Problem 1: Electrostatics

A wire of radius R_1 is insulated with a dielectric of outer radius R_2 that is itself enclosed in a grounded conducting sheath. Let the charge per unit length on the wire be λ.

1. Find an expression for the electric field, $\vec{E}$, on the wire at a radius ρ from the center of the wire. [3 points]
2. Find the voltage, V, between the inner and outer conductors. [2 points]
3. Calculate the force per unit volume on the insulating material in the coaxial cable. [3 points]
4. Estimate the size of the force for $R_1 = 1$ mm, $R_2 = 5$ mm, $\epsilon_r = 2.5$, and $V = 25,000$ volts. Is this force larger than the force of gravity if the dielectric has the same density as water (10^3 *kilograms/meter*^{3})? [2 points]

[Hint: The force per unit volume on a dielectric is given by $\frac{1}{2}(\epsilon - \epsilon_0)\nabla E^2$; also, $\epsilon_0 = 8.85 \times 10^{-12} \mathrm{C}^2/(\mathrm{Nm}^2)$.]

$h_1 = 1 \quad h_2 = s \quad h_3 = 1$

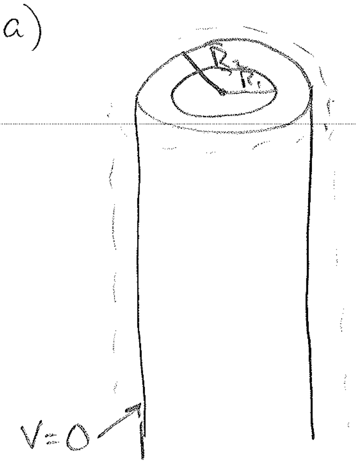
$$\nabla \Phi = \left\langle \frac{1}{h_1}\frac{\partial \Phi}{\partial s}, \frac{1}{h_2}\frac{\partial \Phi}{\partial \varphi}, \frac{1}{h_3}\frac{\partial \Phi}{\partial z} \right\rangle$$

Jan 2018

E + M #1

Gaussian

a)



$\nabla \cdot D = 4\pi \rho_f$

* if $s < R_1$:

$$\int \nabla \cdot E \, dV = \oint E \cdot d\vec{a} = \int 4\pi \rho_f$$

$$E \cdot 2\pi s L = 4\pi \lambda L \frac{s^2}{R_1^2}$$

$$\vec{E} = 2\lambda \frac{s}{R_1^2} \hat{\varphi}$$

* if $R_1 < s < R_2$:

$$\oint \vec{D} \cdot d\vec{a} = \int 4\pi \rho_f$$

$$D \cdot 2\pi s L = 4\pi \lambda L$$

$$\vec{D} = \frac{2\lambda}{s} \hat{\varphi}$$

* Assuming dielectric is linear, $D = \epsilon E$

$\hookrightarrow E = \frac{2\lambda}{\epsilon s} \hat{\varphi}$

* if $s > R_2$:

$\vec{E} = 0$ (identically true for conductors)

#1 (cont.)

b) $$\Delta \Phi = -\int_a^b \vec{E} \cdot dl$$

$$= -\int_{R_2}^{R_1} \vec{E} \cdot dl$$

$$= -\int_{R_2}^{R_1} \frac{2\lambda}{\epsilon s}\, ds$$

$$= -\frac{2\lambda}{\epsilon} \ln(s) \Big|_{R_2}^{R_1}$$

$$= \frac{2\lambda}{c} \ln\left(R_1/R_2\right)$$

c) $$F = qE \quad \Rightarrow \quad \frac{F}{V} = \frac{qE}{V}$$

$$\Rightarrow \quad \frac{F}{V} = \frac{\lambda E}{\pi (R_2^2 - R_1^2)}$$

$$\frac{F}{V} = \frac{2\lambda^2}{\pi \epsilon s (R_2^2 - R_1^2)}$$