Sphere

E&M

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1 Magnetic Materials

Assume the field inside a large piece of magnetic material is $\vec{\mathbf{B}}_0$ so that

$$\vec{\mathbf{H}}_0 = \frac{1}{\mu_0}\vec{\mathbf{B}}_0 - \vec{\mathbf{M}}$$

.

a) Consider a small spherical cavity that is hollowed out of the material. Find the field $\vec{\mathbf{B}}$, at the center of the cavity, in terms $\vec{\mathbf{B}}_0$ and $\vec{\mathbf{M}}$. Also find $\vec{\mathbf{H}}$ at the center of the cavity in terms of $\vec{\mathbf{H}}_0$ and $\vec{\mathbf{M}}$. (3 Points)

b) Do the same calculations for a long needle-shaped cavity running parallel to $\vec{\mathbf{M}}$. (3 Points)

c) Do the same calculations for a thin wafer-shaped cavity perpendicular to $\vec{\mathbf{M}}$. (4 Points)

Hint: Assume the cavities are small enough so that $\vec{\mathbf{M}}$, $\vec{\mathbf{B}}_0$ and $\vec{\mathbf{H}}_0$ are essentially constant. The field of a magnetized sphere is $\vec{\mathbf{B}} = \frac{2}{3}\mu_0\vec{\mathbf{M}}$ and the field inside a long solenoid is $\mu_0 K$ where K is the surface current density.

Part (a)

We can consider the magnetic field at the center of the cavity to be the sum of the field from the material and a magnetized sphere, with magnetization in the opposite direction as the material. So

$$\vec{B}_{cav} = \vec{B}_{tot} + \vec{B}_{sph}$$

$$\vec{B} = \vec{B}_0 + \frac{2}{3}\mu_0(-\vec{M})$$

and

$$\boxed{\vec{B} = \vec{B}_0 - \frac{2}{3}\mu_0\vec{M}}.$$

At the center of the cavity, $\vec{M} = 0$, so

$$\vec{H} = \frac{1}{\mu_0}\vec{B}$$

$$= \frac{1}{\mu_0}\left(\vec{B}_0 - \frac{2}{3}\mu_0\vec{M}\right)$$

$$= \frac{1}{\mu_0}\vec{B}_0 - \frac{2}{3}\vec{M}.$$

but

$$\vec{H}_0 = \frac{1}{\mu_0}\vec{B}_0 - \vec{M}$$

$$\frac{1}{\mu_0}\vec{B}_0 = \vec{H}_0 + \vec{M}$$

so

$$\vec{H} = (\vec{H}_0 + \vec{M}) - \frac{2}{3}\vec{M}$$

$$\boxed{\vec{H} = \vec{H}_0 + \frac{1}{3}\vec{M}}.$$

Part (b)

Again, we can consider the magnetic field at the center of the cavity to be the sum of the field from the material and a long solenoid, with opposite magnetization. So

$$\vec{B}_{cav} = \vec{B}_{tot} + \vec{B}_{sol}$$
$$\vec{B} = \vec{B}_0 + \vec{B}_{sol}.$$

In this case,

$$\vec{B}_{sol} = \mu_0 \vec{K}.$$

We can say

$$\vec{K} = (-\vec{M}) \times \hat{n}$$
$$= -\vec{M}$$

since the cavity is parallel to $\vec{M}$. So

$$\vec{B}_{sol} = -\mu_0 \vec{M}$$

and

$$\boxed{\vec{B} = \vec{B}_0 - \mu_0 \vec{M}}.$$

We know $\vec{M} = 0$ at the center of the cavity, so

$$\vec{H} = \frac{1}{\mu_0} \vec{B}$$
$$= \frac{1}{\mu_0}(\vec{B}_0 - \mu_0 \vec{M})$$
$$= \frac{1}{\mu_0} \vec{B}_0 - \vec{M}.$$

but

$$\frac{1}{\mu_0} \vec{B}_0 = \vec{H}_0 + \vec{M},$$

So

$$\vec{H} = (\vec{H}_0 + \vec{M}) - \vec{M}$$
$$\boxed{\vec{H} = \vec{H}_0}.$$

Part (c)

In this case, we can assume that our current density can be approximated as a current loop. But since the wafer is thin, we can assume that the contribution of the wafer to the magnetic field is negligible. Thus, the field at the center of the cavity is simply

$$\boxed{\vec{B} = \vec{B}_0}.$$

Then

$$\vec{H} = \frac{1}{\mu_0}\vec{B}$$
$$= \frac{1}{\mu_0}\vec{B}_0 .$$

But

$$\frac{1}{\mu_0}\vec{B}_0 = \vec{H}_0 + \vec{M} ,$$

so

$$\boxed{\vec{H} = \vec{H}_0 + \vec{M}}.$$