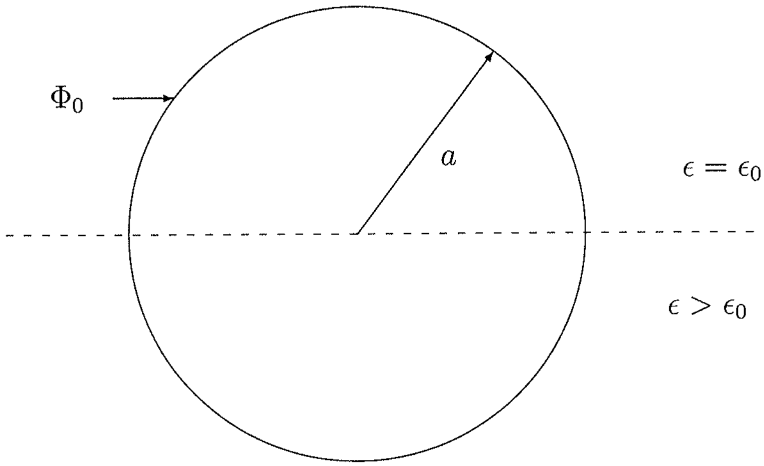
Sphere

SPRING 2012

4. Half Submerged Conducting Sphere



An originally uncharged thin spherical conducting shell of radius a is brought to a potential Φ_0. The shell floats half submerged in a dielectric liquid of dielectric constant $k = \epsilon_r \equiv \epsilon/\epsilon_0$.

Determine the following:

~~(a)~~ (2 pts) The electric potential Φ everywhere **outside** the shell,

~~(b)~~ (2 pts) The electric field $\mathbf{E}$ everywhere **outside** the shell,

~~(c)~~ (2 pts) The free surface charge density σ on the shell,

(d) (4 pts) The net electrostatic force $\mathbf{F}$ acting on the shell.

Part (a)

We assume that the potential outside the sphere is the same as if the sphere were not halfway embedded in a dielectric material.

In general, the potential at $r \geq a$ is

$$\Phi(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}.$$

So at $r = a$,

$$\Phi(a) = \frac{1}{4\pi\varepsilon_0}\frac{q}{a} = \Phi_0,$$

and thus, outside the sphere,

$$\boxed{\Phi(r) = \Phi_0 \frac{a}{r}}.$$

Part (b)

The electric field is given by

$$\vec{E} = -\vec{\nabla}\Phi(r)$$
$$= -\frac{\partial}{\partial r}\Phi(r)\,\hat{r}$$
$$= -\left(-\Phi_0 \frac{a}{r^2}\right)\hat{r}$$
$$\boxed{\vec{E} = \Phi_0 \frac{a}{r^2}\,\hat{r}}.$$

Part (c)

We know

$$\Phi_0 = \frac{1}{4\pi\varepsilon_0}\frac{q}{a},$$

and that

$$\sigma_f = \frac{q}{A} = \frac{q}{4\pi a^2}.$$

So

$$q = 4\pi\varepsilon_0 a \Phi_0$$

and

$$q = 4\pi a^2 \sigma_f,$$

so

$$4\pi a^2 \sigma_f = 4\pi\varepsilon_0 a \Phi_0$$
$$\sigma_f = \frac{4\pi\varepsilon_0 a \Phi_0}{4\pi a^2}$$
$$\boxed{\sigma_f = \frac{\Phi_0 \varepsilon_0}{a}}.$$