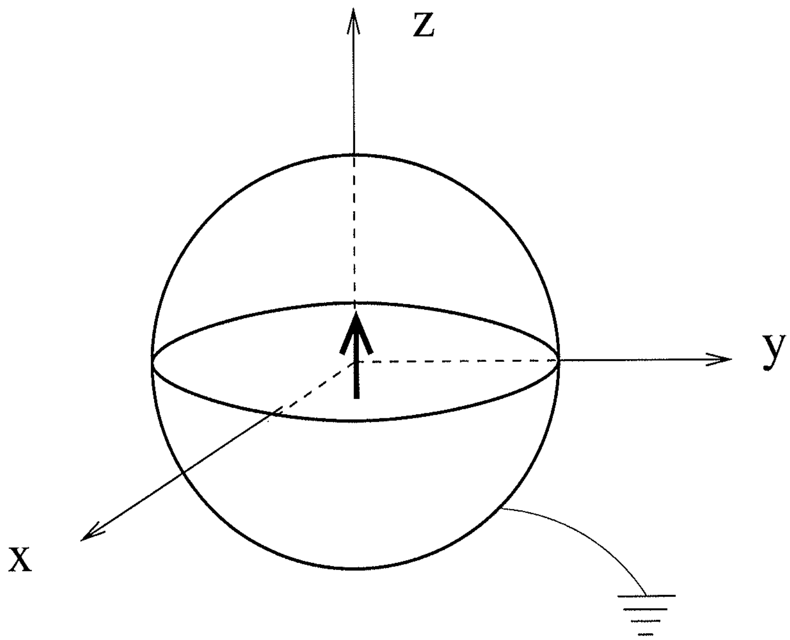
Legendre poly

FALL 2010

3. A point electric dipole with dipole moment $\mathbf{p} = p_0\hat{k}$ is located at the center of a hollow, grounded, conducting sphere.



(a) {2 pts} What are the boundary conditions satisfied by the electric field and electric potential in this problem?

(b) {5 pts} Compute the electrostatic potential inside the sphere.

(c) {3 pts} Compute the charge density σ on the inside surface on the grounded sphere.

Part (a)

The electric field must satisfy

$$E_{out,\perp} - E_{in,\perp} = \frac{\sigma}{\varepsilon_0}$$

$$E_{out,\parallel} - E_{in,\parallel} = 0.$$

Since the sphere is grounded, we must have

$$\Phi_{out}(r = \text{surface}) = \Phi_{in}(r = \text{surface}) = 0.$$

Part (b)

The potential inside the sphere will be a sum of the dipole potential and the potential due to the sphere itself,

$$\Phi_{tot} = \Phi_{sphere} + \Phi_{dipole}.$$

The potential due to the dipole is

$$\Phi_{dipole} = \frac{\vec{p}\cdot(\vec{r}-\vec{r}')}{4\pi\varepsilon_0|\vec{r}-\vec{r}'|^3}$$

$$= \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}$$

$$= \frac{p_0 r\cos\theta}{4\pi\varepsilon_0 r^3}$$

$$\Phi_{dipole} = \frac{p_0\cos\theta}{4\pi\varepsilon_0 r^2}.$$

The potential of the sphere can be written in terms of Legendre polynomials $P_\ell(\cos\theta)$,

$$\Phi_{sphere} = \sum_{\ell=0}^{\infty} A_\ell R^\ell P_\ell(\cos\theta).$$