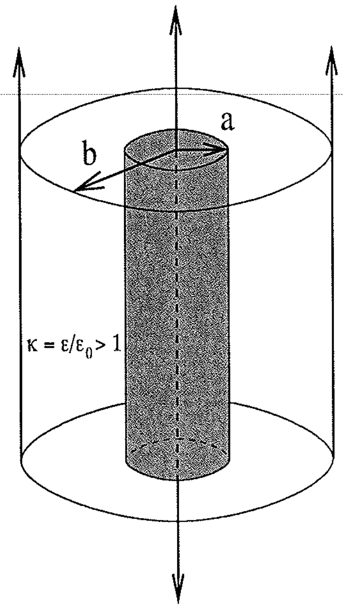

1. A very long conducting wire of radius a, carrying free positive charge per unit length λ, is surrounded by a dielectric coating of outside radius b and relative dielectric constant $\kappa = \epsilon/\epsilon_0$.

   (a) {2 pts} Find the displacement vector **D** everywhere.

   (b) {2 pts} Find the electric field **E** everywhere.

   (c) {2 pts} Find the polarization density **P** everywhere.

   (d) {2 pts} Find the bound volume charge density ρ_b and the bound surface charge density σ_b everywhere.

   (e) {2 pts} Show that the total charge densities, bound and free, produce the same **E** found in (a).

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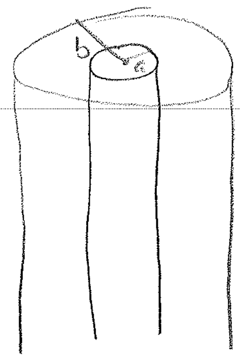
E+M #1



⇒ Long conducting wire of radius a w/ free charge per unit length λ

⇒ dielectric coating of radius b, w/ $\kappa = \epsilon/\epsilon_0$

a) Gauss Law states:

$$\oint_S \vec{D} \cdot d\vec{a} = Q_{enc}, \quad Q_{enc} = \lambda L$$

* Long wire means D has azimuthal symmetry → points along $\hat{r}$

$$\Rightarrow \oint_S \vec{D} \cdot d\vec{a} = \lambda L$$

$$D \int_S d\vec{a} = \lambda L$$

$$D 2\pi r L = \lambda L$$

$$D = \frac{\lambda}{2\pi r}\hat{r} \quad \text{(outside wire)}$$

b/c our wire is conducting, all the charge flows on the surface

assumption of evenly distributed charge means this is an electrostatics problem and any field inside the conductor causes moving charges

$$\Rightarrow D = \begin{cases} 0 & r < a \\ \frac{\lambda}{2\pi r}\hat{r} & r \geq a \end{cases}$$

b) We know that $D = \epsilon E$

$$\Rightarrow E = \begin{cases} \frac{\lambda}{2\pi\epsilon_0 r} & r > b \\ \frac{\lambda}{2\pi\kappa\epsilon_0 r} & a < r < b \\ 0 & r < a \end{cases}$$

#1 (cont.)

c) We know that $\vec{P} = \vec{D} - \epsilon_0\vec{E}$ (from $D = \epsilon_0\vec{E} + \vec{P}$)

$$\Rightarrow P = \begin{cases} 0 & r > b \\ \frac{\lambda(1-\frac{1}{k})}{2\pi r}\hat{r} & a < r < b \\ 0 & r < a \end{cases}$$

d) The bound charge is given by: $\rho_b = -\nabla\cdot\vec{P}$

$$\rho_b = -\nabla\cdot\vec{P}$$
$$= -\frac{1}{s}\frac{\partial}{\partial s}\left(s\,\frac{\lambda(1-\frac{1}{k})}{2\pi s}\right)$$
$$= -\frac{1}{s}\frac{\partial}{\partial s}\left(\frac{\lambda(1-1/k)}{2\pi}\right)$$
$$= 0$$

$\rho_b = 0$ everywhere

The surface bound charge $\sigma_b = \vec{P}\cdot\hat{n}$

↳ at $s=b$, $P = \frac{\lambda(1-\frac{1}{k})}{2\pi b}\hat{s}\cdot\hat{n}$, $\hat{n} = \hat{s}$

$$\Rightarrow \sigma_b\big|_{s=b} = \frac{\lambda(1-\frac{1}{k})}{2\pi b}$$

$$\sigma_b\big|_{s=a} = \frac{-\lambda(1-\frac{1}{k})}{2\pi a}$$ (by similar logic as above, $\hat{n} = -\hat{s}$)

e) According to Gauss Law: $\int\vec{E}\cdot d\vec{a} = Q_{enc}/\epsilon_0$

*if $s > b$: $Q_{enc} = \lambda L - \frac{\lambda(1-\frac{1}{k})}{2\pi a}(2\pi a L) + \frac{\lambda(1-\frac{1}{k})}{2\pi b}(2\pi b L)$

$$= \lambda L - \cancel{\lambda L(1-\tfrac{1}{k})} + \cancel{\lambda L(1-\tfrac{1}{k})}$$
$$= \lambda L$$

↳ $\vec{E} = \frac{\lambda}{2\pi r\epsilon_0}\hat{r}$

*if $a < s < b$: $Q_{enc} = \lambda L - \frac{\lambda(1-\frac{1}{k})}{2\pi a}(2\pi a L)$

$$= \lambda L - (1-\tfrac{1}{k})\lambda L$$
$$= \frac{\lambda L}{k}$$

↳ $= \frac{\lambda}{2\pi r k\epsilon_0}\hat{r}$

*if $s < a$: $Q_{enc} = 0$

↳ $\vec{E} = 0$

All fields match!