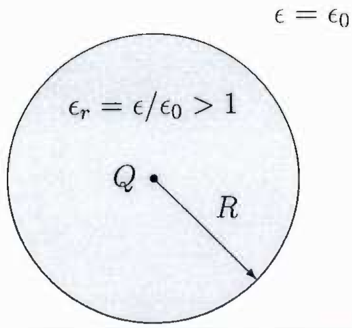
E & M Qualifier

January 9, 2014

To insure that the your work is graded correctly you MUST:

1. use only the blank answer paper provided,
2. write only on one side of the page,
3. put your alias **(NOT YOUR REAL NAME)** on every page,
4. supply 3 numbers for every page (Problem: #, Page: #, of: #). If you took 4 pages to work the 5^{th} problem then the 2^{nd} page would be (Problem: 5, Page: 2, of: 4)
5. start each problem by stating your units e.g., SI or Gaussian,
6. **do not** staple your exam when done.

Use only the reference material supplied (Schaum's Guides).



1. A point charge Q, is embedded at the center of a sphere of linear dielectric material with electric susceptibility χ_e, and radius R. If the sphere is centered on the origin, calculate the following:

   (a) [2 pts] The magnitude and direction of the electric displacement $\mathbf{D}$ inside and outside the sphere.

   (b) [2 pts] The magnitude and direction of the electric field $\mathbf{E}$ inside and outside of the sphere.

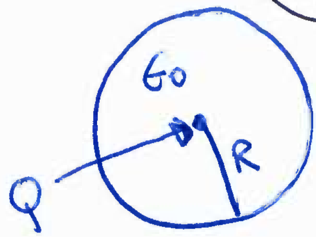
   (c) [2 pts] The magnitude and direction of the electric polarization $\mathbf{P}$ inside and outside the sphere.

   (d) [2 pts] The bound surface charge density σ_b on the sphere and the bound volume charge density ρ_b inside the sphere.

   (e) [2 pts] The total bound charge on the sphere's surface and the total bound charge inside the sphere.

1.

$\epsilon/\epsilon_0 = \epsilon_r > 1$



numerically different

$$\epsilon_r = \epsilon/\epsilon_0 = 1 + \chi_e^{SI} = 1 + \boxed{4\pi}\,\chi_e^{G}$$

↑ relative permittivity!

(a) $\vec{\nabla}\cdot\vec{D} = 4\pi\rho \Rightarrow \boxed{\vec{D} = \frac{Q}{4\pi r^2}\hat{r}}$ for all r

Green = SI

Red = Gaussian

(b)
$$\boxed{\begin{aligned}\vec{E} &= \frac{Q}{(4\pi\epsilon_0)\epsilon_r r^2}\hat{r} && r < R \\ &= \frac{Q}{(4\pi\epsilon_0) r^2}\hat{r} && r > R\end{aligned}}$$

$\vec{D} = \epsilon_0\vec{E} + \vec{P}$ SI

$\vec{D} = \vec{E} + 4\pi\vec{P}$ G

(c) $\vec{P} = \epsilon_0 \chi_e^{SI} \vec{E}$

$\vec{P} = \chi_e^{G} \vec{E}$

$$\vec{P} = \frac{\chi_e^{SI} Q}{4\pi \epsilon_r r^2}\hat{r} \quad r < R$$
$$= 0 \quad r > R$$

$$\vec{P} = \frac{\chi_e^{G} Q}{\epsilon_r r^2}\hat{r}, \quad r < R$$
$$= 0, \quad r > R$$

Same except for units in SI + Gaussian

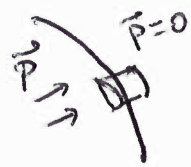
(d) $\rho_b = -\vec{\nabla}\cdot\vec{P}$ follows from $\vec{\nabla}\cdot\vec{D} = 4\pi\rho$

$$\Rightarrow \vec{\nabla}\cdot(\epsilon_0\vec{E} + 4\pi\vec{P}) = 4\pi\rho_{free}$$

$\vec{P} \propto \vec{E}$ and $\vec{\nabla}\cdot\vec{E} = 0$ in the sphere

∴ $\rho_b = 0$ for $0 < r < R$!

(singular at $r=0$ and $r=R$)



$$\int_{V_3} \overbrace{\rho_b}^{-\vec{\nabla}\cdot\vec{P}}\, dvol = \int_{\partial V_3} (-\vec{P})\cdot d\vec{a} = \vec{P}\cdot\Delta\vec{a}$$

∴ $\sigma_b = \vec{P}\cdot\hat{r}$ on the boundary

∴ $$\boxed{\sigma_b = \frac{\chi_e^{SI}}{4\pi}\frac{Q}{\epsilon_r R^2} = \frac{\chi_e^{G} Q}{\epsilon_r R^2}}$$

(e) $$\boxed{\oint \sigma_b\, da = \frac{\chi_e^{SI} Q}{\epsilon_r} = \frac{4\pi\chi_e^{G} Q}{\epsilon_r}}$$

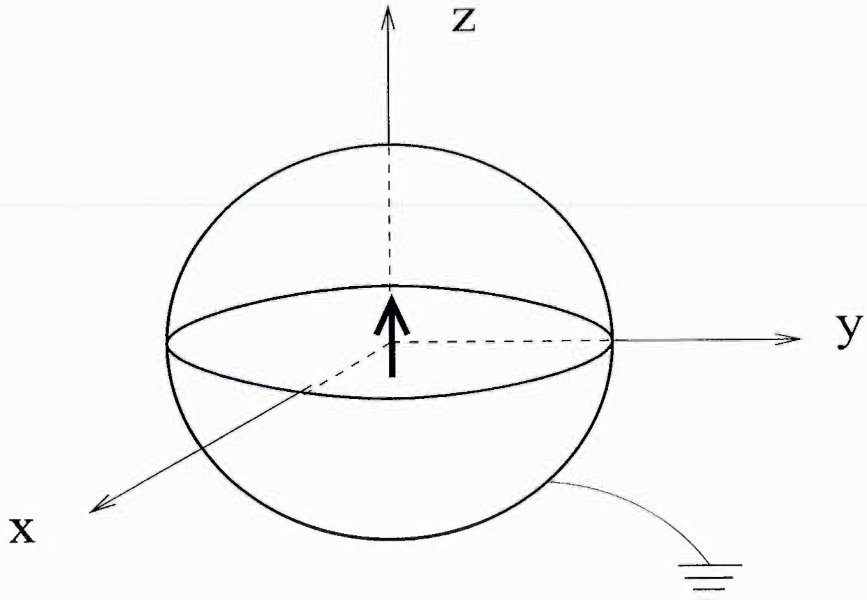
Bound charge on $r=R$!

Bound charge at $r=0$ $$= \int_{r<R} -\vec{\nabla}\cdot\vec{P}\, dvol = \int_{r=\delta>0} -\vec{P}\cdot d\vec{a} = \frac{-\chi_e^{SI} Q\, 4\pi\delta^2}{4\pi\epsilon_r\delta^2}$$

$$\boxed{= -\frac{\chi_e^{SI} Q}{\epsilon_r} = -\frac{4\pi\chi_e^{G} Q}{\epsilon_r}}$$

net $= 0$!

2. A point electric dipole with dipole moment $\mathbf{p} = p_0\,\hat{\mathbf{z}}$ is located at the center of a thin hollow, grounded, conducting sphere of radius R.



(a) [2 pts] For general boundary conditions like those given in the figure, i.e., sources are located at the origin and the potential is given at $r = R$, a solution to the Laplace equation in spherical-polar coordinates with azimuthal symmetry ($\nabla^2\Phi(r,\theta) = 0$ with no ϕ dependence) can be written as a sum of Legendre polynomials

$$\Phi(r,\theta) = \sum_\ell f_\ell(r) P_\ell(\cos\theta).$$

Give the r dependence of each function $f_\ell(r)$.

(b) [2 pts] For the explicit boundary conditions shown in the figure what are the limiting values of the electrostatic potential at $r = R$ and at $r \sim 0$?

(c) [2 pts] What constraints do the boundary conditions in (b) place on the functions $f_\ell(r)$?

(d) [2 pts] Give the electrostatic potential inside the sphere.

(e) [2 pts] Compute the charge density σ on the inside surface on the grounded sphere.

2.

(a) $$\phi(r,\theta) = \sum_{l=0}^{\infty} \underbrace{\left[a_l r^l + \frac{b_l}{r^{l+1}} \right]}_{= f_l(r)} P_l(\cos\theta)$$

SI

(b) $$\phi(r \to 0, \theta) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p}\cdot\vec{r}}{r^3} \overset{①}{=} \boxed{\frac{p_0 \, \overset{\cos\theta}{\overset{\parallel}{P_1(\cos\theta)}}}{4\pi\epsilon_0 \, r^2}}$$

② $\boxed{\phi(r=R,\theta) = 0}$

(c) ① $\Rightarrow$ $b_l = 0$ for $l \neq 1$, $\boxed{b_1 = \frac{p_0}{4\pi\epsilon_0}}$

② $\Rightarrow$ $a_l R^l + \frac{b_l}{R^{l+1}} = 0$

combine to get
$$\boxed{\begin{aligned} a_1 &= -b_1/R^3 = -\frac{p_0}{4\pi\epsilon_0 R^3} \\ a_l &= b_l = 0 \text{ for } l \neq 1 \end{aligned}}$$

(d) $$\boxed{\phi(r,\theta) = \frac{p_0}{4\pi\epsilon_0}\left(-\frac{r}{R^3} + \frac{1}{r^2} \right) \cos\theta}$$

(e) $\vec{\nabla}\cdot\vec{D} = 4\pi\rho \Rightarrow \int \vec{D}\cdot d\vec{a} = 4\pi Q_{in}$

$$\parallel$$

$$-\vec{D}\cdot\hat{r}\,\Delta A = 4\pi\,\sigma\,\Delta A$$

$$\Rightarrow \sigma = -\frac{\epsilon_0 \vec{E}\cdot\hat{r}}{4\pi}$$

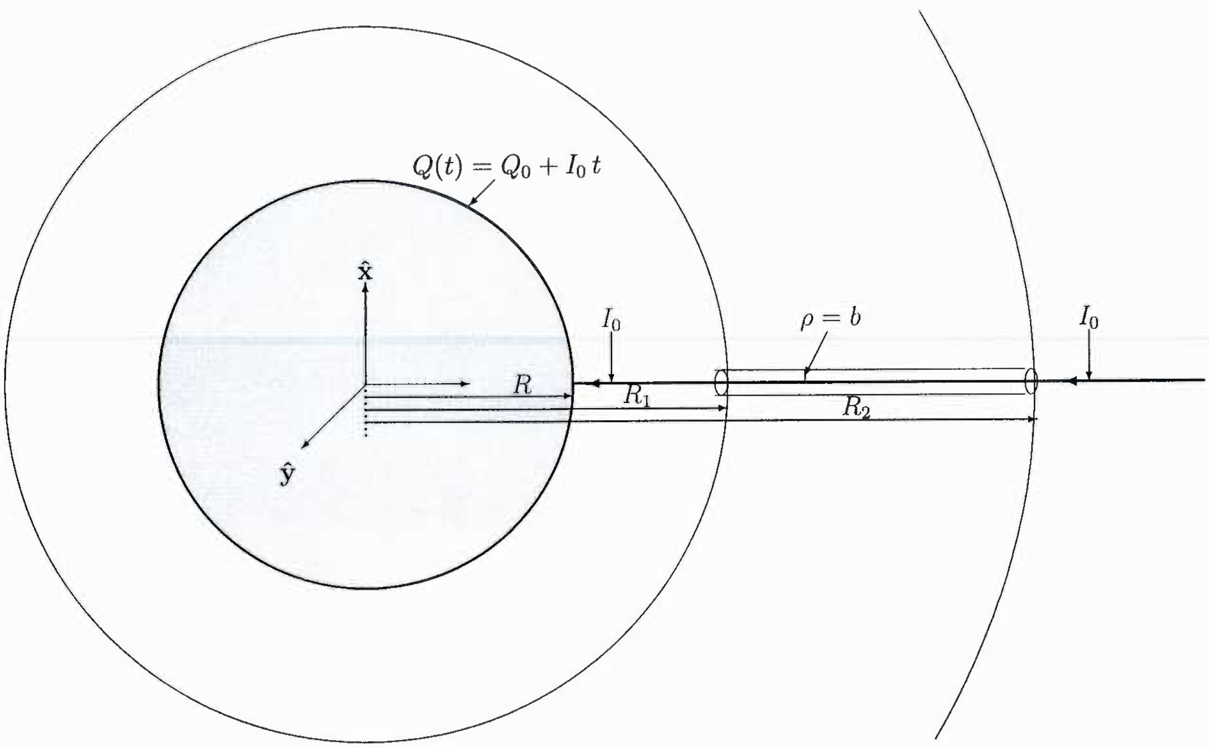
$$E^r = -\frac{\partial \phi}{\partial r}$$

$$\therefore\ \sigma = \frac{\epsilon_0 P_0}{4\pi}\left[-\frac{1}{R^3}-\frac{2}{R^3}\right]\cos\theta$$

$$\boxed{\sigma = -\frac{\epsilon_0}{4\pi}\frac{3P_0}{R^3}\cos\theta}$$

$\epsilon_0 =$ SI

$4\pi =$ Gaussin



3. A very thin straight wire (on the z-axis) carries a constant current I_0 from infinity radially inward to a spherical conducting shell of radius R to which the wire is attached. Assume the time dependent charge on the surface of the shell is uniformly distributed and can be approximated by $Q(t) = Q_0 + I_0 t$. This time dependent charge causes the electric field in the space outside the sphere to increase and hence the energy density stored in electric field to increase. Neglect retardation effects when answering the following.

   (a) [2 pts] Calculate the electric field $\mathbf{E}(t, r)$ as a function of distance for $r > R$ from the center of the spherical shell due to $Q(t)$.

   (b) [2 pts] Calculate the energy $U(t)$ stored in the electric field in the region $R_1 \leq r \leq R_2$ where $R < R_1$.

   (c) [2 pts] Calculate the magnetic field $\mathbf{B}(r, \rho)$ caused by I_0 at the surface of a thin cylinder ($\rho = b \ll R$) that surrounds the wire on the z-axis ($z > R$) .

   (d) [2 pts] Use your $\mathbf{E}(t, r)$ and $\mathbf{B}(r, \rho)$ fields to calculate the Poynting vector on the surface of the cylinder. Assume the cylinder is so small in diameter that the electric field is approximately tangent to the cylinder's surface.

   (e) [2 pts] Use the Poynting vector to show that the rate energy leaves the part of the wire between $R_1 \leq r \leq R_2$ equals the rate of change of the energy stored in the electric field calculated in (b).

Y1

#3 (a) $\vec{\nabla}\cdot\vec{D} = 4\pi\rho \Rightarrow \boxed{\vec{E} = \frac{Q(t)}{4\pi\epsilon_0 r^2}\hat{r}}$

(c) $\vec{\nabla}\times\vec{H} = \frac{4\pi}{c}\vec{J} \Rightarrow \boxed{\vec{B} = \frac{\mu_0}{4\pi c}\frac{2I_0}{\rho}(-\hat{\phi})}$

Green = SI
Red = Gaussian

(b) $u = \frac{1}{4\pi}\frac{\vec{E}\cdot\vec{D}}{2} = \frac{\epsilon_0}{4\pi}\frac{E^2}{2}$

$$U(t) = \int u\, dvol = \frac{\epsilon_0}{4\pi\, 2}\frac{Q^2(t)}{(4\pi\epsilon_0)^2}\int_{R_1}^{R_2}\frac{4\pi r^2\, dr}{r^4}$$

$$\boxed{U(t) = \frac{Q^2(t)}{(4\pi\epsilon_0)\, 2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

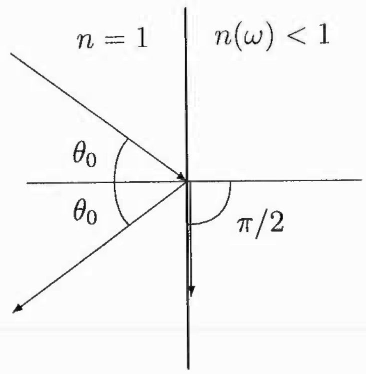
$Q(t) = Q_0 + I_0 t$

(d) $\boxed{\vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{H} = \frac{1}{(4\pi\epsilon_0)}\frac{Q(t)I_0}{2\pi r^2\rho}\hat{\rho}}$

(e) $\int \vec{S}\cdot d\vec{a} = \frac{1}{(4\pi\epsilon_0)}\frac{Q(t)I_0}{\cancel{2\pi\rho}}\int_{R_1}^{R_2}\frac{dr}{r^2}\cancel{2\pi\rho}$

$$= \boxed{\frac{1}{(4\pi\epsilon_0)}Q(t)I_0\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}$$

$= \frac{d}{dt}U(t)$ from (b)



4. X-rays of a fixed angular frequency ω which are incident on a metallic surface at an angle θ (relative to the normal) that is greater than a critical angle $\theta_0(\omega)$ are totally reflected.

   (a) [2 pts] Assume you know the index of refraction of the metal is $n(\omega)$ and that it's value is less than 1. Use Snell's law to calculate the critical angle $\theta_0(\omega)$.

   (b) [3 pts] Assume the electrons in the metal behave as if they are as completely free as electrons in a plasma. As the wave penetrates into the metal each free electron is accelerated by the x-ray's time dependent electric field (a wave $\propto E_0 e^{-i\omega t}$). Calculate the electron's steady state motion as a function of time in response to x-ray's electric field.

   (c) [2 pts] Knowing the steady steady state motion of the electrons from (b) calculate each electron's contribution to the polarization density as a function of the electric field.

   (d) [3 pts] Assuming the metal contains n_e free electrons per unit volume, you can now calculate the index of refraction $n(\omega)$ caused by the free electrons. Assume that $\mu = \mu_0$ and that only the electrons are contributing to ϵ.

#4

Snell's law

(a) $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$1 \sin\theta_0(\omega) = n(\omega)\, 1$$

$$\therefore \boxed{\theta_0(\omega) = \mathrm{ArcSin}(n(\omega))}$$

(b) $m\ddot{\vec{r}} = -e\vec{E} = -eE_0 e^{-i\omega t}\hat{x}$

$$\therefore \ddot{x} = -\frac{e}{m} e^{-i\omega t} \Rightarrow x = \frac{e/m}{\omega^2} e^{-i\omega t} E_0$$

$$\therefore \boxed{\vec{r} = \frac{e/m}{\omega^2}\vec{E}}$$

(c) $\vec{p} = -e\vec{r} = \boxed{\frac{-e^2/m}{\omega^2}\vec{E}}$

$\vec{D} = \epsilon_0\vec{E} + 4\pi\vec{P}$

$\vec{D} = \epsilon\vec{E}$

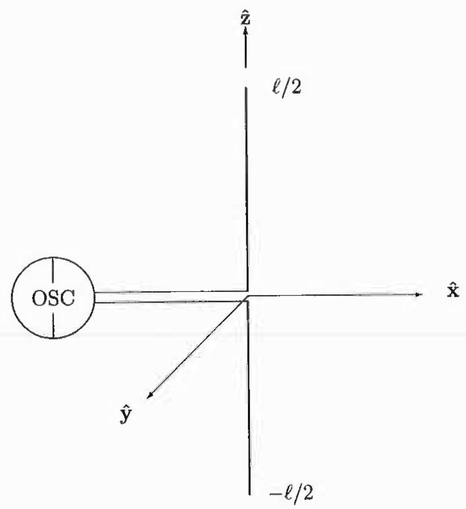
$\vec{P} = n_e\vec{p}$

$$\epsilon_r = \epsilon/\epsilon_0 = \left(1 - \frac{4\pi n_e e^2}{\epsilon_0 m \omega^2}\right)$$

Plasma frequency $= \omega_p$

$$= 1 - \omega_p^2/\omega^2, \quad \omega_p^2 \equiv \frac{4\pi n_e e^2}{\epsilon_0 m}$$

(d) $n = \sqrt{\epsilon_r \mu_r} = \sqrt{\epsilon_r} = \sqrt{1 - \omega_p^2/\omega^2}$



5. Consider a half-wave antenna (length $\ell = \lambda/2$, see the figure) centered on the origin and aligned with the z-axis . The antenna is driven by an alternating signal ($\omega/k = \nu\lambda = c$)applied to its center which produces a current in the antenna given by

$$\begin{aligned} I(z,t) &= I_0 \cos(kz)\sin(\omega t), |z| \leq \ell/2 \\ &= 0, |z| > \ell/2, \end{aligned}$$

(a) [2 pts] Give a 1-dimensional integral expression for the retarded vector potential $\mathbf{A}(t,\mathbf{r}) = A^z(t,r,\theta)\,\hat{\mathbf{z}}$ for this antenna using

$$\mathbf{A}(t,\mathbf{r}) = \left(\frac{\mu_0}{4\pi}\right)\int \frac{\mathbf{J}(t-|\mathbf{r}-\mathbf{r}'|/c,\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^3r', \qquad \left(\frac{\mu_0}{4\pi}\right) \longrightarrow \frac{1}{c} \text{ in Gaussian units.}$$

(b) [3 pts] Evaluate your integral from (a) assuming $|\mathbf{r}| \gg \ell$,i.e., assume

$$|\mathbf{r}-\mathbf{r}'| = r - z'\cos\theta + \mathcal{O}\left(\frac{1}{r}\right), \text{ and } \frac{1}{|\mathbf{r}-\mathbf{r}'|} = \frac{1}{r} + \mathcal{O}\left(\frac{1}{r^2}\right).$$

You only need to keep these terms if you want to find the radiation part of $\mathbf{B}$. The form of the integral you will need to evaluate is

$$\int_{-\pi/2}^{\pi/2} \cos(x)\sin(a+bx) = \frac{2\cos(\frac{\pi}{2}b)\sin(a)}{1-b^2}.$$

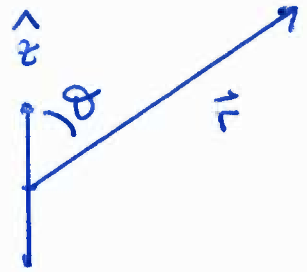
(c) [3 pts] Calculate the radiation part of $\mathbf{B}$ using $\mathbf{B} = \nabla\times\mathbf{A} = (\nabla A^z)\times\hat{z}$ and the radiation part of $\mathbf{E}$ using

$$\mathbf{E}_{\text{rad}} = (c)\mathbf{B}_{\text{rad}} \times \hat{\mathbf{r}}, \qquad (c) \longrightarrow 1 \text{ in Gaussian units.}$$

Remember that the radiation parts are those that are $\propto 1/r$ for large $r \gg \ell$. Also recall that $\nabla\theta \propto 1/r$ and $\nabla(1/r) \propto 1/r^2$

(d) [2 pts] Calculate the time average of the Poynting vector $\langle\mathbf{S}\rangle$ as a function of (r,θ) for $r \gg \ell$ and plot (sketch) $|\langle\mathbf{S}\rangle|$ as a function of θ with $r = $ constant.

#5



$k = 2\pi/\lambda = \pi/\ell = \omega/c$

Green = SI   Red = Gaussian

$$\vec J\left(t-\frac{|\vec r-\vec r\,'|}{c},\vec r\,'\right)d^3r' = I\left(z', t-\frac{|\vec r - z'\hat z|}{c}\right)dz'\,\hat z$$

(a)

$$\therefore\ \vec A(t,\vec r) = \frac{\mu_0}{4\pi}\left(\frac{1}{c}\right)\int_{-\ell/2}^{+\ell/2} \frac{I_0\cos(kz')\sin\left[\omega\left(t-\frac{|\vec r-z'\hat z|}{c}\right)\right]dz'}{|\vec r-z'\hat z|}\,\hat z$$

SI   G

$$\vec A(t,\vec r)\cong \frac{\mu_0}{4\pi c}\frac{I_0}{r}\int_{-\ell/2}^{+\ell/2}\cos kz'\,\sin\left[\omega\left(t-\frac{r}{c}+\frac{z'\cos\theta}{c}\right)\right]dz'\,\hat z$$

rad part only

$kz' = x\,,\quad a = \omega(t-r/c)\,,\quad b = \cos\theta$

(b)

$$\therefore\ \vec A(t,\vec r)\cong \frac{\mu_0}{4\pi c}\frac{I_0}{r}\int_{-\pi/2}^{\pi/2}\cos(x)\sin(a+bx)\,\frac{dx}{k}\,\hat z$$

$$\boxed{\vec A(t,\vec r) = \frac{\mu_0}{4\pi c}\frac{I_0\,\ell}{\pi\, r}\,\frac{2\cos\left(\frac{\pi}{2}\cos\theta\right)\sin\left[\omega(t-r/c)\right]}{\sin^2\theta}\,\hat z}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$= \frac{\mu_0}{4\pi c} \frac{2 I_0 \ell}{\pi} \vec{\nabla}\left(\frac{\cos(\frac{\pi}{2}\cos\theta) \sin[\omega(t - r/c)]}{r \sin^2\theta} \right) \times \hat{z}$$

$$\cos\theta = z/r \qquad \vec{\nabla}\theta = \frac{1}{r}\hat{z} - z\vec{r}/r^3 \propto 1/r$$

$$\vec{\nabla}\frac{1}{r} = -\vec{r}/r^3 \propto 1/r^2$$

$$\vec{B}_{rad} \cong \frac{\mu_0}{4\pi c} \frac{2 I_0 \ell}{\pi} \frac{\cos(\frac{\pi}{2}\cos\theta)}{r \sin^2\theta} \cos[\omega(t - r/c)] \left(\frac{\omega \hat{r}}{c} \times \hat{z}\right)$$

$$\downarrow$$

$$+\frac{\omega}{c} \sin\theta \, \hat{\phi}$$

$$\boxed{\vec{B}_{rad} = \frac{\mu_0}{4\pi c} 2 I_0 \frac{\cos(\frac{\pi}{2}\cos\theta)}{r \sin\theta} \cos[\omega(t - \frac{r}{c})] \hat{\phi}}$$

(c)

$$\boxed{\vec{E}_{rad} \cong \frac{\mu_0 c \, 2 I_0}{4\pi c} \frac{\cos(\frac{\pi}{2}\cos\theta)}{r \sin\theta} \cos[\omega(t - r/c)] \hat{\theta}}$$

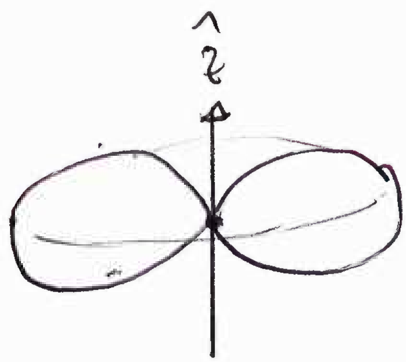
(d) S~~ketch a graph of $\langle|\vec{S}|\rangle$ as a fn of θ~~

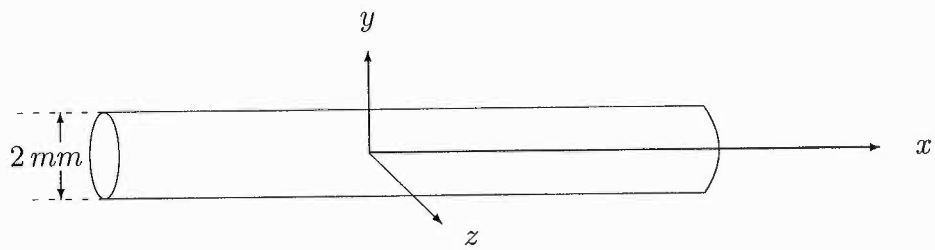
$$\vec{S} = \frac{c}{4\pi} \vec{E} \times \vec{H} = \frac{c}{4\pi} \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

$$\vec{S} = \frac{1}{4\pi c}\frac{c\mu_0}{(4\pi)^2}\left[\frac{2I_0\cos(\frac{\pi}{2}\cos\theta)}{r\sin\theta}\right]^2 \cos^2[\omega(t-r/c)]\,\hat{r}$$

$$\boxed{\langle\vec{S}\rangle = \frac{1}{4\pi c}\frac{1}{4\pi\epsilon_0}\frac{2I_0^2\cos^2(\frac{\pi}{2}\cos\theta)}{r^2\sin^2\theta}\,\hat{r}}$$

$$\left.\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right|_{\theta\to 0} = 0 \qquad \left.\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right|_{\theta\to \pi/2} = 1$$





6. A green laser pointer (wavelength = 550 nano-meters) has a power of 50 milliwatts with a 2 millimeter beam diameter. Assume the beam can be represented by a plane electromagnetic wave with fields

$$E^y = E_0 \sin(kx - \omega t),$$
$$B^z = B_0 \sin(kx - \omega t),$$

confined to a cylinder of diameter 2 mm. As with any wave the wave number k is related to wavelength λ by $k = 2\pi/\lambda$ and ω is related to the frequency f by $\omega = 2\pi f$.

(a) [2 pts] If the beam travels in vacuum use Maxwell's equations to find the numerical value of f and to relate B_0 to E_0.

(b) [3 pts] Compute the Poynting vector and the energy density at time t and position $\mathbf{r}$ for the above wave as a function of E_0.

(c) [2 pts] What is the time average of the above two quantities at a position within the beam?

(d) [3 pts] Use one or both of the above time-averages and the known 50 milliwatt power of the laser to determine the amplitudes E_0 and B_0.

1/2

6.

(a) $\lambda f = c \Rightarrow f = c/\lambda = \dfrac{3\times10^{8}\,\text{m/s}}{550\times10^{-9}\,\text{m}} = \boxed{5.45\times10^{14}\,\text{s}^{-1}}$

these come from Max eqns below !

$B_0 = \dfrac{E_0}{c}$

$\vec\nabla\times\vec E + \frac{1}{c}\frac{\partial\vec B}{\partial t} = 0$ $\qquad$ $\vec\nabla\times\vec H - \frac{1}{c}\frac{\partial\vec D}{\partial t} = 0 \leftarrow J=0$

$k E_0 \cos(kz-\omega t)\,\hat z$ $\qquad$ $-\frac{B_0}{\mu_0} k \cos(kx-\omega t)\,\hat y$

$+\frac{1}{c}(-\omega) B_0 \cos(kz-\omega t)\,\hat z = 0$ $\qquad$ $-\frac{1}{c}(-\omega)\epsilon_0 E_0 \cos(kx-\omega t)\,\hat y = 0$

$\therefore \boxed{k E_0 = \dfrac{\omega B_0}{c}}$ $\qquad$ $\therefore \boxed{\dfrac{B_0 k}{\mu_0} = \dfrac{\omega \epsilon_0 E_0}{c}}$

combine to get

$\dfrac{k}{\omega} c = \dfrac{\omega}{k c c^2} \Rightarrow \boxed{\dfrac{\omega}{k} = c}$

$f\lambda$

Green = SI

Red = Gaussian

$$\boxed{\frac{B_0}{E_0} = \frac{k}{\omega} c = \frac{1}{c}}$$

(b) $\vec S = \dfrac{c}{4\pi}\vec E\times\vec H = \boxed{\dfrac{c}{4\pi}\epsilon_0 E_0^2 \sin^2(kx-\omega t)\,\hat x}$

$u = \dfrac{1}{4\pi}\dfrac{(E\cdot D + H\cdot B)}{2} = \dfrac{\epsilon_0 E_0^2 + B_0^2/\mu_0}{(4\pi)\;2}\sin^2(kx-\omega t)$

$u = \boxed{\dfrac{\epsilon_0 E_0^2}{4\pi}\sin^2(kx-\omega t)}$

(c)

$$\langle \vec{S} \rangle = \frac{c\,\epsilon_0 E_0^2}{4\pi \; 2}\,\hat{x}$$

$$\langle u \rangle = \frac{\epsilon_0 E_0^2}{4\pi \; 2}$$

(d)

$$\frac{c\,\epsilon_0\,E_0^2}{4\pi \; 2} = \frac{50\,(mW)}{(m)^2} = 50 \text{ ergs/sec/cm}^2$$

$$E_0 = \sqrt{\frac{100\,(10^{-3}\,W)/m^2}{3\times10^{8}\,m/s\;\epsilon_0}} \quad \text{in SI}$$

$$\boxed{E_0} = \sqrt{12\pi}\ \text{volt/m} = \boxed{6.14\ \text{volt/m in SI}}$$

$$E_0 = \sqrt{\frac{4\pi\;100\,(10^{-3}W)/m^2}{(3\times10^{10}\,cm/s)}}$$

$$\boxed{E_0 = \sqrt{12\pi}\;\frac{10^{-4}}{3}\;(\text{statvolt/cm})} \quad \text{in Gaussian}$$

$$\boxed{\begin{array}{l} B_0 = E_0/c = 2.0\times10^{-8}\ \text{Tesla} \quad \text{in SI} \\ B = E_0 = 2.0\times10^{-4}\ \text{Gauss} \end{array}}$$