4a^2} = \frac{2mE}{\hbar^2} \rightarrow \boxed{E = \frac{n^2\pi^2\hbar^2}{8ma^2}}$

? depends on what happens with y

D here $E = \frac{\pi^2\hbar^2}{8ma^2}(n_x^2 + n_y^2)$

$\boxed{\text{No way to make } n_x^2 + n_y^2 = 3}$

F-2006

PROBLEM 1: Infinite Square Well

For a particle moving in an infinite square well of width $2a$, the potential energy is

$$V(x) = \begin{cases} 0 & \text{for } |x| < a,\ a > 0, \text{ and} \\ \infty & \text{for } |x| \geq a\ . \end{cases}$$

Its wave function at time $t = 0$ is

$$\psi(x, 0) = \frac{1}{\sqrt{2}}\left[u_1(x) + u_2(x)\right]$$

where $u_1(x)$ and $u_2(x)$ are the normalized ground state and first excited state wave functions respectively and they are orthogonal to each other.

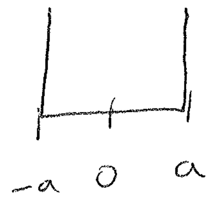
(a) Determine the energy eigenvalues E_1 and E_2 then find the wave function $\psi(x,t)$ as a function of time. *(2 points)*

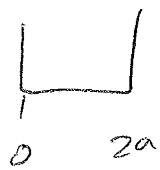
(b) Find the expectation value of its kinetic energy $\langle T \rangle$ with $\psi(x,t)$. *(3 points)*

(c) What is the expectation value of its total energy ($\langle E \rangle$)? Explain the relationship between this result and what you found in Part (b). *(2 points)*

(d) Evaluate ΔX in this state with $\psi(x,t)$. *(3 points)*

F-2006

1 

A Assume 

Then Translate it

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \cancel{\phi} = E\psi \rightarrow \frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi$$

$$= -k^2\psi$$

$kx = n\pi$

$$\psi = A\sin kx + B\cos kx$$

$$\rightarrow \psi = A\sin\frac{n\pi}{2a}x$$

$$\rightarrow 1 = \int_0^{2a} A^2\sin^2\frac{n\pi}{2a}x\,dx = \frac{A^2}{2}\int_0^{2a}dx - \cancel{\frac{A^2}{2}\int_0^{2a}\cos\frac{n\pi}{2a}x} \rightarrow \phi$$

$$= 1 = A^2 a \rightarrow A = \frac{1}{\sqrt{a}}$$

$$\rightarrow \psi = \frac{1}{\sqrt{a}}\sin\left(\underbrace{\frac{n\pi}{2a}\left[x+a\right]}_{x=\pm a \rightarrow \sin = \phi}\right) = \frac{1}{\sqrt{a}}\left[\sin\frac{n\pi}{2a}x\cos\frac{n\pi}{2} + \cos\frac{n\pi}{2a}x\sin\frac{n\pi}{2}\right]$$

$$\boxed{\begin{array}{l}\text{when } n \text{ is even} \\ \frac{1}{\sqrt{a}}\sin\frac{n\pi}{2a}x \\ \text{when } n \text{ is odd} \\ \frac{1}{\sqrt{a}}\cos\frac{n\pi}{2a}x\end{array}}$$

(A) Continued

(2)

$$E_n = 2ka = n\pi \rightarrow \frac{\sqrt{2mE}}{\hbar} = \frac{n\pi}{2a} \rightarrow E_n = \frac{\hbar^2 n^2 \pi^2}{8a^2 m}$$

$$\rightarrow \boxed{E_1 = \frac{\hbar^2 \pi^2}{8a^2 m}, \quad E_2 = \frac{\hbar^2 \pi^2}{2a^2 m}}$$

$$u|\psi\rangle = e^{\frac{-iHt}{\hbar}}|1\rangle + e^{-iHt/\hbar}|2\rangle$$

$$\boxed{\psi(x,t) = \frac{1}{\sqrt{a}}\left[\exp\left(-\frac{i\hbar\pi^2}{8a^2 m}\right)\cos\left(\frac{\pi}{2a}x\right) + \exp\left(-\frac{i\hbar\pi^2}{2a^2 m}\right)\sin\left(\frac{\pi x}{a}\right)\right]}$$

(B) $$\langle T\rangle = \frac{1}{\sqrt{a}}\int_{-a}^{a}\left[\exp\left(\frac{i\hbar\pi^2}{8a^2 m}\right)\cos\left(\frac{\pi}{2a}x\right) + \exp\left(\frac{i\hbar\pi^2}{2a^2 m}\right)\sin\left(\frac{\pi x}{a}\right)\right]\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right)$$

$$\underbrace{\left[\exp\left(\frac{-i\hbar\pi^2}{8a^2 m}\right)\cos\left(\frac{\pi}{2a}x\right) + \exp\left(-\frac{i\hbar\pi^2}{2a^2 m}\right)\sin\left(\frac{\pi x}{a}\right)\right]}$$

$\rightarrow$

I'm NOT going to Finish

this, if you have time, you should

~~M~~A S-2008

Problem 1: The Infinite Square Well: (10 Points)

A single particle is in a one dimensional infinitely deep potential well of width L where $V(x)$ is given by:

$$V(x) = \begin{cases} 0, & \text{if } 0 \leq x \leq L \\ \infty, & \text{otherwise} \end{cases}$$

1. Find the allowed energies (E_n) and the normalized eigenfunctions ($\Psi(x)$) to Schrodinger's Equation for this potential. Show all your work. **(2 Points)**

2. Sketch the wave functions for the first three stationary states for this potential. **(2 Points)**

3. Now, if four spin-1/2 identical particles of mass m are placed in this potential, calculate the three lowest values for the total energy of the system of particles. **(3 Points)**

4. Determine the degeneracy for each of the three energy states found in part 3. **(3 Points)**

each energy level can only be occupied by 1 Fermion

→ Therefore,

$n_1 = 1$

$n_2 = 2$

$n_3 = 3$

$n_4 = 4$

No
can
be
~~[illegible]~~
The same

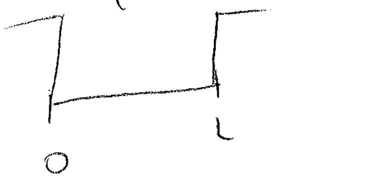
2nd energy level
$n = 2, 3, 4, 5$
etc

degeneracy in that any of the particles can have this n_1 non-specific

S-2008

1 $V(x) = \begin{cases} \phi & 0 \le x \le L \\ \infty & \text{elsewhere} \end{cases}$

1A



inside well $\rightarrow$ $\left[\frac{p^2}{2m} + V(x)\right]\psi(x) = E(\psi)$

$\rightarrow p = -i\hbar \frac{d}{dx} \rightarrow \frac{-\hbar^2}{2m}\frac{d^2}{dx}\psi(x) = E(\psi)$

$\rightarrow \frac{d^2}{dx}\psi(x) = \frac{-2m}{\hbar^2}E\,\psi(x)$

$\rightarrow k^2 = \frac{2mE}{\hbar^2} \rightarrow \frac{d^2}{dx}\psi(x) = -k^2\psi(x)$

$\rightarrow \left(\frac{d^2}{dx} + k^2\right)\psi(x) = \phi$

$\rightarrow \psi(x) = Ae^{ikx} + Be^{-ikx}$

or $A\sin(kx) + B\cos(kx)$ ✓

$\rightarrow$ at $x = \phi \rightarrow \cos(\phi) \neq \phi \rightarrow B = \phi$

$\rightarrow \psi(x) = A\sin(kx)$

$\rightarrow$ zeros of sin $\rightarrow kx = n\pi$

$\rightarrow kL = n\pi \rightarrow k = \frac{n\pi}{L}$

$\rightarrow \psi(x) = A\sin\left(x\frac{n\pi}{L}\right)$

②

(A) - Continued

$$\int_0^L A^2 \sin^2\left(\frac{x n\pi}{L}\right) dx = 1 \rightarrow \sin^2\left(\frac{x n\pi}{L}\right) = \frac{1}{2} - \frac{1}{2}\cos\left(\frac{2xn\pi}{L}\right)$$

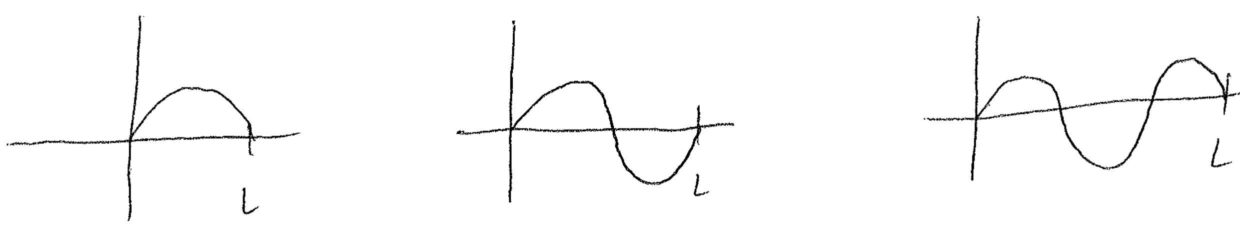
$$\rightarrow \int_0^L A^2\left[\frac{1}{2} - \frac{1}{2}\cos\left(\frac{2xn\pi}{L}\right)\right] dx = A^2 \frac{x}{2}\Big|_0^L - \varnothing$$ (integrates to sin)

$$\rightarrow \frac{A^2 L}{2} = 1 \rightarrow A = \sqrt{\frac{2}{L}}$$

$$\rightarrow \boxed{\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{x n\pi}{L}\right)}$$

$$\rightarrow \frac{n^2\pi^2}{L^2} = \frac{2mE}{\hbar^2} \rightarrow \boxed{E = \frac{\hbar^2\pi^2 n^2}{2mL^2}, \; n = 1, 2, 3}$$

(B) $n-1$ nodes since $n \geq 1$



(C) $$E_{tot} = \frac{\hbar^2\pi^2}{2mL^2}\left[\underbrace{n_1^2 + n_2^2 + n_3^2 + n_4^2}_{n}\right]$$

$\rightarrow$ spin $\frac{1}{2}$ $\rightarrow$ fermions

$\rightarrow$ spin up & down so 2 can have same n

$\rightarrow$ minimum $n = 1 + 1 + 2^2 + 2^2 = 10 \rightarrow E_1 = \frac{5\hbar^2\pi^2}{mL^2}$

$\rightarrow n = 15 = 1 + 1 + 2^2 + 3^2 = 15 \rightarrow E_2 = \frac{15}{2}\frac{\hbar^2\pi^2}{mL^2}$

$\rightarrow n = 18 = 1 + 2^2 + 2^2 + 3^2 = 18 \rightarrow E_3 = \frac{9\hbar^2\pi^2}{mL^2}$

? 15 This is right

(C)

(D) DO