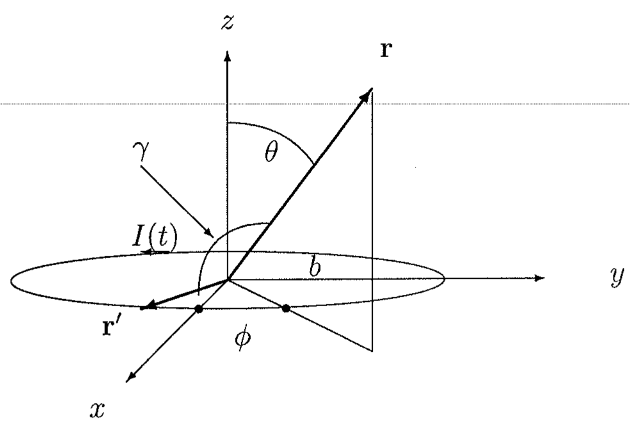

6. A circular current loop of radius b lies in the x-y plane and is centered on the origin. If the current varies harmonically with time as $I(t) = I_0 \cos(\omega t)$, use the following to carry out steps (a) through (e):

$$\begin{aligned}
\mathbf{r}' &= b\,\{\sin\theta'(\cos\phi'\hat{\imath} + \sin\phi'\hat{\jmath}) + \cos\theta'\hat{k}\} \\
&= b\,(\cos\phi'\hat{\imath} + \sin\phi'\hat{\jmath}), \\
\mathbf{r}\cdot\mathbf{r}' &= b\,r\,\cos\gamma \\
&= b\,r\,\{\cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi-\phi')\} \\
&= b\,r\,\sin\theta\cos(\phi-\phi').
\end{aligned}$$

In the above $\mathbf{r}'$ is a point on the current loop with spherical-polar coordinates $r' = b,\ \theta' = \pi/2,\ 0 \le \phi' \le 2\pi$, and γ is the angle between $\mathbf{r}$ and $\mathbf{r}'$.

(a) {2 pts} Compute the time dependent magnetic dipole moment $\mathbf{m}(t)$ of the current loop. Recall that $\mathbf{m}_{Gaussian} = \mathbf{m}_{SI}/c$.

(b) {2 pts} Give an integral expression for the retarded vector potential $\mathbf{A}(t, \mathbf{r})$.

(c) {2 pts} Approximate the integral found in (b) for $\mathbf{A}$ assuming $b << r$ and $b << c/\omega$. If you have made no mistakes your answer should agree with the potential for a point magnetic dipole, i.e., with:

$$\begin{aligned}
\mathbf{A} &= \frac{\mu_0}{4\pi}\nabla\times\left\{\frac{\mathbf{m}(t-r/c)}{r}\right\}, && SI \\
\mathbf{A} &= \nabla\times\left\{\frac{\mathbf{m}(t-r/c)}{r}\right\}. && Gaussian
\end{aligned}$$

(d) {2 pts} From your results for (c) or from the point magnetic dipole result, compute the radiation (far field) part of $\mathbf{E}$ by assuming $b << c/\omega << r$.

(e) {2 pts} Using only the radiation part, i.e., the part $\propto 1/r$, of $\mathbf{E}$ and $\mathbf{B}$, and the Poynting vector, compute the time averaged electromagnetic energy flux radiated away by the dipole as a function of the spherical polar coordinates (r, θ, ϕ). Recall that

$$\begin{aligned}
\mathbf{B} &= \frac{1}{c}\hat{r}\times\mathbf{E}, && (SI) \\
\mathbf{B} &= \hat{r}\times\mathbf{E}, && (Gaussian)
\end{aligned}$$

for radiation coming from a source at the origin.

Aug 2010

E+M #6

a) $m(t) = \frac{1}{2c}\int \vec{r} \times \vec{J}$

$= \frac{I(t)}{2c} \oint \vec{r} \circ d\vec{r}$

$*$ if $\vec{r} = b(\cos\hat{i} + \sin$