

# E&M

Fall 2009

## 1 Magnetic Materials

Assume the field inside a large piece of magnetic material is  $\vec{B}_0$  so that

$$\vec{H}_0 = \frac{1}{\mu_0} \vec{B}_0 - \vec{M}$$

- a) Consider a small spherical cavity that is hollowed out of the material. Find the field  $\vec{B}$ , at the center of the cavity, in terms of  $\vec{B}_0$  and  $\vec{M}$ . Also find  $\vec{H}$  at the center of the cavity in terms of  $\vec{H}_0$  and  $\vec{M}$ . (3 Points)
- b) Do the same calculations for a long needle-shaped cavity running parallel to  $\vec{M}$ . (3 Points)
- c) Do the same calculations for a thin wafer-shaped cavity perpendicular to  $\vec{M}$ . (4 Points)

Hint: Assume the cavities are small enough so that  $\vec{M}$ ,  $\vec{B}_0$  and  $\vec{H}_0$  are essentially constant. The field of a magnetized sphere is  $\vec{B} = \frac{2}{3}\mu_0\vec{M}$  and the field inside a long solenoid is  $\mu_0 K$  where  $K$  is the surface current density.

## 2 Space-charge-limited Thermionic Planar Diode

Consider a planar diode with a grounded, hot metallic cathode at  $x = 0$  and a metallic anode plate at  $x = H$ , which is held at an electrical potential of  $V_p$  relative to ground. [Cathode and anode plates are infinite in the  $y$  and  $z$  directions.] The cathode is very hot and emits copious electrons such that the diode is "space charge limited", that is: the electric field **at the cathode is zero**. The current density  $J$  is constant and in the  $-x$  direction. [Ignore any transient effects.]

In this problem let :  $V(x)$  be the electric potential,  $E(x)$  be the electric field,  $s(x)$  be the velocity of an electron,  $\rho(x)$  be the charge density, and  $m$  and  $-e$  be the mass and charge of an electron respectively.

- a) State whether you are using MKS or cgs units. (1 Point)
- b) Find  $\rho(x)$  as a function of  $V(x)$  and any other relevant variables. (2 Points)
- c) Use Poisson's equation to find the differential equation for  $V(x)$ . (2 Points)
- d) State the boundary conditions for  $E(x)$  at  $x = 0$  and  $V(x)$ , at  $x = 0$  and  $x = H$ . (2 Points)

**Work e) or f) on a separate sheet of paper and submit only the one you wish to be graded.**

- e) Solve for  $V(x)$  in terms of  $V_p$  and  $H$  using results of c) and d). (3 Points for part e or f)

*Hint: multiply both sides of your differential equation by  $dV(x)/dx$  and recall that:  $(dV/dx)(d^2V/dx^2) = \frac{1}{2}d(dV/dx)^2/dx$*

**If you have trouble using the above hint to complete part e), then try f).**

- f) Assume  $V(x)$  is of the form:  $Ax^n + Bx + C$  and solve for  $V(x)$  in terms of  $V_p$  and  $H$  using parts c) and d) above. Find the current density  $J$  in terms of  $V_p$  and  $H$ . (3 Points for part e or f)

### 3 Wire

An infinitely-long, thin wire (radius  $b$ ) is coated with a dielectric (relative dielectric constant  $k = \epsilon/\epsilon_0$  with radius  $a > b$ ). The metal wire has charge per unit length  $\lambda$

- a) Find the electric displacement  $\vec{D}$  everywhere. (2 points)
- b) Find the electric field  $\vec{E}$  everywhere. (2 points)
- c) Find the polarization  $\vec{P}$  everywhere. (3 points)
- d) Find **all** the bound charge everywhere. (3 points)

## 4 Electromagnetic Waves

Consider a plane electromagnetic wave with propagation vector  $\vec{k}$  and angular frequency  $\omega$ . Construct the four-vector  $k^\mu = (\omega/c, \vec{k})$ . Use the metric  $g_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$

a) Verify that  $k_\mu k^\mu = 0$ . (2 points)

b) In terms of the position four-vector  $x^\mu = (ct, \vec{r})$ , show that the plane wave propagation factor is

$$e^{ik_\mu x^\mu} = e^{i(\vec{k} \cdot \vec{r} - \omega t)}.$$

(2 points)

c) Now use Lorentz transformations to show that radiation of frequency  $\omega$  propagating at an angle  $\theta$  with respect to the z-axis, will, to an observer moving with relative velocity  $v = \beta c$  along the z axis, have the frequency

$$\omega' = \frac{1}{\sqrt{1 - \beta^2}} \omega (1 - \beta \cos \theta).$$

(2 points)

d) Further show that the moving observer sees the radiation propagating at an angle  $\theta'$  with respect to the z-axis, where

$$\cos \theta' = \frac{\cos \theta - \beta}{1 - \beta \cos \theta},$$

which is aberration. (2 points)

e) Find  $\theta'$  explicitly if  $|\beta| < 1$ . (2 points)

## 5 Thin Infinite Sheet

- a) Compute the 4-current  $J^\alpha(x^\beta)$  and the E&M fields for a stationary, thin, and infinite sheet of charge located at  $z = 0$  in the lab. Assume the surface charge density is a constant  $\sigma_0$ . (4 points)
- b) Now assume you move with speed  $v < c$  in the  $x$ -direction relative to the lab. What is the 4-current  $J'^\alpha(x^\beta)$  and E&M field in your frame? (6 points)

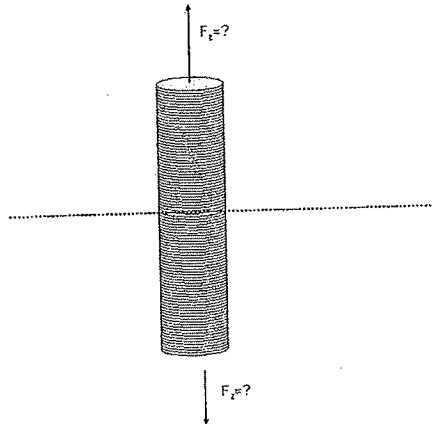


Figure 1: Stack of Disks for Stress Tensor Problem. Problem 6

## 6 Stress Tensor

Consider a long cylinder of radius  $a$  and length  $L$  made up of a stack of infinitesimally thin discs (See Figure). Assume the disks alternate between disks with charge density  $\rho$  and angular velocity  $\omega\hat{z}$  and disks with charge density  $-\rho$  and angular velocity  $-\omega\hat{z}$ .

- Specify the system of units you will be using. (1 points)
- write down an expression for the charge and current density in any small volume (of dimension larger than the infinitesimal thickness of the disks). (1 points)
- Find the electromagnetic field everywhere. (2 points)
- Find the Maxwell Stress Tensor everywhere. (2 points)
- Use your answer to part c to find the force of the top half of the cylinder on the bottom half. (2 points)
- Is the force attractive or repulsive? (2 points)