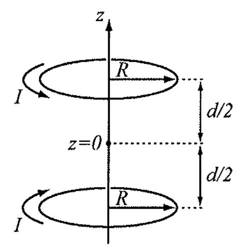
Problem 2: Magnetostatics

Consider a circular loop of radius R which carries a steady current I.

(a) Calculate the magnetic field a distance z above the center of the loop. [3 points]

(b) Now consider a configuration composed of two circular loops a distance d apart and with currents flowing in opposite directions, as shown in the figure. This configuration is know as anti-Helmholtz coils. Calculate the magnetic field along the z-axis as a function of z. [2 points]



(c) For what value of z will the the magnetic field due to the anti-Helmholtz coils be equal to zero? Give a physical explanation for your result. [2 points]

(d) Calculate the magnetic dipole moment of the configuration. [3 points]

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E+M #2

a) We calculate the magnetic field according to the Biot-Savart Law

$$\vec{B} = \frac{4\pi}{c}\int \frac{\vec{J}\times(\vec{r}-\vec{r}')}{|r-r'|^3}dr'$$

$$= \frac{4\pi}{c} I_0 \int \frac{dl \times (\vec{r}-\vec{r}')}{|r-r'|^3}$$

$$= \frac{4\pi}{c} I_0 \int \frac{R\,dl}{\sqrt{(R^2+z^2)^3}}\hat{z}$$ (from evaluation of cross product + symmetry of problem, leaving only z-component non-zero)

$$= \frac{4\pi}{c} I_0 \frac{2\pi R^2}{(R^2+z^2)^{3/2}}\hat{z}$$

$$= \frac{8\pi^2 I_0 R^2}{c(R^2+z^2)^{3/2}}\hat{z}$$

b) In our current configuration, we call the loop with the CCW current loop 1, and the loop with CW current loop 2

$$\Rightarrow B_t = B_1 + B_2$$

$-I_0$ b/c CW current

$\pm d/2$ added to reflect shift from origin

$$= \frac{8\pi^2 I_0 R^2}{c(R^2+[z-d/2]^2)^{3/2}}\hat{z} + \frac{8\pi^2(-I_0)R^2}{c(R^2+[z+d/2]^2)^{3/2}}\hat{z}$$

$$= \frac{8\pi^2 I_0 R^2}{c}\left[\frac{1}{(R^2+[z-d/2]^2)^{3/2}} - \frac{1}{(R^2+[z+d/2]^2)^{3/2}}\right]\hat{z}$$

c) $\vec{B}=0$ at $z=0$. This is due to the fields generated by each coil pointing in the opposite direction

d) The formula for the magnetic dipole moment is:

$$\vec{m} = I\int d\vec{a}$$

$$\hookrightarrow \vec{m}_{tot} = \vec{m}_1 + \vec{m}_2$$

$$= I_0\pi R^2\hat{z} - I_0\pi R^2\hat{z}$$

$$= 0\ ? \quad \text{or} \quad 2I_0\pi R^2$$ (depending on if we have positive or negative normal on m_2 integral)