2. Consider a linear, homogeneous, isotropic, and non-dissipative dielectric (i.e., a dielectric where $\mathbf{D} = \epsilon \mathbf{E}$ and ϵ is a constant) in the shape of a sphere of radius R with a point charge Q embedded at its center.

   (a) {2 pts} Find the electric displacement vector $\mathbf{D}$, the electric field $\mathbf{E}$, and the polarization density $\mathbf{P}$ inside the dielectric.

   (b) {2 pts} Find the bound charge volume density ρ_D inside the dielectric.

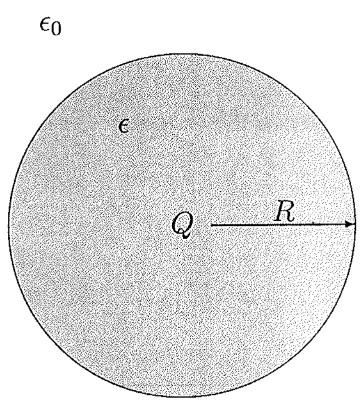
   (c) {1 pts} Find the total bound charge Q_D on the $r = R$ boundary of the dielectric.

   (d) {2 pts} Find the net charge (free plus bound) at the center of the dielectric.

   (e) {1 pts} Find the electric displacement vector $\mathbf{D}$, the electric field $\mathbf{E}$, and the polarization density $\mathbf{P}$, outside the dielectric sphere.

   (f) {2 pts} Are $\mathbf{D}$ and $\mathbf{E}$ continuous at $r = R$? If not explain why.

   (If you use Gaussian units you can put $\epsilon_0 = 1$.)



Jan 2010

E+M #2

Gaussian

a) *As a reminder, our electrostatic Maxwell's law in media are:

$$\nabla \cdot D = 4\pi \rho \qquad D = \vec{E} + 4\pi \vec{P} \quad , \quad D = \epsilon \vec{E}$$

$$\nabla \times E + \frac{1}{c}\frac{\partial B}{\partial t} = 0 \qquad P = \chi_e E \qquad \hookrightarrow \ \epsilon = 1 + 4\pi \chi_e$$

*We solve for D using Gauss Law

$$\int \nabla \cdot D = \int 4\pi \rho$$

$$D \cdot 4\pi r^2 = 4\pi Q$$

$$D = \frac{Q}{r^2}\hat{r}$$

$$\Rightarrow \vec{E} = \frac{D}{\epsilon}$$

$$= \frac{Q}{\epsilon r^2}\hat{r}$$

$$\Rightarrow \vec{P} = \frac{\vec{D} - \vec{E}}{4\pi}$$

$$= \frac{1}{4\pi}\left(\frac{Q}{r^2} - \frac{Q}{\epsilon r^2}\right)\hat{r}$$

$$= \frac{Q}{4\pi r^2}\left(1 - \frac{1}{\epsilon}\right)\hat{r}$$

b) The bound charge density is defined as:

$$\rho_b = -\vec{\nabla} \cdot \vec{P}$$

$$= -\frac{1}{r^2}\left(\frac{\partial}{\partial r} r^2 \frac{Q}{4\pi r^2}\left(1 - \frac{1}{\epsilon}\right)\right)$$

$$= Q\,\delta(\vec{r})\left(1 - \frac{1}{\epsilon}\right)$$

c) To find the total surface bound charge,

$$\sigma_b = P \cdot \hat{n}$$

$$= P \cdot \hat{r}$$

$$= \frac{Q}{4\pi r^2}\left(1 - \frac{1}{\epsilon}\right)\Big|_{r=R} = \frac{Q}{4\pi R^2}\left(1 - \frac{1}{\epsilon}\right)$$

#2 (cont.)

c) We must integrate over the surface to find:

$$Q_D = Q\left(1 - \frac{1}{6}\right)$$

d)