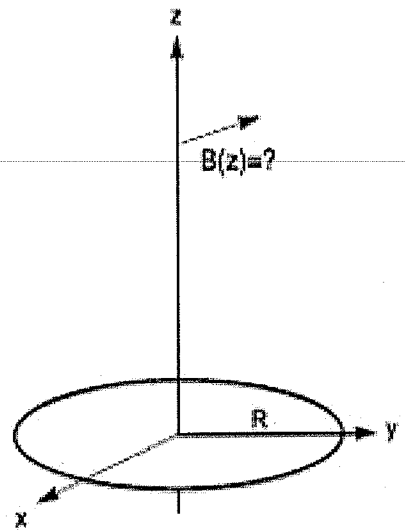
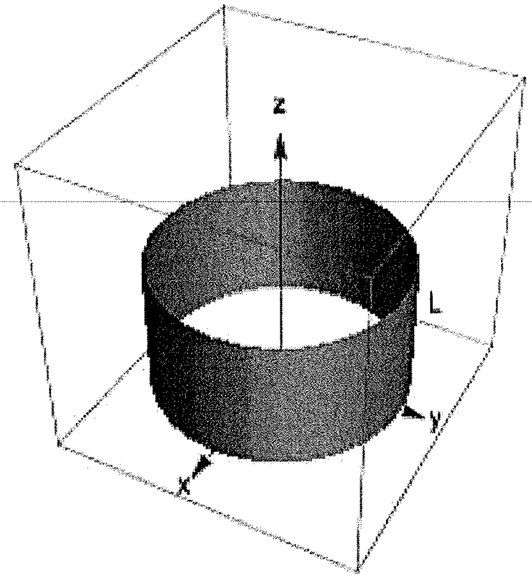
3.





(a) [5 pts] A circular loop of wire of radius R carries a current I as shown in the first figure. Find the magnitude and direction of the magnetic induction $\mathbf{B}(z)$ **on the axis** of the loop as a function of z.

(b) [5 pts] Use the result of part (a) to find $\mathbf{B}(z)$ along the axis of a solenoid of radius R and length L wound with n turns per unit length (total turns $N = n \times L$).

Hint:

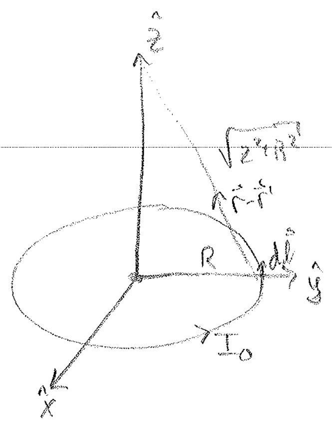
$$\int \frac{dx}{[a^2 + x^2]^{3/2}} = \frac{x}{a^2[a^2 + x^2]^{1/2}} + \text{constant}.$$

Aug 2015

E+M #3

Gaussian

a)



$|r-r'| = \sqrt{z^2+R^2}$

* We can find $\vec{B}$ via the Biot-Savart Law

$$\Rightarrow \vec{B} = \frac{1}{c}\int \frac{\vec{J}\times(\vec{r}-\vec{r}')}{|r-r'|^3}\,d^3r'$$

$$= \frac{I_0}{c}\int \frac{d\vec{l}'\times(r-r')}{|r-r'|^3}$$

$$= \frac{I_0}{c}\int \frac{dl\,R}{(z^2+R^2)^{3/2}}$$

$$= \frac{2\pi R^2 I_0}{c(z^2+R^2)^{3/2}}\,\hat{z}$$ (any non $\hat{z}$ component cancels due to circular symmetry about axis)

b)