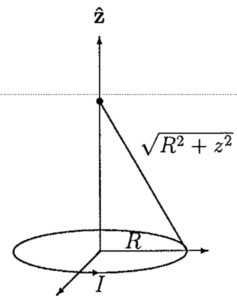

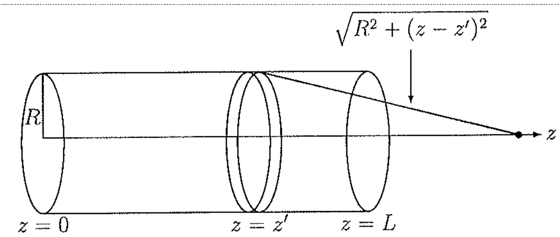

2. (a) [3 pts] A circular loop of radius R, centered on the origin and $z=0$ plane, carries a current I. Find the magnetic field B on the axis of the loop as a function of the distance z from the center of the loop.

   (b) [4 pts] Use the result of part (a) to find B along the axis of a solenoid of radius R and length L, uniformly wound with $n = N/L$ turns per unit length.

   (c) [3 pts] Assume that instead of a solenoid you had a cylinder of radius R and length L made out of a piece of uniformly magnetized iron with magnetization $\mathbf{M}$ pointing along the axis of the solenoid. Use the solution of part (b) to calculate the magnetic field strength $\mathbf{H}$ and the magnetic induction $\mathbf{B}$ along the axis of the cylinder, both inside and outside.

HINTS:

$$\int \frac{dw}{[R^2+w^2]^{3/2}} = \frac{w}{\sqrt{R^2+w^2}} + \text{constant}.$$

The bound volume and surface current densities associated with a smooth magnetization density are respectively

$$\mathbf{J}_b|_{SI} = \nabla \times \mathbf{M},$$

and

$$\mathbf{K}_b|_{SI} = \mathbf{M} \times \mathbf{n},$$

where $\mathbf{n}$ is the outward unit normal at the magnet's boundary. The Gaussian expressions for $\mathbf{J}_b$ and $\mathbf{K}_b$ contain an additional factor of c in the numerators.

Aug 2013

E+M #2

SI

a) We can use Biot-Savart Law to determine $\vec{B}$

$$B = \frac{\mu_0}{4\pi}\int \frac{\vec{J}\times(\vec{r}-\vec{r}')}{|r-r'|^3}\,dr'$$

$$B = \frac{\mu_0}{4\pi}\int \frac{I\,dl\,\hat{\varphi}\times(r-r')}{(R^2+z^2)^{3/2}}$$

* evaluate cross product:

$$\begin{vmatrix} \hat{r} & \hat{\varphi} & \hat{z} \\ 0 & dl & 0 \\ R & 0 & z \end{vmatrix} = \langle z\,dl,\ 0,\ R\,dl\rangle$$

$$B = \frac{\mu_0}{4\pi}\int \frac{IR\,dl}{(R^2+z^2)^{3/2}}\hat{z}$$ (ignore $\hat{r}$ compoment due to symmetry

$$= \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{(R^2+z^2)^{3/2}}\hat{z}$$

$$= \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}\hat{z}$$

b) * Typically, when going from current loop to solenoid, $I \to In\,dz$. Also, because our solenoid is of a finite length, $z \to z-z'$

$$\hookrightarrow dB = \frac{\mu_0 n I\,dz'\,R^2}{2(R^2-(z-z')^2)^{3/2}}\hat{z}$$

$$B = \frac{\mu_0 n I}{2}\int_0^L \frac{R^2\,dz'\,\hat{z}}{(R^2-(z-z')^2)^{3/2}}$$

$$= \frac{\mu_0 n I R^2}{2}\left[\frac{z'}{(R^2+(z-z')^2)}\Bigg|_0^L\right]$$

$$= \frac{\mu_0 n I R^2 L}{2(R^2+(z-L)^2)^{1/2}}$$

#2 (cont.)

c)