Problem 6: Relativity

(a) Write down both the homogeneous and inhomogeneous Maxwell's equations in manifestly Lorentz covariant form using the 2nd rank field strength tensor $F_{\mu\nu}$ and its dual $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$. (State which system of units you are using) [2 points]

(b) Write down the components (in matrix form) of $F_{\mu\nu}$. [2 points]

(c) Lorentz transformations on a four-vector are given by $x'_\mu = \Lambda^\nu_\mu x_\nu$. Write down the form of Λ^μ_ν for a boost to velocity $\beta_x = v_x/c$ along the x-direction. [2 points]

(d) Using Λ^μ_ν, calculate the Lorentz transformation relations for $\vec{E}$ and $\vec{B}$ for a boost along the x-direction. [2 points]

(e) Depict the lines of electric field $\vec{E}$ from a point charge a) at rest and b) moving with some large velocity β_x. [2 points]

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E+M #6

a) The manifestly covariant form of Maxwell's eqns are (in SI units)

$$\partial_\alpha F^{\alpha\beta} = \mu_0 J^\beta$$

$$\partial_\alpha \left(\tfrac{1}{2}\epsilon^{\alpha\beta\gamma\delta} F_{\gamma\delta}\right) = 0$$

* Rest of problem in Gaussian

b) $$F_{\mu\nu} = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{bmatrix}$$

c) $$\Lambda^\mu_\nu = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d) $F'^{\alpha\beta} = \Lambda^\mu_\alpha \Lambda^\nu_\beta F_{\mu\nu} \iff F' = \Lambda F \Lambda^T$ in matrix form

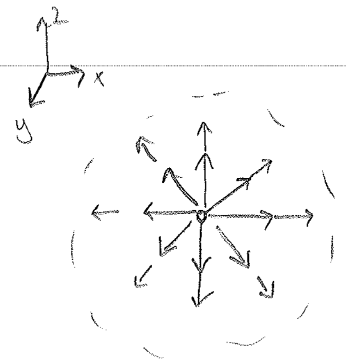
$$F' = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{bmatrix} \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \beta\gamma E_x & \gamma E_x & \gamma E_y + \beta\gamma B_z & \gamma E_z - \beta\gamma B_y \\ -\gamma E_x & -\beta\gamma E_x & -\beta\gamma E_y - \gamma B_z & -\beta\gamma E_z + \gamma B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{bmatrix} \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & (\cancelto{1}{-\beta^2\gamma^2 + \gamma^2})E_x & \gamma E_y + \beta\gamma B_z & \gamma E_z - \beta\gamma B_y \\ (\cancelto{-1}{-\gamma^2 + \beta^2\gamma^2})E_x & 0 & -\beta\gamma E_y - \gamma B_z & -\beta\gamma E_z + \gamma B_y \\ -\gamma E_y - \beta\gamma B_z & \beta\gamma E_y + \gamma B_z & 0 & -B_x \\ -\gamma E_z + \beta\gamma B_y & \beta\gamma E_z - \gamma B_y & B_x & 0 \end{bmatrix}$$

#6 (cont.)

e) For a point charge at rest



For a rapidly moving point charge