Sphere

Electricity & Magnetism Qualifier

For each problem state what system of units you are using.

1. Imagine that a spherical balloon is being filled with a charged gas in such a way that the *rate* of charge being introduced is constant; $\dot{q} \equiv \frac{dq}{dt} =$ constant. Furthermore, assume that the elasticity of the balloon is such that the volume charge *density*, which you can assume to be uniform, remains constant; $\rho(t) =$ constant. For a fixed point a inside the balloon, $a \leq R$, where R is the balloon radius, calculate the following quantities as a function of the given variables, a, R, ρ, and $\dot{q}$:

   (a) [3 points] The electric field $\vec{\mathbf{E}}$.

   (b) [3 points] The electric potential V.

   (c) [2 points] The time rate of change of $\vec{\mathbf{E}}$.

   (d) [2 points] The time rate of change of V.

Part (a)

Using Gauss's law, with our Gaussian surface being a sphere of radius 'a', we have

$$\oint \vec{E} \cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0}.$$

The enclosed charge is simply

$$q_{enc} = \rho \left(\frac{4}{3}\pi a^3\right),$$

so

$$\oint \vec{E} \cdot d\vec{a} = \frac{4\pi a^3 \rho}{3\varepsilon_0}$$

$$E \oint da = \frac{4\pi a^3 \rho}{3\varepsilon_0}$$

$$E\left(4\pi a^2\right) = \frac{4\pi a^3 \rho}{3\varepsilon_0}$$

$$\boxed{\vec{E}_{in} = \frac{a\rho}{3\varepsilon_0}\hat{r}}.$$

Part (b)

In order to find the electric potential within the balloon, we first need to determine the electric field outside of the balloon. Again, using Gauss's law,

$$\oint \vec{E} \cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0}$$

$$E \oint da = \frac{1}{\varepsilon_0} \rho \left(\frac{4}{3}\pi R^3\right)$$

$$E\left(4\pi r^2\right) = \frac{4\pi R^3 \rho}{3\varepsilon_0}$$

$$\vec{E}_{out} = \frac{R^3 \rho}{3\varepsilon_0 r^2}\,\hat{r}\,.$$

Then

$$V = -\int_{\infty}^{R} E_{out}\, dr - \int_{R}^{a} E_{in}\, dr$$

$$= -\int_{\infty}^{R} \frac{R^3 \rho}{3\varepsilon_0 r^2}\, dr - \int_{R}^{a} \frac{r\rho}{3\varepsilon_0}\, dr$$

$$= \frac{R^3 \rho}{3\varepsilon_0 r}\Big|_{\infty}^{R} - \frac{\rho r^2}{6\varepsilon_0}\Big|_{R}^{a}$$

$$= \frac{R^2 \rho}{3\varepsilon_0} - \frac{\rho}{6\varepsilon_0}\left(a^2 - R^2\right)$$

$$\boxed{V = \frac{R^2 \rho}{2\varepsilon_0} - \frac{\rho a^2}{6\varepsilon_0}}\,.$$

Part (c)

The time rate of change of the electric field is

$$\dot{E} = \frac{dE}{dt}$$

$$= \frac{d}{dt}\left(\frac{a\rho(t)}{3\varepsilon_0} \right)$$

$$\dot{E} = \frac{a}{3\varepsilon_0}\,\dot{\rho}(t).$$

but $\rho(t)$ = constant, so $\dot{\rho}(t) = 0$ and

$$\boxed{\dot{E} = 0}.$$