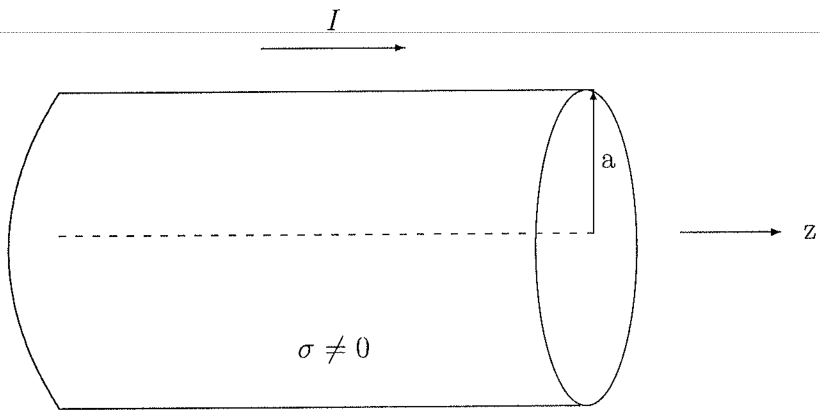
3. Poynting Vector



A straight metal wire of conductivity σ and cross-sectional area $A = \pi a^2$ carries a uniform, steady current I.

(a) (2 pts) Calculate $\mathbf{E}$ at the surface of the wire.

(b) (2 pts) Calculate $\mathbf{B}$ at the surface of the wire.

(c) (1 pts) Calculate the direction and magnitude of the Poynting vector at the surface of the wire.

(d) (3 pts) Integrate the normal component of the Poynting vector over the surface of the wire for a segment of length L.

(e) (2 pts) compare your result for (d) with the Joule heat produced in this segment.

The Poynting vector is

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}, \qquad (SI)$$

$$\mathbf{S} = \frac{c}{4\pi}\mathbf{E} \times \mathbf{H}. \qquad (Gaussian)$$

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E+M #3

a) From Ohms Law, we know $\vec{J} = \sigma\vec{E}$, and that the total current I is related to $\vec{J}$ via ~~$I = \oint \vec{J} \cdot d\vec{a}$~~

$$\Rightarrow \quad I = \oint \sigma E \cdot d\vec{a}$$

$$I = \sigma \oint E \cdot d\vec{a}$$

$$I = \sigma E \cdot \pi a^2$$

$$\frac{I}{\sigma \pi a^2}\hat{z} = \vec{E}$$

b) We can calculate $\vec{B}$ using Ampere's Law

$$\oint \vec{B} \cdot dl = \frac{4\pi}{c} I_{enc}$$

$$B \cdot 2\pi a = \frac{4\pi}{c} I$$

$$\vec{B} = \frac{2I}{ca}\hat{\varphi} \quad \text{(from RHR)}$$

c) In Gaussian units, $\vec{S} = \frac{c}{4\pi}\vec{E} \times \vec{H}$, $B = H + 4\pi M$

$$\vec{S} = \frac{c}{4\pi}\begin{vmatrix} \hat{\rho} & \hat{\varphi} & \hat{z} \\ 0 & 0 & \frac{I}{\pi\sigma a^2} \\ 0 & \frac{2I}{ca} & 0 \end{vmatrix}$$

$$= \langle -\frac{2I^2}{\pi c \sigma a^2}, 0, 0 \rangle$$

$$= \frac{-2I^2}{\pi c \sigma a^2} \cdot \frac{c}{4\pi}$$

$$= \frac{-I^2}{2\pi^2 \sigma a^3}\hat{\rho}$$

#3 (cont.)

d) We want to calculate $\oint S \cdot d\vec{a}$

$$\oint \vec{S} \cdot d\vec{a} = S \oint d\vec{a}$$

$$= \frac{-I^2}{2\pi^2 \sigma a^3} (2\pi a L)$$

$$= \frac{-I^2 L}{\sigma a^2}$$