1. Dielectric Sphere

   A dielectric sphere of radius R is polarized so that $\mathbf{P} = (K/r)\hat{\mathbf{r}}$ where $\hat{\mathbf{r}}$ is the unit radial vector. Assume the sphere is in an empty vacuum and that the sphere's dielectric material is linear and isotropic, calculate

   (a) (3 pts) the volume and the surface densities of bound charge,

   (b) (2 pts) the volume density of free charge,

   (c) (2 pts) the electric field inside the sphere,

   (d) (3 pts) the electric field outside the sphere.

   Your answers should be given in terms of K, χ_E, ϵ_0, ϵ, and/or ϵ_r. Recall that for linear isotropic materials:

   In SI units,

$$\mathbf{D} = \epsilon\mathbf{E} = \epsilon_0\mathbf{E} + \mathbf{P}$$

$$\mathbf{P} = \epsilon_0\chi_E\mathbf{E}$$

$$\epsilon_r \equiv \frac{\epsilon}{\epsilon_0} = 1 + \chi_E$$

   In Gaussian units,

$$\mathbf{D} = \epsilon\mathbf{E} = \mathbf{E} + 4\pi\mathbf{P}$$

$$\mathbf{P} = \chi_E\mathbf{E}$$

$$\epsilon = 1 + 4\pi\chi_E = \epsilon_r$$

Red = Gaussian, Green = SI

#1)

$$\vec{P} = \frac{k}{r}\hat{r} = \frac{\epsilon_0}{4\pi}(\epsilon_r - 1)\vec{E}$$

$$= \epsilon_0 \chi_e \vec{E} = \frac{\chi_e}{\epsilon_r}\vec{D}$$

Maxwell's eqn (Gauss's law)

$$\vec{\nabla}\cdot\vec{D} = 4\pi\rho_F$$

$$\vec{\nabla}\cdot\left(\epsilon_0\vec{E} + 4\pi\vec{P}\right)$$

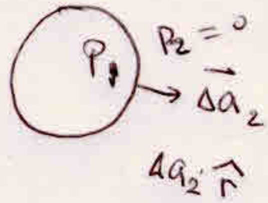
$$\Rightarrow \vec{\nabla}\cdot\vec{E} = \frac{4\pi}{\epsilon_0}\left(\rho_F - \vec{\nabla}\cdot\vec{P}\right)$$

$$\therefore \boxed{\rho_{Bound} = \rho_b = -\vec{\nabla}\cdot\vec{P}}$$

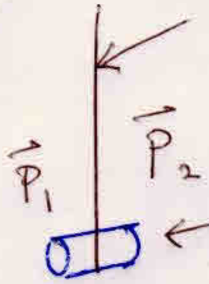
(a) #1

$$-\vec{\nabla}\cdot\vec{P} = -K\vec{\nabla}\cdot\left(\frac{\vec{r}}{r^2}\right) = -\frac{K}{r^2}$$

$$\therefore \boxed{\rho_b = -\frac{K}{r^2}}_{r<R}$$ bound volume charge density.



Junction – jump in $\vec{P}$



← apply Gauss's law to

$$\text{Bound charge in vol} = \int_{vol} -\vec{\nabla}\cdot\vec{P}\,d\text{vol} = -\oint \vec{P}\cdot d\vec{a}$$

$$= -(\vec{P}_2 - \vec{P}_1)\cdot\vec{\Delta a}_2$$

$$= \sigma_b\,\Delta a_2$$

$$\Rightarrow \boxed{\sigma_b = \vec{P}_1\cdot\hat{r} = \frac{K}{R}}$$

Surface bound charge density at $r = R$!

#1

(b) $$\rho_F = \frac{\vec{\nabla}\cdot\vec{D}}{4\pi} = \frac{\vec{\nabla}\cdot\left(\frac{\frac{K}{r}\hat{r}}{\frac{\chi_e}{\epsilon_r}}\right)}{4\pi} = \frac{-\rho_{Bound}}{4\pi\ \chi_e/\epsilon_r}$$

$$\boxed{\rho_F = \frac{K\ \epsilon_r}{4\pi\ r^2\ \chi_e} = \frac{\epsilon}{4\pi\ \epsilon_0\chi_e}\ \frac{K}{r^2}}$$

Gaussian has the 4π SI has the ϵ_0

(c) $$\vec{E} = \frac{\vec{P}}{\frac{\epsilon_0}{4\pi}(\epsilon_r-1)} = \frac{4\pi\left(\frac{K}{r}\hat{r}\right)}{\epsilon_0(\epsilon_r-1)} = \frac{4\pi\frac{K}{r}\hat{r}}{(\epsilon-\epsilon_0)}$$

$$= \frac{\frac{K}{r}\hat{r}}{\epsilon_0\ \chi_e} \qquad r<R$$

(d) Outside $\vec{E} = \vec{D}/\epsilon_0 = \dfrac{4\pi\ Q_{in}}{\epsilon_0\ 4\pi r^2}\hat{r}$

↑ from Gauss's law

$$\therefore\ \vec{E} = \frac{Q_{in}}{4\pi\epsilon_0\ r^2}$$

$$Q_{in} = \int \rho_f\, d\mathrm{vol} = \int_0^R \frac{\epsilon}{4\pi\epsilon_0\chi_e}\frac{K}{r^2}\ 4\pi r^2 dr$$

$$= \frac{\epsilon K\ 4\pi R}{4\pi\ \epsilon_0\ \chi_e} \Rightarrow \boxed{\vec{E} = \frac{\epsilon\ KR}{\epsilon_0^2\ \chi_e r^2}\hat{r}}$$

2. Gauge Transformation

   (a) (2 pts)

   Define the vector potential $\mathbf{A}$ and the scalar potential Φ using Maxwell's equations. (i.e. give their relationships to the $\mathbf{E}$ and $\mathbf{B}$ fields.)

   (b) (3 pts) Show that when $\mathbf{A}$ and Φ undergo the gauge transformations,

$$\mathbf{A}' = \mathbf{A} + \nabla\Lambda, \qquad (SI) \text{ and } (Gaussian)$$

$$\Phi' = \Phi - \frac{\partial\Lambda}{\partial t}, \qquad (SI)$$

   or

$$\Phi' = \Phi - \frac{1}{c}\frac{\partial\Lambda}{\partial t}, \qquad (Gaussian)$$

   where Λ is an arbitrary scalar, $\mathbf{B}$ and $\mathbf{E}$ are unaffected.

   (c) Two gauges used in solid-state physics for static, uniform magnetic fields $\mathbf{B}$ (i.e., constant in direction, magnitude, and time) are the Landau gauge and the circular gauge. Examples for $\mathbf{B} = B_0\hat{z}$ of each gauge respectively are:

$$\mathbf{A} = (A_x, A_y, A_z) = (0, B_0\,x, 0)$$

   and

$$\mathbf{A}' = (A'_x, A'_y, A'_z) = (-B_0 y/2, B_0 x/2, 0),$$

   with

$$\Phi = 0,$$

   for both gauges.

      i. (2 pts) Show that $\mathbf{A}$ and $\mathbf{A}'$ with $\Phi = \Phi' = 0$ describe the same $\mathbf{E}$ and $\mathbf{B}$ fields.

      ii. (3 pts) Find the scalar function Λ that produces the gauge transformation from $\mathbf{A}$ to $\mathbf{A}'$ in part (c).

2 (a) $\nabla \cdot \vec{B} = 0 \Rightarrow \boxed{\vec{B} = \nabla \times \vec{A}}$

$$\therefore \nabla \times \vec{E} = -\frac{\partial \vec{B}}{c\,\partial t} = -\frac{\partial}{c\,\partial t}(\nabla \times \vec{A}) = -\nabla \times \frac{\partial \vec{A}}{c\,\partial t}$$

$$\Rightarrow \nabla \times \left[\vec{E} + \frac{\partial \vec{A}}{c\,\partial t}\right] = 0$$

red is Gaussian

$$\Rightarrow \vec{E} + \frac{\partial \vec{A}}{c\,\partial t} = -\nabla V$$

$$\Rightarrow \boxed{\vec{E} = -\nabla V - \frac{\partial \vec{A}}{c\,\partial t}}$$

(b) $\vec{B} = \nabla \times \vec{A} \rightarrow \nabla \times (\vec{A} + \nabla\psi) = \nabla \times \vec{A} + \cancel{\nabla \times \nabla\psi}^{\,0}$

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{c\,\partial t} \longrightarrow -\nabla\left(V - \frac{\partial \psi}{c\,\partial t}\right) - \frac{\partial}{c\,\partial t}\left(\vec{A} + \nabla\psi\right)$$

$$= -\nabla V + \frac{\partial \nabla\psi}{c\,\partial t} - \frac{\partial \vec{A}}{c\,\partial t} - \frac{\partial \nabla \psi}{c\,\partial t}$$

$$= -\nabla V - \frac{\partial \vec{A}}{c\,\partial t}$$

(c) $\vec{E} = -\nabla V - \frac{\partial \vec{A}}{c\,\partial t} = 0 = -\nabla V' - \frac{\partial \vec{A}'}{c\,\partial t}$

$$\vec{B} = \nabla \times \vec{A} = \hat{z}\frac{\partial (Bx)}{\partial x} = B\hat{z}$$

$$\vec{B} = \nabla \times \vec{A}' = \hat{z}\left(\frac{\partial (Bx/2)}{\partial x} + \frac{\partial (By/2)}{\partial y}\right) = \hat{z}\left(\frac{B}{2} + \frac{B}{2}\right) = B\hat{z}$$

#2 ~~Drag~~

(d) $\vec{A}' = \vec{A} + \nabla\psi \Rightarrow \nabla\psi = \vec{A}' - \vec{A}$

$$\Rightarrow \nabla\psi = \left(-\frac{By}{2}, \frac{Bx}{2}, 0\right) - (0, Bx, 0) = \left(-\frac{By}{2}, -\frac{Bx}{2}, 0\right)$$

But $\nabla\psi = \left(\frac{\partial\psi}{\partial x}, \frac{\partial\psi}{\partial y}, \frac{\partial\psi}{\partial z}\right)$

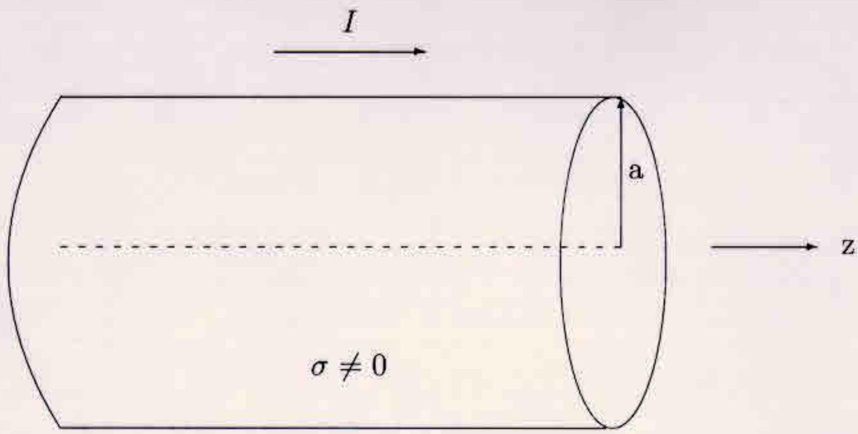
$$\Rightarrow \frac{\partial\psi}{\partial x} = -\frac{By}{2} \Rightarrow \psi = -\frac{Bxy}{2} + f(y,z)$$

$$\frac{\partial\psi}{\partial y} = -\frac{Bx}{2} \Rightarrow \psi = -\frac{Bxy}{2} + g(x,z)$$

$$\frac{\partial\psi}{\partial z} = 0 \Rightarrow \psi = h(x,y)$$

$$\therefore \psi = -\frac{Bxy}{2} + \text{constant.}$$

3. Poynting Vector



A straight metal wire of conductivity σ and cross-sectional area $A = \pi a^2$ carries a uniform, steady current I.

(a) (2 pts) Calculate $\mathbf{E}$ at the surface of the wire.

(b) (2 pts) Calculate $\mathbf{B}$ at the surface of the wire.

(c) (1 pts) Calculate the direction and magnitude of the Poynting vector at the surface of the wire.

(d) (3 pts) Integrate the normal component of the Poynting vector over the surface of the wire for a segment of length L.

(e) (2 pts) compare your result for (d) with the Joule heat produced in this segment.

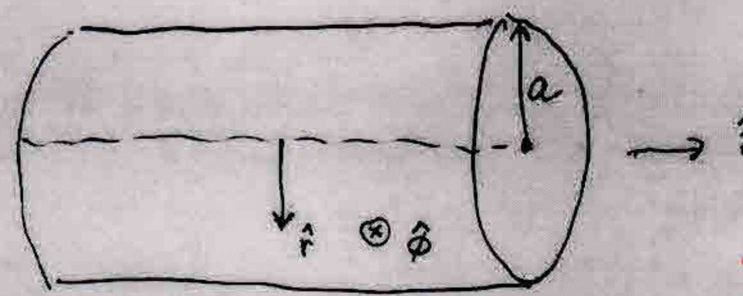
The Poynting vector is

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}, \qquad (SI)$$

$$\mathbf{S} = \frac{c}{4\pi}\mathbf{E} \times \mathbf{H}. \qquad (Gaussian)$$

(3) Use cylindrical coordinates (r, ϕ, z)

(a)



$$B_{surf}(2\pi a) = \frac{4\pi}{c}\frac{I}{\mu_0} \longrightarrow \vec{B}_{surf} = \frac{4\pi}{c}\frac{I}{2\pi a \mu_0}\hat{\phi}$$

$$\sigma\vec{E} = \vec{J} = \frac{I}{A}\hat{z} \Rightarrow \vec{E} = \frac{I\hat{z}}{A\sigma}$$

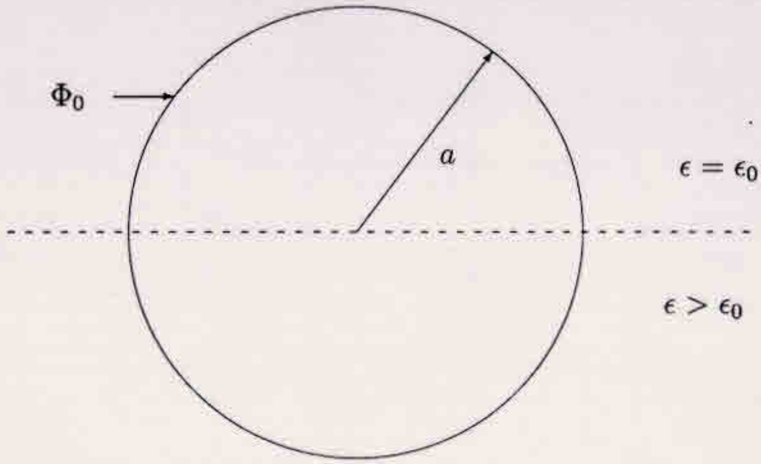
$$\vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{H} = \frac{I}{A\sigma}\hat{z}\times\mu_0\left(\frac{I}{2\pi a\mu_0}\right)\hat{\phi} \qquad \frac{4\pi}{c}\ast\frac{c}{4\pi} = 1$$

$$= -\frac{I^2}{2\pi a A\sigma}\hat{r} \quad \cancel{= -\frac{I^2}{2\sqrt{\pi}A^{3/2}\sigma}\hat{r}}$$

(b) $$\left|\int\vec{S}\cdot d\vec{a}\right| = S(2\pi a L) = \frac{I^2}{2\pi a A\sigma}(2\pi a L) = \frac{I^2 L}{A\sigma}$$

$$= I^2 R \quad \text{(Joule heat!)}$$

4. Half Submerged Conducting Sphere



An originally uncharged thin spherical conducting shell of radius a is brought to a potential Φ_0. The shell floats half submerged in a dielectric liquid of dielectric constant $k = \epsilon_r \equiv \epsilon/\epsilon_0$.

Determine the following:

(a) (2 pts) The electric potential Φ everywhere **outside** the shell,

(b) (2 pts) The electric field $\mathbf{E}$ everywhere **outside** the shell,

(c) (2 pts) The free surface charge density σ on the shell,

(d) (4 pts) The net electrostatic force $\mathbf{F}$ acting on the shell.

4

(a) Since $\Phi = \Phi_0$ on $r=a$ and $=0$ at $r=\infty$

and there are no sources at $r>a$

$$\Phi = \frac{\text{con}}{r} = \frac{\Phi_0 a}{r} \quad \text{for } r \geq a \,!$$

(same in I + II)

(b) $$\vec{E} = -\vec{\nabla}\phi = \frac{\Phi_0 a}{r^2}\,\hat{r} \quad \text{for } r \geq a$$

(Same in I + II)

(c) $$\vec{\nabla}\cdot\vec{D} = 4\pi\rho \Rightarrow E_n = \frac{4\pi\sigma}{\epsilon}$$

$$\Rightarrow \boxed{\sigma_I = \frac{E_I\,\epsilon_0}{4\pi} = \frac{\epsilon_0\,\Phi_0}{4\pi\,a}}$$

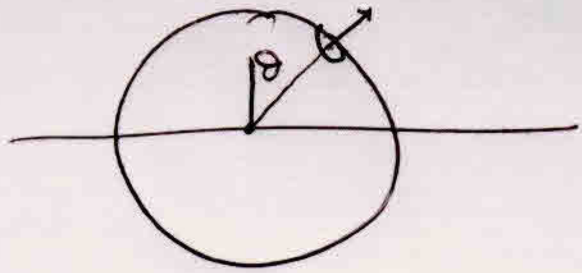
$$\boxed{\sigma_{II} = \frac{E_{II}\,\epsilon}{4\pi} = \frac{\epsilon\,\Phi_0}{4\pi\,a}}$$

Red = Gaussian

Green = SI

#4

d)



$\vec{F} = \frac{1}{2}\int \vec{E}\,\sigma\, da$

$\propto \hat{z}$ from symmetry

$$F_z = \frac{1}{2}\int_0^{\pi}\int_0^{2\pi} E\,\sigma \cos\theta\, a^2 \sin\theta\, d\theta\, d\phi$$

$$= \frac{E\,a^2}{2}\, 2\pi \int_0^{\pi} \sigma \cos\theta \sin\theta\, d\theta$$

$$= \Phi_0\, a\, \pi \left[\sigma_I \int_0^{\pi/2} \cos\theta \sin\theta\, d\theta + \sigma_{II} \int_{\pi/2}^{\pi} \cos\theta \sin\theta\, d\theta \right]$$

$$= \Phi_0\, a\, \pi\, (\sigma_I - \sigma_{II})\, \frac{1}{2}$$

$$F_z = \frac{\Phi_0^2\, \pi/2\, (\epsilon_0 - \epsilon)}{4\pi}$$

$= K = \epsilon/\epsilon_0$

$$\boxed{F_z = -\frac{\epsilon_0\, \Phi_0^2\, \pi/2\, (\epsilon_r - 1)}{4\pi}}$$

5. Capacitor Plates

Consider a very large parallel plate capacitor with the positive plate at $z = d/2$, the negative plate at $z = -d/2$ and no dielectric material in between. If the respective surface charge densities are $\pm\sigma$ compute the *force/area on the positive plate* in the following two ways:

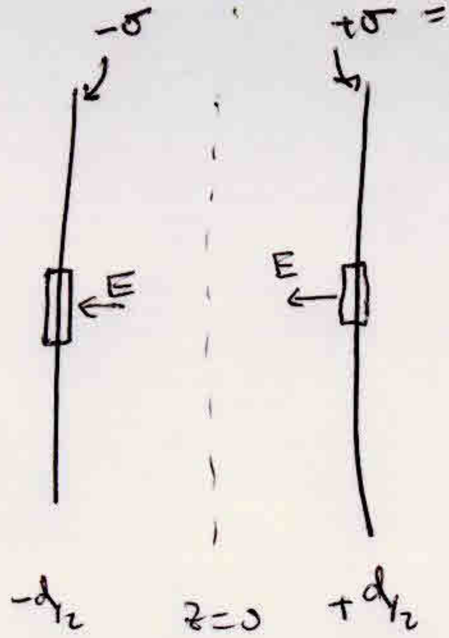
(a) (4 pts) Calculate it directly from σ and the electric field $\mathbf{E}$. Give a logical explanation of why your answer is correct.

(b) (6 pts) Calculate it using the Maxwell stress tensor

$$T_M^{ij} = \epsilon_0 \left[E^i E^j - \frac{1}{2}\delta^{ij} \vec{E} \cdot \vec{E} \right], \qquad (SI)$$

$$T_M^{ij} = \frac{1}{4\pi} \left[E^i E^j - \frac{1}{2}\delta^{ij} \vec{E} \cdot \vec{E} \right]. \qquad (Gaussian)$$

#5



Red = Gaussian

Green = SI

$$\vec{\nabla}\cdot\vec{D} = 4\pi\rho$$

$$\Rightarrow E_m = \frac{4\pi\,\sigma_f}{\epsilon_0}$$

E Total at positive plate surface

(a) $$\frac{\Delta F}{\Delta \text{Area}} = \frac{\sigma_f E_m}{2} = \boxed{\frac{4\pi\,\sigma_f^2}{2\epsilon_0}} = \sigma_{right} * E_{left}$$

E from left plate only.

Half of E_m comes from the + plate itself and half from the other plate!

(b) $$T_m^{ij} = \frac{\epsilon_0}{4\pi}\left[\delta^i_z\delta^j_z E_m^2 - \frac{1}{2}\delta^{ij}E_m^2\right]$$

$$= \frac{\epsilon_0}{4\pi}\left[\delta^i_z\delta^j_z - \frac{1}{2}\delta^{ij}\right]\left(\frac{4\pi\,\sigma_f}{\epsilon_0}\right)^2$$

$$= \frac{4\pi}{\epsilon_0}\sigma_f^2\left[\delta^i_z\delta^j_z - \frac{1}{2}\delta^{ij}\right]$$

$$\frac{\Delta F^i}{\Delta A} = \frac{\int T^{ij}_m\, dA^j}{\Delta A} = \frac{4\pi}{\epsilon_0}\sigma_f^2\,\delta^i_z\,\frac{1}{2} = \boxed{\frac{4\pi}{\epsilon_0}\frac{\sigma_f^2}{2}\,\hat{z}}$$

6. E&M Waves

A monochromatic, plane polarized, plane electromagnetic wave traveling in the z-direction in the lab (in a vacuum) can be written in the following 3+1 dimensional form:

$$\mathbf{E} = E_0\ \hat{\mathbf{x}}\ e^{i(k\,z-\omega\,t)},$$

$$\mathbf{B} = B_0\ \hat{\mathbf{y}}\ e^{i(k\,z-\omega\,t)}.$$

(a) (3 pts) Combine this **E** and **B** into a single electromagnetic field tensor $F^{\alpha\beta}$ and use Maxell's equations in the 4-dimensional form

$$\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0,$$

$$\partial_\alpha F^{\alpha\beta} = 0$$

to find all constraints on the 4 constants E_0, B_0, k, and ω (i.e., the above wave won't satisfy Maxwell's equations for arbitrary values of all four of these parameters). Depending on your choice of conventions: $x^\alpha = (x^0, x^1, x^2, x^3)$ with $x^0 = ct$ or $x^\alpha = (x^1, x^2, x^3, x^4)$ with $x^4 = ct$ and $x^1 = x, x^2 = y, x^3 = z$.

(b) (1 pts) What are the values of the invariants $F^{\alpha\beta}F_{\alpha\beta}$ and $\epsilon^{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta}$ for this wave?

(c) (3 pts) Use a Lorentz boost to find $F'^{\alpha\beta}$ in a frame moving in the $+z$ direction with a speed v. Don't forget to express your answer in terms of the moving coordinates ct' and x', y', z'.

(d) (2 pts) What is the frequency and the wavelength of this wave in the moving frame?

(e) (1 pts) How have the electric and magnetic fields changed in direction and/or magnitude?

RK5 ∴ (Black = Gaussian) 1/5

#1

Green = SI

F^{01} + F^{13} are the terms

(a)

$$F^{\alpha\beta} = -1 \times \begin{pmatrix} 0 & -E_0/c & 0 & 0 \\ E_0/c & 0 & 0 & B_0 \\ 0 & 0 & 0 & 0 \\ 0 & -B_0 & 0 & 0 \end{pmatrix} e^{i(kz-\omega t)}$$

+2

$$\partial_0 \underline{F_{13}} + \partial_1 F_{30} + \partial_3 \underline{F_{01}} = 0$$

$$\frac{1}{c}\frac{\partial}{\partial t}\left[B_0 e^{i(kz-\omega t)}\right] + \frac{\partial}{\partial x}[0] + \frac{\partial}{\partial z}\left(+\frac{E_0}{c} e^{i(kz-\omega t)}\right) = 0$$

$$-i\frac{\omega}{c}B_0 e^{i(kz-\omega t)} + ik\frac{E_0}{c} e^{i(kz-\omega t)} = 0$$

(1) $\Rightarrow \boxed{\frac{\omega}{c}B_0 = k\frac{E_0}{c}}$ +3

$$\partial_\alpha F^{\alpha 1} = 0 \Rightarrow$$

$$\partial_0 \underline{F^{01}} + \partial_1(0) + \partial_2(0) + \partial_z \underline{F^{31}} = 0$$

$$\frac{1}{c}\frac{\partial}{\partial t}\left(-\frac{E_0}{c} e^{i(kz-\omega t)}\right) + \frac{\partial}{\partial z}\left(-B_0 e^{i(kz-\omega t)}\right) = 0$$

$$+\frac{i\omega}{c}\frac{E_0}{c} e^{i(kz-\omega t)} - ikB_0 e^{i(kz-\omega t)} = 0$$

(2) $\Rightarrow \boxed{\frac{\omega}{c}\frac{E_0}{c} = kB_0}$ +3

combine (1) + (2) $\frac{\omega}{c}\left(\frac{\frac{\omega}{c}E_0}{k}\right) = kE_0$

$$\Rightarrow \boxed{k = \pm \omega/c} \qquad ② \Rightarrow \boxed{\frac{E_0}{c} = \pm B_0}$$

+ = traveling to right (+z)

− = " " left (−z)

choice $\boxed{k = \omega/c \,,\, \frac{E_0}{c} = B_0}$ 2 remaining parameters!

arbitrary frequency

arbitrary amplitude

(b)

$$B = \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix}$$

$X' = BX$

$$F' = B^T F B$$

$$B^T = B$$

RKC 3/5

$$B^T F = \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix}^{-1} \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

$$\star \frac{E_0}{c} e^{i(kz-\omega t)}$$

$$= \begin{pmatrix} 0 & (-\gamma+\beta\gamma) & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & (\beta\gamma-\gamma) & 0 & 0 \end{pmatrix}^{-1} \frac{E_0}{c} e^{i(kz-\omega t)}$$

$$B^T F B = \begin{pmatrix} \\ \\ \\ \end{pmatrix}^{-} \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix}$$

$$= \frac{E_0}{c} e^{i(kz-\omega t)} \begin{pmatrix} 0 & -\gamma(1-\beta) & 0 & 0 \\ \gamma(1-\beta) & 0 & 0 & \gamma(1-\beta) \\ 0 & 0 & 0 & 0 \\ 0 & -\gamma(1-\beta) & 0 & 0 \end{pmatrix}$$

RK5

4/5 !

$$F' = -\frac{E_0}{c}\gamma(1-\beta)e^{i(kz-\omega t)}\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

+10

$$kz-\omega t = -k_\sigma x^\sigma = -\left(k_\sigma B^{-1\,\sigma}{}_\lambda\right)x'^\lambda = -k'_\lambda x'^\lambda$$

$$\downarrow$$

$$B^{-1\,\sigma}{}_\lambda x'^\lambda$$

$$\therefore \quad k'_\lambda = k_\sigma B^{-1\,\sigma}{}_\lambda$$

$$k = \omega/c \Rightarrow \quad k_\sigma = k(1,0,0,-1)$$

$$k'_\lambda = k(1,0,0,-1)\begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix}$$

$$= k\left(\gamma(1-\beta) \quad 0 \quad 0 \quad -\gamma(1-\beta)\right)$$

$$k'_\lambda = k\gamma(1-\beta)(1,0,0,-1) = k'(1,0,0,-1)$$

$$\boxed{k' = k\gamma(1-\beta), \quad \frac{\omega'}{c} = \frac{\omega}{c}\gamma(1-\beta)}$$

RK 5

5/5

For F' see top of page 4/

where $e^{i(kz-\omega t)} = e^{i(k'z'-\omega' t')}$

where $k' = \frac{\omega'}{c} = k\gamma(1-\beta) = k\sqrt{\frac{1-\beta}{1+\beta}}$

$$\vec{E}' = \hat{i}' E_0 \gamma(1-\beta) e^{i(k'z'-\omega' t')}$$

$$\vec{B}' = \hat{j}' \frac{E_0}{c} \gamma(1-\beta) e^{i(k'z'-\omega' t')}$$

Directions are the same but magnitudes

(d) have changed $E_0' = E_0\gamma(1-\beta)$ +6

$B_0' = \frac{E_0}{c}\gamma(1-\beta)$

(c) Frequency $\omega' = \omega\sqrt{\frac{1-\beta}{1+\beta}}$

+7

$\lambda \propto 1/k$ $\lambda' = \lambda\sqrt{\frac{1+\beta}{1-\beta}}$