#1

(a) $C \equiv q/V = C_x + C_{l-x} = \boxed{\frac{\epsilon w}{d}(x) + \frac{\epsilon_0 w}{d}(l-x) = \frac{w}{d}\left[(\epsilon-\epsilon_0)x + \epsilon_0 l\right]}$

$$q = V\,C$$

$$U_{cap} = \tfrac{1}{2}Vq = \tfrac{1}{2}V^2 C = \tfrac{1}{2}q^2/C$$

(b) For fixed $V = V_0$

energy change

$$\Delta U_{cap} = F_{ext}\,\Delta x + \underbrace{V_0\,\Delta q}_{\text{work by battery}} \qquad \text{(no change in KE)}$$

$$\begin{gathered} \Delta U_{cap} \\ \| \\ \tfrac{1}{2}V_0\Delta q \end{gathered} \quad \Rightarrow \quad F_{ext} = -\tfrac{1}{2}V_0\frac{\Delta q}{\Delta x} = -\tfrac{1}{2}V_0^2\frac{\Delta C}{\Delta x}$$

$$F_{ext} = -\tfrac{1}{2}V_0^2\,\frac{w}{d}(\epsilon-\epsilon_0)$$

to decrease x – ie to left

$$\boxed{\therefore F_{elec} = +\tfrac{1}{2}V_0^2\,\frac{w}{d}(\epsilon-\epsilon_0) \quad \text{to right } +x}$$

pulls dielectric into capacitor

(c) From the energy change eqn above

$$-\tfrac{1}{2}V_0\frac{\Delta q}{\Delta t} = F_{ext}\,\Delta x = \left(-\tfrac{1}{2}V_0^2\,\frac{w}{d}(\epsilon-\epsilon_0)\right)\frac{\Delta x}{\Delta t}$$

$$\therefore \boxed{i = V_0\,\frac{w}{d}(\epsilon-\epsilon_0)\,v_0}$$

easier or from $\frac{\Delta q}{\Delta t} = V_0\frac{\Delta C}{\Delta t} = \boxed{V_0\,\frac{w}{d}(\epsilon-\epsilon_0)\,v_0}$

#1

(d) It oscillates from 1/2 sticking out on the left to 1/2 of the slab sticking out on the right.

$$V_0 \Delta q = \Delta U + \Delta KE = \tfrac{1}{2} V_0^2 \left(C_{x=l} - C_{x=l/2} \right) + \tfrac{1}{2} m v^2$$

$$\downarrow$$

$$V_0^2 \Delta C$$

$$\therefore \quad \tfrac{1}{2} m v^2 = +\tfrac{1}{2} V_0^2 \left(C_{x=l} - C_{x=l/2} \right)$$

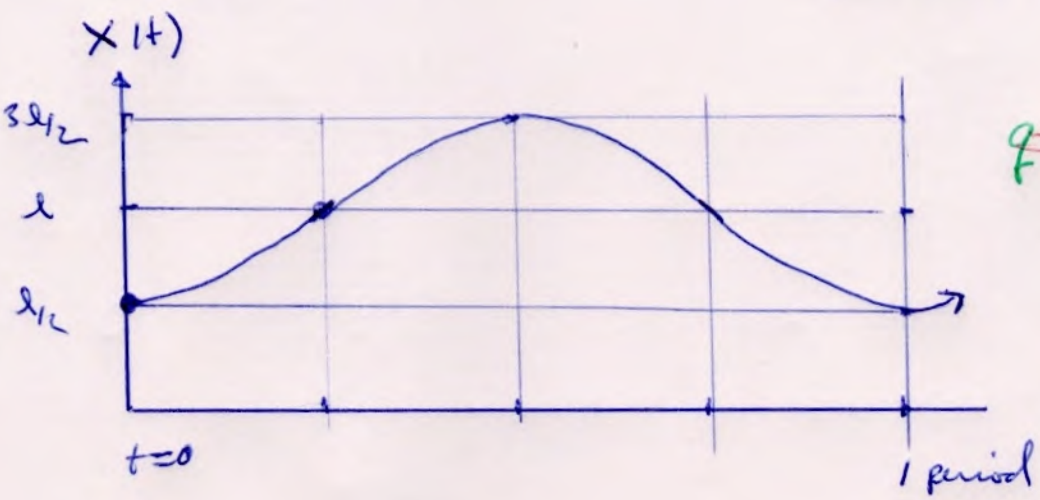
$$v = \sqrt{\frac{V_0^2}{m}\left(\frac{\epsilon w l}{d} - \frac{(\epsilon+\epsilon_0) w l}{2d} \right)}$$

$$\boxed{v_{max} = V_0 \sqrt{\frac{wl}{2md}(\epsilon - \epsilon_0)}}$$

(e) F is a constant pointing to $x = l$ $\therefore$ $a = \text{const}$ else

with $v = at + v_0$ and $x = \tfrac{1}{2} a t^2 + v_0 t + x_0$

parabolic motion



~~$q = V_0 C(x)$~~

2.

moves with target

(a) $\underline{X}' = A\,\underline{X} \qquad \underline{X} = A^{-1}\,\underline{X}'$

$$A = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad A^{-1} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$ct' = \gamma(ct - \beta x) \qquad ct = \gamma(ct' + \beta x')$$

$$x' = \gamma(-\beta ct + x) \qquad x = \gamma(\beta ct' + x')$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

(b) phase $= \vec{k}_I \cdot \vec{r} - \omega_I t = \frac{\omega_I}{c}\left[-\cos\theta_I\, x + \sin\theta_I\, y\right] - \omega_I t$

$$= \frac{\omega_I}{c}\left[-\cos\theta_I\, \gamma(\beta c t' + x') + \sin\theta_I\, y'\right] - \omega_I \gamma\left(t' + \frac{\beta}{c} x'\right)$$

$$= \frac{\omega_I}{c}\left[\gamma(-\cos\theta_I - \beta)x' + \sin\theta_I\, y'\right] - \omega_I\gamma(1 + \beta\cos\theta_I)t'$$

$$= \vec{k}'_I \cdot \vec{r}' - \omega'_I t'$$

$$\Rightarrow \boxed{\begin{aligned} \vec{k}'_I &= \frac{\omega_I}{c}\left[\gamma(-\cos\theta_I - \beta)\hat{\imath} + \sin\theta_I\, \hat{\jmath}\right] \\ \omega'_I &= \omega_I\gamma(1 + \beta\cos\theta_I) \end{aligned}}$$

\#2 $\vec{k}'_I = \frac{\omega'_I}{c}\left[-\cos\theta'_I\,\hat{i} + \sin\theta'_I\,\hat{j}\right]$

(c)

$$\Rightarrow \quad \left.\begin{aligned} -\frac{\omega'_I}{c}\cos\theta'_I &= \frac{\omega_I}{c}\gamma(-\cos\theta_I - \beta) \\ \frac{\omega'_I}{c}\sin\theta'_I &= \frac{\omega_I}{c}\sin\theta_I \\ \omega'_I &= \omega_I\gamma(1+\beta\cos\theta_I) \end{aligned}\right\} \text{from (b)}$$

$$\boxed{\therefore \quad \cos\theta'_I = \frac{\cos\theta_I + \beta}{1+\beta\cos\theta_I} \quad \text{or} \quad \sin\theta'_I = \frac{\sin\theta_I}{\gamma(1+\beta\cos\theta_I)}}$$

usual aberration of angles

(d) $\vec{k}'_R = \frac{\omega'_I}{c}\left[\cos\theta'_I\,\hat{i} + \sin\theta'_I\,\hat{j}\right]$

$$\omega'_R = \omega'_I \qquad \text{i.e. } \theta'_R = \theta'_I$$

$$\vec{k}_R\cdot\vec{r} - \omega_R t = \vec{k}'_R\cdot\vec{r}' - \omega'_R t'$$

$$= \frac{\omega'_I}{c}\left[\cos\theta'_I\,x' + \sin\theta'_I\,y'\right] - \omega'_I t'$$

#2

$$\vec{k}_R \cdot \vec{r} - \omega_R t = \frac{\omega_I'}{c}\left[\cos\theta_I' (\gamma)(\beta c t + x) + \sin\theta_I\, y \right]$$

$$- \omega_I' \gamma \left(t - \frac{\beta}{c} x\right)$$

$$\therefore \underline{\omega_R = \omega_I' \gamma \left[\beta \cos\theta_I' + 1 \right]}$$

$$\omega_R = \omega_I \gamma (1 + \beta \cos\theta_I) \gamma \left[1 + \beta \frac{(\cos\theta_I + \beta)}{1 + \beta \cos\theta_I} \right]$$

$$\boxed{\omega_R = \omega_I \gamma^2 \left[1 + \beta^2 + 2\beta \cos\theta_I \right]}$$

Doesema #

#3

Solution Qual #1

A. With origin at the center of the square:

$$V = \sum_i \frac{q_i}{4\pi\epsilon_0 r_i} = 4 \times \frac{-q}{4\pi\epsilon_0 (s/\sqrt{2})}$$

$$= \frac{-\sqrt{2}\, q}{\pi\epsilon_0 s}$$

B. $$W = (-q)V = -q\left(\frac{-\sqrt{2}\, q}{\pi\epsilon_0 s}\right) = \frac{\sqrt{2}\, q^2}{\pi\epsilon_0 s}$$

C. $$W = \frac{1}{2}\sum_i q_i V_i = \frac{1}{2}\sum_i (-q)\left(\frac{-2q}{4\pi\epsilon_0 s} - \frac{q}{4\pi\epsilon_0 \sqrt{2} s}\right)$$

$$= \frac{1}{2}\sum_i \frac{q^2}{4\pi\epsilon_0 s}\left(2 + \frac{1}{\sqrt{2}}\right) = \frac{1}{2}(4)\frac{q^2}{4\pi\epsilon_0 s}\left(2 + \frac{1}{\sqrt{2}}\right)$$

$$= 2(2 + 1/\sqrt{2})q^2 / 4\pi\epsilon_0 s$$

OR: $$W = 0 + (-q)\left(-\frac{q}{4\pi\epsilon_0 s}\right) + (-q)\left(-\frac{q}{4\pi\epsilon_0 s} - \frac{q}{4\pi\epsilon_0 \sqrt{2} s}\right)$$

$$+ (-q)\left(-\frac{2q}{4\pi\epsilon_0 s} - \frac{q}{4\pi\epsilon_0 \sqrt{2} s}\right)$$

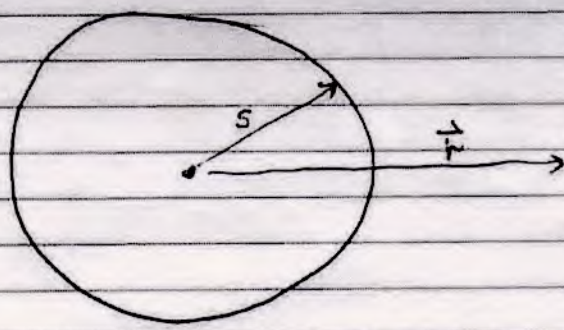
$$= \frac{q^2}{4\pi\epsilon_0 s} + \frac{q^2}{4\pi\epsilon_0 s} + \frac{q^2}{4\pi\epsilon_0 \sqrt{2} s} + \frac{2q^2}{4\pi\epsilon_0 s} + \frac{q^2}{4\pi\epsilon_0 \sqrt{2} s}$$

$$= \frac{2(2 + 1/\sqrt{2})\, q^2}{4\pi\epsilon_0 s}$$

Doezema

#3 2/2

D.



Use Gauss: $\vec{E}_{outside} = \frac{(-4q)}{4\pi\epsilon_0 r^2}\hat{r}$

$\vec{E}_{inside} = 0$

$$\Rightarrow V(r=0) = -\int_{\infty}^{0} \vec{E}\cdot d\vec{l} = -\int_{\infty}^{s} E_{outside}\,dr - \int_{s}^{0} E_{inside}\,dr$$

$$= -\int_{\infty}^{s} \frac{(-4q)}{4\pi\epsilon_0 r^2}\,dr - 0$$

$$= \frac{-4q}{4\pi\epsilon_0 s} = \frac{-q}{\pi\epsilon_0 s}$$

then $W = (-q)V(0) = \frac{q^2}{\pi\epsilon_0 s}$

Shaffer

1/2

#4 (3)

$$\Phi(r,\theta) = \sum_n (A_n r^m + B_n r^{-m-1}) P_n(\cos\theta)$$

a.

$r < R \quad B_n = 0$

$$\text{so} \quad \Phi(r,\theta) = \sum_n A_n r^m P_n(\cos\theta)$$

$r > R \quad A_n = 0 \quad \lim r \to \infty \quad \Phi(r,\theta) = 0$

$$\text{so} \quad \Phi(r,\theta) = \sum_n B_n r^{-m-1} P_n(\cos\theta)$$

b. $\lim_{r\to\infty} \Phi(r,\theta) = 0$

$\Phi(R,\theta) = V_0 \cos\theta$

$\Phi(0,\theta)$ finite.

potential is uniquely detemed because we know the potential on the boundaries. This satisfies one of the uniqueness conditions.

c. $r < R \quad \Phi(R,\theta) = \sum_n A_n R^m P_n(\cos\theta) = V_0 \cos\theta$

$$P_1(\cos\theta) = \cos\theta$$

$$A_1 R \cos\theta = V_0 \cos\theta$$

$$A_1 = \frac{V_0}{R}$$

$$\Phi(r,\theta) = \frac{V_0}{R} r P_1(\cos\theta) = V_0\left(\frac{r}{R}\right)\cos\theta$$

Shaffer 2/2

#4 (c) $r > R$

$$\Phi(R,\theta) = \sum_n B_n R^{-n-1} P_n(\cos\theta) = V_0 \cos\theta$$

$$B_1 R^{-2} \cos\theta = V_0 \cos\theta$$

$$B_1 = V_0 R^2$$

$$\Phi(r,\theta) = V_0 \left(\frac{R^2}{r^2}\right) \cos\theta$$

d $\quad \underline{E} = -\nabla\Phi = -\frac{\partial\Phi}{\partial r}\hat{r} - \frac{1}{r}\frac{\partial\Phi}{\partial\theta}\hat{\theta}$

$r < R$

$$\underline{E} = -\frac{V_0}{R}\cos\theta\,\hat{r} + \frac{V_0}{R}\sin\theta\,\hat{\theta}$$

$r > R \quad \underline{E} = \frac{2V_0R^2}{r^3}\cos\theta\,\hat{r} + \frac{1}{r}\frac{V_0R^2}{r^2}\sin\theta\,\hat{\theta}$

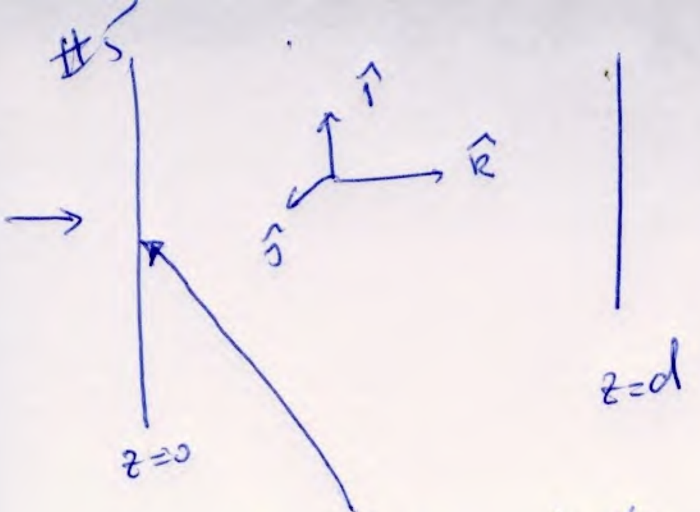
$$\underline{E}(r=R) = \frac{2V_0}{R}\cos\theta\,\hat{r} + \frac{V_0}{R}\sin\theta\,\hat{\theta}$$

e. No $\quad E_\parallel \neq 0$

f. $\quad \frac{\sigma}{\epsilon_0} = \frac{2V_0}{R}\cos\theta + \frac{V_0}{R}\cos\theta$

$$\sigma = \frac{3\epsilon_0 V_0}{R}\cos\theta$$

#5



$\hat{j} = \sin\phi_0 \hat{e}_s - \cos\phi_0 \hat{e}_f$

1/3

$$\vec{E} = E_0 \hat{j} e^{-i\omega t} \quad (z=0_+) = E_0\left(\sin\phi_0 \hat{e}_s + \cos\phi_0 \hat{e}_f\right) e^{-i\omega t}$$

$$\therefore \vec{E}(z=d) = E_0\left[\sin\phi_0 \hat{e}_s e^{i(k_s d-\omega t)} + \cos\phi_0 \hat{e}_f e^{i(k_f d-\omega t)}\right]$$

$k_s = \frac{\omega}{c} n_s \qquad n_f = n_s - \delta_n \qquad \bar{n} = \frac{n_f + n_s}{2}$

$k_f = \frac{\omega}{c} n_f \qquad n_s = \bar{n} + \frac{\delta n}{2}$

$n_f = \bar{n} - \frac{\delta n}{2}$

$\therefore k_s = \bar{k} + \frac{\delta k}{2} \qquad \delta k = k_s - k_f = \frac{\omega}{c}(\delta n)$

$k_f = \bar{k} - \frac{\delta k}{2} \qquad \bar{k} = \frac{k_s + k_f}{2} = \frac{\omega}{c}\left(\frac{n_s + n_f}{2}\right)$

$$\vec{E}(z=d) = E_0\left[\sin\phi_0 \hat{e}_s e^{\frac{i\delta k d}{2}} + \cos\phi_0 \hat{e}_f e^{-\frac{i\delta k d}{2}}\right] e^{i(\bar{k}d-\omega t)}$$

$\hat{e}_s \to \cos\phi_0 \hat{i} + \sin\phi_0 \hat{j} \qquad \hat{e}_f \to -\sin\phi_0 \hat{i} + \cos\phi_0 \hat{j}$

$$\vec{E}(z=d) = E_0\left[\sin\phi_0\cos\phi_0\left(e^{\frac{i\delta k d}{2}} - e^{-\frac{i\delta k d}{2}}\right)\hat{i} + \left(\sin^2\phi_0 e^{\frac{i\delta k d}{2}} + \cos^2\phi_0 e^{-\frac{i\delta k d}{2}}\right)\hat{j}\right] e^{i(\bar{k}d-\omega t)}$$

#5

$$\Delta \equiv \frac{\delta k d}{2} = \frac{\omega \delta n\, d}{2c}$$

use
$\cos^2\phi_0 = \frac{1}{2}(1+\cos 2\phi_0)$
$\sin^2\phi_0 = \frac{1}{2}(1-\cos 2\phi_0)$

$$\therefore \vec{E}(z=d) = E_0\Big[i \sin 2\phi_0 \sin\left(\frac{\delta k d}{2}\right)\hat{i}$$

$$+\Big\{\frac{1}{2}\left(e^{i\frac{\delta k d}{2}} + e^{-i\frac{\delta k d}{2}}\right)$$

$$-\frac{\cos 2\phi_0}{2}\left(e^{i\frac{\delta k d}{2}} - e^{-i\frac{\delta k d}{2}}\right)\Big\}\hat{j}\Big] e^{i(\bar{k}d-\omega t)}$$

(a)

$$\vec{E}(z=d) = E_0\Big[\left(i \sin 2\phi_0 \sin\Delta\right)\hat{i}$$

$$+\left(\cos\Delta - i\cos 2\phi_0 \sin\Delta\right)\hat{j}\Big] e^{i(\bar{k}d-\omega t)}$$

(b) $$|E_x|^2 + |E_y|^2 = E_0^2$$

to be circularly polarized

$$E_x\hat{i} + E_y\hat{j} = E_0 e^{i\alpha}\frac{(\hat{i} \pm i\hat{j})}{\sqrt{2}}$$

$$\therefore \frac{E_y}{E_x} = \pm i = -\frac{i\cot\Delta}{\sin 2\phi_0} - \cot 2\phi_0$$

$\therefore\ 2\phi_0 = \pm\pi/2$ and

$$\Delta = \pi/4$$

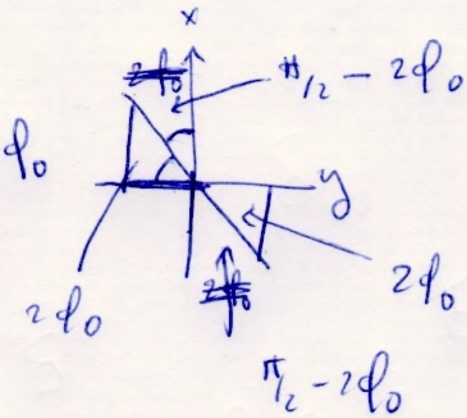
#5

(c) $\cos\delta = 0$

$$\frac{\omega d}{c\,2}\delta_m = \delta = \pi/2$$

$$\boxed{d_{min} = \frac{c\pi}{\omega\,\delta_m}}$$

$$\frac{E_x}{E_y} = -\tan 2\varphi_0$$



Green = S I   Red = Gaussian

b

$$\vec{H} = \frac{\vec{B}_0}{\mu_0} - 4\pi \vec{M} \longleftarrow \boxed{\vec{m} = m_0 \hat{k} \text{ for } r \le a \,!}$$

Ampère's law $\boxed{\vec{\nabla} \times \vec{H} = 0 \Rightarrow \vec{H} = -\vec{\nabla}\phi_M}$,

Junction condition

$B_n = \text{cont}$ at $r=a$, $H_t = \text{cont}$ at $r=a$ $\Leftrightarrow$ $\phi_M = \text{cont}$ at $r=a$

For $r>a$ $\vec{B} = -\mu_0 \vec{\nabla}\Phi_M$

$\boxed{\phi_M = \frac{1}{4\pi}\int \frac{\vec{m}\cdot d\vec{A}'}{|\vec{r}-\vec{r}'|}}$ could use but didn't

$$\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \nabla^2 \Phi_M = 0$$

$$\left(\begin{matrix}\text{axial symmetry} \\ + \\ \text{vanish at } \infty\end{matrix}\right) \Rightarrow \Phi_M = \sum_{\ell=0}^{\infty} \frac{b_\ell P_\ell(\cos\theta)}{r^{\ell+1}}$$

For $r<a$ $\frac{\vec{B}}{\mu_0} = -\vec{\nabla}\Phi_m + 4\pi\vec{M}$

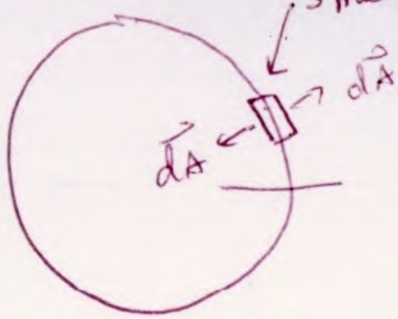
$$\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \nabla^2\Phi_m = 4\pi\vec{\nabla}\cdot\vec{M} = 0 \left(\text{except on } r=a\,!\right)$$

$$\therefore \Phi_m = \sum_{\ell=0}^{\infty} a_\ell r^\ell P_\ell(\cos\theta) \quad \text{for } r<a$$

Φ_m cont at $r=a$

$$\Rightarrow \boxed{\frac{b_\ell}{a^{\ell+1}} = a_\ell a^\ell \quad \text{all } \ell\,!}$$

#6 cont



$$\int_{V_3} \nabla^2 \Phi_m \, d^3r = \oint_{\partial V_3} \vec{\nabla} \Phi_m \cdot d\vec{A}$$

$$= 4\pi \oint_{\partial V_3} \vec{M} \cdot d\vec{A} \qquad = \left[\left.\frac{\partial \Phi_m}{\partial r}\right|_{a+} - \left.\frac{\partial \Phi_m}{\partial r}\right|_{a-} \right] dA$$

Bound surface magnetic charge $\rightarrow$

inside only $\rightarrow$ $4\pi M_0 \oint_{\partial V_3} \hat{r} \cdot d\vec{A}$

outside $= 0$

$$-4\pi M_0 \cos\theta = \left.\frac{\partial \Phi_m}{\partial r}\right|_{r=a_1}^{r=a_+}$$

(B_{normal} = cont at $r = a$
$\Rightarrow \Delta H_{normal} = -4\pi \Delta M_{normal}$)

$\uparrow$ jump in H_{normal}

$$\therefore \quad \boxed{-4\pi M_0 P_1(\cos\theta) = \sum_l \left(\frac{-(l+1) b_l}{a^{l+2}} - l a_l r^{l-1} \right) P_l(\cos\theta)}$$

$$\boxed{\begin{aligned} &\Rightarrow -4\pi M_0 = -2\frac{b_1}{a^3} - a_1 \\ &0 = \frac{-(l+1)b_l}{a^{l+2}} - l a_l a^{l-1} \qquad l \neq 1 \end{aligned}}$$

Combine with Φ_m cont gives

$a_l = b_l = 0 \quad l \neq 1$

$$-4\pi M_0 = -\frac{2b_1}{a^3} - \frac{b_1}{a^3} = -3\frac{b_1}{a^3} = -3a_1$$

#6 cont

$\therefore$

$$\Phi_{M_{in}} = a_1 r \cos\theta = \frac{4\pi M_0}{3} r\cos\theta \qquad r<a$$

$$\Phi_{M_{out}} = \frac{b_1}{r^2}\cos\theta = 4\pi\frac{M_0 a^3}{3r^2}\cos\theta \qquad r>a$$

$\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$

$$\boxed{\vec{H}_{in} = -\vec{\nabla}\phi = -4\pi\frac{M_0}{3}\hat{k} \qquad r<a}$$

$$\boxed{H_{out} = -4\pi\frac{M_0}{3}a^3\vec{\nabla}\left(\frac{z}{r^3}\right) \qquad r>a}$$

$$\boxed{\vec{H}_{out} = 4\pi\frac{M_0}{3}a^3\left(\frac{-\hat{k}}{r^3} + \frac{3z\,\hat{r}}{r^4}\right) \qquad r>a}$$

$$\left(\frac{2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}}{r^3}\right)$$

$$\vec{B} = \mu_0\left[\vec{H} + 4\pi\vec{M}\right] =$$

$$\boxed{\vec{B}_{in} = \mu_0 4\pi M_0\left[\frac{2}{3}\hat{k}\right] \qquad r<a}$$

$$\boxed{\vec{B}_{out} = \mu_0\vec{H} = \mu_0 4\pi\frac{M_0}{3}a^3\left[\frac{-\hat{k}}{r^3} + \frac{3z\,\hat{r}}{r^4}\right] \qquad r>a}$$