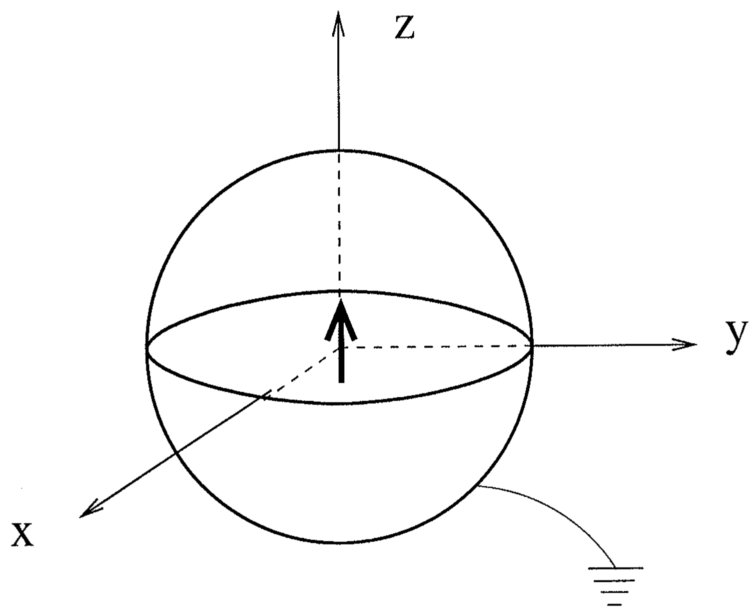
3. A point electric dipole with dipole moment $\mathbf{p} = p_0\hat{k}$ is located at the center of a hollow, grounded, conducting sphere.



(a) {2 pts} What are the boundary conditions satisfied by the electric field and electric potential in this problem?

(b) {5 pts} Compute the electrostatic potential inside the sphere.

(c) {3 pts} Compute the charge density σ on the inside surface on the grounded sphere.

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E+M #3

a) $\Phi = 0$ at the radius of the sphere (due to grounding) and at infinity

$\vec{E} \to 0$ at infinity

b) $\Phi = \frac{\vec{p}\cdot\vec{r}}{4\pi\epsilon_0 r^3} + \frac{1}{4\pi\epsilon_0}\sum_{l=0} A_l r^l P_l(\cos\theta)$

$\Phi(\vec{r}=r) = \frac{pr\cos\theta}{4\pi\epsilon_0 r^3} + \frac{1}{4\pi\epsilon_0}\sum_l A_l r^l P_l(\cos\theta)$

*Since only $P_{l=1}$ is guaranteed to go to 0 at $\vec{r}=r$, A_l, $l \neq 1 = 0$

$$0 = \frac{p\cos\theta}{4\pi\epsilon_0 r^2} + \frac{1}{4\pi\epsilon_0} A_1 r\cos\theta$$

$$0 = \frac{p}{r^2} + A_1 r$$

$$A_1 = \frac{-p}{r^3}$$

$$\Rightarrow \Phi = \frac{1}{4\pi\epsilon_0}\left(\frac{p}{r^2} - \frac{r}{a^3}\right)\cos\theta$$

c) $\int \vec{D}\cdot d\vec{a} = \sigma\Delta A$

$\vec{D}\Delta A = \sigma\Delta A$

$\hookrightarrow \sigma = -\epsilon_0 E(r=a)$

$$E = -\nabla\Phi$$

$$= \frac{p}{4\pi\epsilon_0}\left(\frac{2}{a^3} + \frac{1}{a^3}\right)\cos\theta$$

$$= \frac{3p\cos\theta}{4\pi\epsilon_0 a^3}$$

$$\hookrightarrow \sigma = -\frac{3p\cos\theta}{4\pi a^3}$$