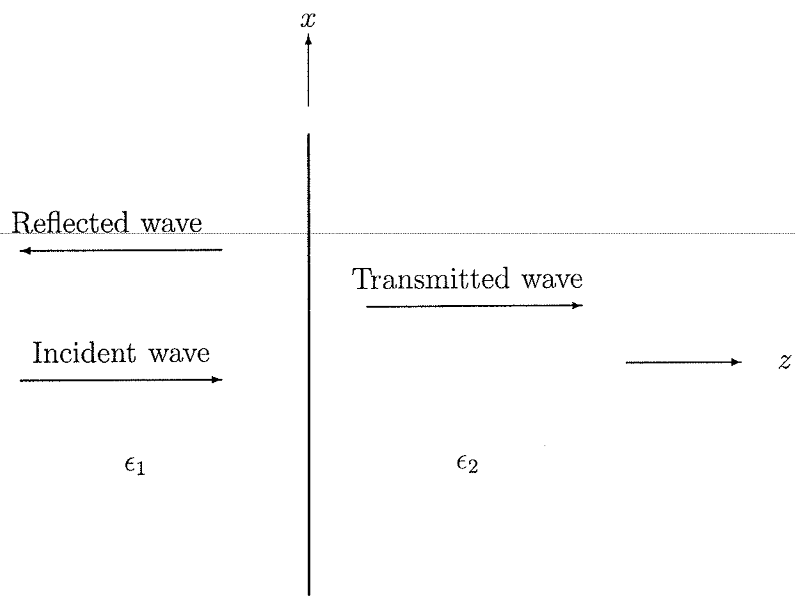

5. A linearly-polarized harmonic ($e^{-i\omega t}$) plane electromagnetic wave traveling to the right in a homogeneous dielectric medium described by a real dielectric constant ϵ_1, strikes a second homogeneous dielectric material described by another real dielectric constant $\epsilon_2 > \epsilon_1$ (see the figure). Assume that both materials have no magnetic susceptibility, $\chi_m = 0$, and that the incidence angle is 0^o (i.e., the wave is traveling perpendicular to the junction). Assume the incoming wave is polarized in the $\hat{x}$ direction and that its electric field amplitude is E_0, i.e., assume the incoming electric field is the real part of

$$\mathbf{E} = E_0\, e^{i(kz-wt)}\, \hat{x}.$$

(a) [2 pts] Give the direction of the magnetic induction $\mathbf{B}$ associated with the above incoming wave and give its amplitude B_0 as a function of E_0. Also give k as a function of ω.

(b) [2 pts] Give similar expressions for $\mathbf{E}$ and $\mathbf{B}$ of the reflected and transmitted waves. Use E_0'' and E_0' for the respective electric field amplitudes of the reflected and transmitted waves.

(c) [3 pts] Apply the boundary conditions at the junction/interface between the dielectrics to the incoming, reflected, and transmitted wave to compute E_0'' and E_0' as functions of E_0 and the two dielectric constants ϵ_1 and ϵ_2.

(d) [3 pts] Evaluate the reflection and transmission coefficients, R and T, for above waves. Recall that R and T are computed from ratios of time averaged Poynting vectors which are defined by

$$\mathbf{S} \equiv \mathbf{E} \times \mathbf{H}, \qquad (SI)$$

$$\mathbf{S} \equiv \frac{c}{4\pi}\mathbf{E} \times \mathbf{H}. \qquad (Gaussian)$$

Aug 2014 $\qquad$ E+M #5 $\qquad$ Gaussian

a) * To find relationship b/w k and ω, we derive wave equation

$$\cancel{\nabla \times \left(\nabla \times E + \frac{1}{c}\frac{\partial B}{\partial t} \right) = 0}$$

$$\nabla \times (\overset{0}{\cancel{\nabla \cdot \vec{E}}}) - \nabla^2 E + \frac{1}{c}\frac{\partial}{\partial t}(\nabla \times B) = 0$$

$$-\nabla^2 E + \frac{1}{c}\frac{\partial}{\partial t}(\nabla \times \mu H) = 0$$

$$-\nabla^2 E + \frac{\mu}{c}\frac{\partial}{\partial t}\left(-\frac{1}{c}\frac{\partial D}{\partial t}\right) = 0$$

$$-\nabla^2 E + \frac{\mu\epsilon}{c^2}\frac{\partial^2}{\partial t^2} E = 0$$

* Given $\vec{E} = E_0 \exp[i(kz - \omega t)]\hat{x}$

$$k^2 E_0 \exp[i(kz - \omega t)] - \frac{\mu\epsilon}{c^2}\omega^2 E_0 \exp[i(kz - \omega t)] = 0$$

$$k^2 - \frac{\mu\epsilon}{c^2}\omega^2 = 0$$

$$\hookrightarrow k = \frac{\sqrt{\mu\epsilon}}{c}\omega$$

$$\vec{B} = c\int \nabla \times E \, dt$$

$$\nabla \times E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ E_0 & 0 & 0 \end{vmatrix} = \langle 0, \partial_z E_0, 0 \rangle = ikE_0 \exp[ikz - \omega t]$$

$$\vec{B} = \int ikc E_0 \exp[i(kz - \omega t)] \, dt \, \hat{y}$$

$$= \frac{+kc}{\omega} E_0 \exp[i(kz - \omega t)]\hat{y}$$

b) * For the reflected wave:

$$\vec{E}_r = E_0'' \exp[i(-k''z - \omega t)]\hat{x} \qquad \vec{B}_r = -\frac{k''c}{\omega}E_0'' \exp[i(-k''z - \omega t)]\hat{y}$$

* For the transmitted wave:

$$\vec{E}_T = E_0' \exp[i(k'z - \omega t)]\hat{x} \qquad \vec{B}_T = \frac{k'c}{\omega}E_0' \exp[i(k'z - \omega t)]\hat{y}$$

#5 (cont.)

c) * In general, our boundary conditions are:

$$① \; D_1^{\perp} - D_2^{\perp} = 4\pi \sigma_f \qquad ③ \; B_1^{\perp} - B_2^{\perp} = 0$$

$$② \; E_1^{\parallel} - E_2^{\parallel} = 0 \qquad ④ \; H_1^{\parallel} - H_2^{\parallel} = \frac{4\pi}{c} \vec{K}_f$$

* Using conditions ① and ④ we generate the following system of equations

$$① \quad E_0 + E_0'' = E_0'$$

$$② \quad \frac{ck}{\omega} E_0 - \frac{ck}{\omega} E_0'' = \frac{ck'}{\omega} E_0'$$

$$* \; k = \frac{\sqrt{\epsilon}}{c} \omega$$

$$\sqrt{\epsilon_1} E_0 - \sqrt{\epsilon_1} E_0'' = \sqrt{\epsilon_2} E_0'$$

$$\sqrt{\epsilon_1} E_0 - \sqrt{\epsilon_1}(E_0' - E_0) = \sqrt{\epsilon_2} E_0'$$

$$2\sqrt{\epsilon_1} E_0 - \sqrt{\epsilon_1} E_0' = \sqrt{\epsilon_2} E_0'$$

$$2\sqrt{\epsilon_1} E_0 = (\sqrt{\epsilon_1} + \sqrt{\epsilon_2}) E_0' \quad \Rightarrow \quad E_0' = \frac{2\sqrt{\epsilon_1} E_0}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$

$$\Rightarrow E_0'' = E_0' - E_0$$

$$= E_0 \left(\frac{2\sqrt{\epsilon_1}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} - 1 \right)$$

d) $$R = \frac{\langle |S''| \rangle}{\langle |S| \rangle} \qquad T = \frac{\langle |S'| \rangle}{\langle |S'| \rangle}$$

* in both cases E, H are scaled by factors found in part c, therefore R, T are those ratio's squared

$$R = \left(\frac{2\sqrt{\epsilon_1}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} - 1 \right)^2 \qquad T = \left(\frac{2\sqrt{\epsilon_1}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} \right)^2$$