Problem 2: Magnetostatics

Consider a sphere of radius R composed of magnetic material with a magnetization given by $\vec{M} = M_0\hat{z}$.

1. Starting with the Maxwell equations for a static magnetic field $\vec{H}$ and a static magnetic induction $\vec{B}$, prove that the magnetic field $\vec{H}$ can be written in terms of a scalar magnetic potential. From this derive the Poisson equation that solves the potential. In addition, derive expressions for the magnetic volume charge density and the bound current density from the Maxwell equations. [2 points]

2. Derive the boundary conditions on $\vec{H}$ and $\vec{B}$. Be sure to clearly define the effective surface magnetic charge density and the surface magnetic current density. [2 points]

3. Derive the fields inside and outside the sphere. *You can assume that $\vec{B}$ and $\vec{M}$ are parallel.* [6 points]

Jan 2017 # E+M #2 Gaussian

a) Maxwell's equations are

$$\nabla \cdot B = 0$$

$$\nabla \times H = \frac{4\pi}{c} \vec{J}_f \qquad \left(\nabla \times B = \frac{4\pi}{c} \vec{J}_f + \vec{J}_b \right)$$