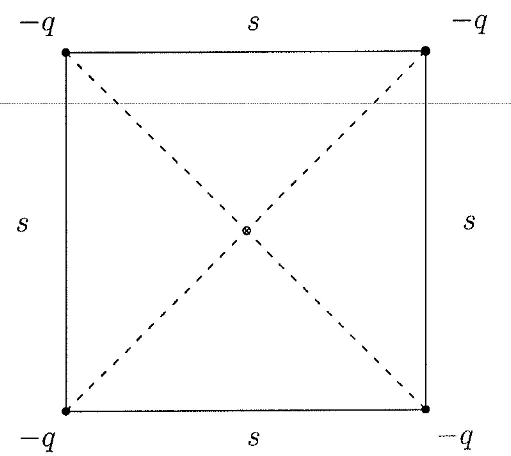


3. Consider a square with sides of length s and charges -q at the corners as shown:

   (a) {2 pts} What is the potential at the center of the square if the potential is zero at ∞?

   (b) {2 pts} How much work does it take to bring in another charge -q from ∞ to the center of the square?

   (c) {3 pts} How much work does it take to assemble the original configuration of 4 negative charges (no charge at center)?

   (d) {3 pts} Now suppose that instead of the 4 charges being located at the corners of a square, a net charge of $-4q$ is distributed uniformly on the surface of a sphere of radius s. How much work does it take to bring in another charge q from ∞ to the center of the sphere?

Jan 2011

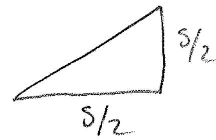
E+M #3

Gaussian

a) We will assume the origin lies at the center of the square

$$\Rightarrow V = \sum_i \frac{q_i}{r_i}$$

$$= 4 \frac{-q\sqrt{2}}{s}$$



$$= \sqrt{2\left(\frac{s}{2}\right)^2} = \sqrt{\frac{s^2}{2}}$$

b) $W = \int \vec{F} \cdot d\vec{l}$

$$= q\,(V(b) - V(a))$$

$$= -q\left(-\frac{\sqrt{2}q}{s}\right)$$

$$= \frac{\sqrt{2}q^2}{s}$$

c) The work necessary to assemble the charge distribution is:

$W = \frac{1}{2}\sum_i q_i V_i$ where i is each individual charge

$$= \frac{1}{2}\left(0 + (-q)\left(\frac{-q}{s}\right) + (-q)\left(\frac{-q}{s} + \frac{-q}{\sqrt{2}s}\right) + -q\left(\frac{-q}{s} + \frac{-q}{s} + \frac{-q}{\sqrt{2}s}\right)\right)$$

$$= \frac{1}{2}\left(\frac{q^2}{s} + \frac{q^2}{s} + \frac{q^2}{\sqrt{2}s} + \frac{q^2}{s} + \frac{q^2}{s} + \frac{q^2}{\sqrt{2}s}\right.$$

$$= \frac{2q^2}{s} + \frac{q^2}{\sqrt{2}s}$$

d) From symmetry, and the fact that the charge is distributed evenly over the surface of the sphere,

$$\vec{E}^{\,s} = \begin{cases} -\frac{4q}{r^2} & r > s \\ 0 & r < s \end{cases}$$

* We also know that

$$V = -\int_{\infty}^{0} E \cdot d\vec{l} = -\left[\int_{\infty}^{s} E_{out} \cdot dl - \int_{s}^{0} E_{in} \cdot dl\right]$$

#3 (cont.)

d) $V = -\int_{\infty}^{s} \frac{-4q}{r^2}\, dr$

$= -\left[-4q\left(\left.\frac{-1}{r}\right|_{\infty}^{s}\right)\right]$

$= 4q\left(\frac{-1}{s} - \cancel{\frac{-1}{\infty}}^{\,0}\right)$

$= \frac{-4q}{s}$

$W = -q\, V(0)$

$= -q\left(\frac{-4q}{s}\right)$

$= \frac{4q^2}{s}$