2. The magnetic field of a plane wave in vacuum is

$$\mathbf{B} = B_0 \sin(kx - \omega t)\,\hat{\mathbf{y}},$$

where $\hat{\mathbf{y}}$ is a unit vector pointing in the positive y-direction.

(a) [1 pts] Give the wavelength λ of this wave as a function of k, ω and/or c (the speed of light).

(b) [2 pts] Write an expression for the wave part of the electric field $\mathbf{E}$ associated with the above magnetic field.

(c) [1 pts] What is the direction and magnitude of the Poynting vector $\mathbf{S}$ associated with this wave?

(d) [1 pts] Assume this plane wave is totally reflected by a thin conducting sheet lying in the y-z plane at x=0. What is the resulting time averaged radiation pressure on the sheet? Recall that the momentum density $\mathbf{g}$ and the Poynting vector of the incoming and reflected waves are related by $\mathbf{g} = \mathbf{S}/c^2$

(e) [2 pts] The component of an electric field parallel to the surface of an ideal conductor must be zero on the surface. Using this fact, find expressions for the reflected electric and magnetic fields. Recall that the electric and magnetic fields vanish within an ideal conductor.

(f) [3 pts] An oscillating surface current $\mathbf{K}$ flows in the thin conducting sheet as a result of this reflection. Along which axis does $\mathbf{K}$ point and what is its amplitude? Hint: To find $\mathbf{K}$ use an Amperian loop with one side just inside the conducting sheet and one side just outside the sheet.

Aug 2015 # E+M #2 Gaussian

a) $c = \frac{\omega}{k}$ $k = 2\pi / \lambda$

$\hookrightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi c}{\omega}$

b) $\vec{B} = B_0 \sin(kx - \omega t)\,\hat{y}$

* To determine $\vec{E}$, we use $\nabla \times B - \frac{1}{c}\frac{\partial E}{\partial t} = \vec{J}$

$$\nabla \times B = \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & B_0 \sin(kx-\omega t) & 0 \end{bmatrix}$$

$$= \langle 0, 0, \frac{\partial}{\partial x} B_0 \sin(kx - \omega t) \rangle$$

$$= k B_0 \cos(kx - \omega t)\,\hat{z}$$

* Since we are in a vacuum, $\vec{J} = 0$

$\hookrightarrow k B_0 \cos(kx - \omega t)\hat{z} = \frac{1}{c}\frac{\partial E}{\partial t}$

$ck B_0 \cos(kx - \omega t)\hat{z} = \frac{\partial E}{\partial t}$

$-\frac{ck}{\omega} B_0 \sin(kx - \omega t)\,\hat{z} = E$

$\hookrightarrow E = -B_0 \sin(kx - \omega t)\,\hat{z}$

c) * In Gaussian units $\vec{S} = \frac{c}{4\pi}(E \times H)$, $H = \vec{B} - 4\pi \vec{M}$

$\hookrightarrow \vec{S} = \frac{c}{4\pi}(\vec{E} \times \vec{B})$

$= \frac{c}{4\pi} B_0^2 \sin^2(kx - \omega t)\,\hat{x}$

#2 (cont.)

d) Radiation pressure is related to $\vec{g}$ via: $P = 2\langle cg \rangle$

$$\hookrightarrow P = 2\langle c\vec{g} \rangle$$
$$= \frac{2}{c}\langle \vec{S} \rangle$$
$$= \frac{2}{c}\frac{B_0^2}{4\pi}\hat{x}$$

e)