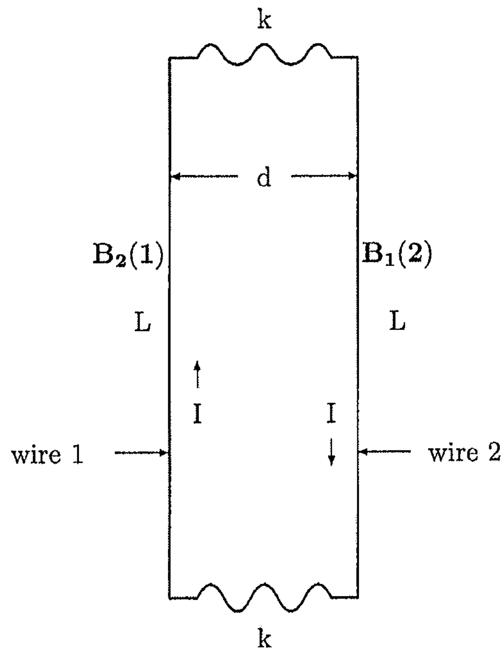
Wire / Cylinder

SPRING 2013




4. A current balance consists of two very long rigid parallel wires of lengths L that are connected at each end by springs (see the figure). The spring constant of both springs is $k$ and the equilibrium distance between the wires, d(I), depends on the current. Assume $d << L$.

   (a) [2 pts] If a current I flows through the closed circuit of the 2 wires and 2 springs, find an expression for the magnetic induction $B_1(2)$ created by the first wire at the location of the second. What is the direction of this $\mathbf{B}_1(2)$ field (give the direction as up, down, left, right, into, or out of the page)?

   (b) [2 pts] Find an expression for the magnetic force $\mathbf{F}_1(2)$ on the second wire due to the $\mathbf{B}_1(2)$. What is the direction of this force?

   (c) [2 pts] Find an expression for the magnetic force $\mathbf{F}_2(1)$ on the first wire due to the magnetic induction created by the second. What is the direction of this force?

   (d) [4 pts] Are the springs stretched or compressed from equilibrium? Using the above results, find an expression for the current as a function of the amount the springs are stretch/compressed.

## Part (a)

Using Ampère's law,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc},$$

where $I_{enc} = I$. So

$$B \oint dl = \mu_0 I$$

$$B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}.$$

At the position of wire 2, $r = d$ so

$$\boxed{B_1 = \frac{\mu_0 I}{2\pi d}}.$$

Using the right-hand rule, the magnetic field is pointing <u>into</u> the page.

## Part (b)

The magnetic force on wire 2 due to the magnetic field produced by wire 1 is

$$\vec{F}_1 = I\vec{l} \times \vec{B}_1$$

$$|\vec{F}_1| = I l B_1$$

$$= I(L)\left(\frac{\mu_0 I}{2\pi d}\right)$$

$$\boxed{F_1 = \frac{\mu_0}{2\pi} \frac{L I^2}{d}}$$

This force is directed to the left, toward wire 1.

## Part (c)

The magnetic field due to wire 2 at the position of wire 1 has the same magnitude as the field from wire 1, and is also pointing into the page. So the force on the first wire is

$$\boxed{F_2 = \frac{\mu_0}{2\pi} \frac{L I^2}{d}}$$

This force is directed to the right, toward wire 2.

So the wires are being pulled closer.

## Part (d)

Since the wires are being pulled closer, the springs should be compressed from equilibrium.

The magnitude of the spring force must be equal to the force from the current-carrying wires. So if $x$ is the amount the spring is compressed from the force due to one wire, then

$$kx = \frac{\mu_0}{2\pi}\frac{LI^2}{d}$$

$$\frac{LI^2}{d} = \frac{2\pi k x}{\mu_0}$$

$$I^2 = \frac{2\pi}{\mu_0}\frac{kxd}{L}$$

$$\boxed{I = \sqrt{\frac{2\pi}{\mu_0}\frac{kd}{L}x}}$$