Classical Mech Main Points

Qualifier

Winter 2012

1 Newtonian Mechanics

- Set $F_N = 0$ to find the point when two objects separate (ex. ball rolls off hemisphere)
- Momentum ($p = mv$, $L = I\omega$) is conserved for all collisions; energy is conserved for elastic collisions
- Force $= -\nabla U$
- For periodic motion, if the equation of motion is $\ddot{x} + \xi x = 0$, the frequency is $\omega = \sqrt{\xi}$. If the equation has a term linear in $\dot{x}$, that is a damping term.
- Power: $P = \frac{dE}{dt} = \frac{\Delta W}{\delta t} = \vec{F} \cdot \vec{v} = \vec{\tau} \cdot \vec{\omega}$

1.1 Angular Motion

- Use $v = \omega r$, $x = \theta r$, $a = \alpha r$ for basic angular motion
- Circular motion: $ma = \frac{mv^2}{r} = m\omega^2 r$
- **Torque**: $\frac{dL}{dt} = \tau = \vec{r} \times \vec{F} = I\alpha = Fd\sin\theta$
- Period $T = \frac{2\pi}{\omega}$
- Remember: it's often easier to find $d\sin\theta$ than to find d and θ separately
- To derive moment of inertia: $I = \int r^2 dm$; solve for dm in terms of dr
- Can still also use $\Sigma F = ma$ if it helps. Consider all forces acting at same point (point particle)
- **Orbits**: $\frac{\partial^2 V_{eff}}{\partial r^2} > 0$ for **stable orbits**. Use $\frac{\partial V}{\partial r} = 0$ for circular orbits
- **Parallel Axis Theorem**: $I_{new} = I_{original} + MR^2$

Helpful moments of inertia:

- **sphere**: $I = \frac{2}{5}MR^2$
- **disc**: $I = \frac{1}{2}MR^2$

Rocket Ships: Use m = mass of ship, dm'=ejected mass, v=velocity of ship, $-u$=ejected mass velocity relative to ship. Then we have:

$$p_i = p_f \rightarrow 0 = (m - dm')(v + dv) + dm'(v - u) \tag{1}$$

Set $v = 0$ for simplicity, and $dm = -dm'$. After that it's mostly algebra/calculus.

2 Virtual Work

The principle of virtual work presents an alternative to Newtonian solutions for force problems. This method uses the equations:

$$\delta W = \sum_i \vec{F}_i^a \cdot \delta \vec{r}_i = 0 \qquad \delta W = \sum_i Q_i^a \delta q_i = 0 \tag{2}$$

In these equations, $\vec{F}_i^a$ represent the net applied forces, and Q_i^a represent the differentiated constraint equations. Transform the Q_i^a equation into the generalized (simplest) coordinates, and solve the resulting equations.

For example, if the constraint equation is for two blocks connected by a massless rod: $x^2 + y^2 - l^2 = 0$, with $x = l\cos\theta$ and $y = l\sin\theta$:

$$\delta W = \sum_i Q_i^a \delta q_i = 0 \rightarrow 2x\delta x + 2y\delta y = 0 \rightarrow \delta x \cos\theta + \delta y \sin\theta = 0 \tag{3}$$

2.1 D'Alembert's Principle

The virtual work method given previously works for systems in static equilibrium. To generalize this method to dynamic systems, D'Alembert introduced a new "force of inertia" that modifies the virtual work equation that governs forces:

$$\delta W = \sum_i \left[\vec{F}_i^a - m_i \ddot{\vec{r}}_i\right] \cdot \delta \vec{r}_i = 0 \tag{4}$$

3 Lagrangian & Hamiltonian

3.1 Lagrangian

- $L = T - U$
- Euler Lagrange Equation:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0 \tag{5}$$

- We can always add a total time derivative of a function to the Lagrangian for free (without changing equations of motion):

$$L' = L + \frac{dF(q, \dot{q}, t}{dt} \tag{6}$$

  This kind of trick can give a simplified Hamiltonian, even making it a constant of the motion.

- A variable q_i is **cyclic** if it does not appear in the Lagrangian. In that case, the associated momentum p_i is conserved/constant, and subtracting the associated $p_i\dot{q}_i$ transforms the Lagrangian into the Routhian:

$$p_i = \frac{\partial L}{\partial \dot{q}_i} = \alpha_i \quad \rightarrow \quad R = L - \alpha_i \dot{q}_i \tag{7}$$

3.2 Hamiltonian

- Legendre Transformation: $H = p\dot{q} - L$
- $p_q = \frac{\partial L}{\partial \dot{q}}$
- Hamilton's equations of motion: $\dot{p}_q = -\frac{\partial H}{\partial q}$ and $\dot{q} = \frac{\partial H}{\partial p_q}$
- Solve for q(t) using the E-L equation or Hamilton's equations of motion (take $\frac{d\dot{q}}{dt}$ and plug in for $\dot{p}_q$)
- We can see that H is conserved (thus representing the total energy) if $\frac{\partial H}{\partial t} = 0$ and if it includes no terms that depend *linearly* on a momentum variable (only quadratically).

- We can go farther, and write a momentum-space "Lagrangian", similar to how we did the first Legendre transform: $K(p, \dot{p}, t) = q_i \dot{p}_i + H(q, p, t)$
- $KE_{cylindrical} = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\phi}^2 + \dot{z}^2\right)$
- $KE_{spherical} = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2\right)$

3.3 Undetermined Multipliers

If we can't include some constraints when writing the Lagrangian, we have to take these constraints into account in the Euler-Lagrange equation as undetermined multipliers:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i^a + \sum_{j=1}^{m} \lambda_j a_{ji} \tag{8}$$

Each λ_j corresponds to each constraint equation f_j, and each a_{ji} corresponds to $\frac{\partial f_j}{\partial q_i}$.
Each Q_i^a corresponds to applied forces that cannot be written as part of the potential energy:

$$Q_i = \frac{\partial \vec{r}_j}{\partial q_i} \cdot \vec{F}_j \tag{9}$$

A constraint is *holonomic* if:

$$\frac{\partial}{\partial y}\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\frac{\partial f}{\partial y} \tag{10}$$

3.4 Canonical Transformations

"Guess" the Q & P to transform into in order to make $\frac{\partial H}{\partial t} = 0$. Show canonical by $[Q, P]_{q,p} = 1$

Use existing p and q definitions to find generating functions:

$$p = \frac{\partial F_1(q, Q)}{\partial q} \qquad P = -\frac{\partial F_1(q, Q)}{\partial P} \tag{11}$$

$$p = \frac{\partial F_2(q, P)}{\partial q} \qquad Q = \frac{\partial F_2(q, P)}{\partial P} \tag{12}$$

$$q = -\frac{\partial F_3(Q, p)}{\partial p} \qquad P = -\frac{\partial F_3(Q, p)}{\partial Q} \tag{13}$$

$$q = -\frac{\partial F_4(p, P)}{\partial q} \qquad Q = \frac{\partial F_4(p, P)}{\partial P} \tag{14}$$

The generating function(s) result in a new Hamiltonian:

$$K(Q, P, t) = H(q, p, t) + \frac{\partial F_2}{\partial t} \tag{15}$$

The new Hamiltonian results in corresponding new equations of motion:

$$\dot{P} = -\frac{\partial K}{\partial Q} \qquad \dot{Q} = \frac{\partial K}{\partial P} \tag{16}$$

$H = T + U$ if $\frac{\partial H}{\partial t} = 0$, no explicit time dependence, AND no terms linear in momentum/velocity

3.5 Small Oscillations with Effective Potentials

To find frequency of small oscillations:

1. Write the Hamiltonian and find the effective potential, V_{eff} (all terms that depend on q)
2. Find $\frac{\partial^2 V_{eff}}{\partial q^2}|_{q=qmin}$ where q represents the variable with small oscillations
3. Write the V matrix as:
$$V = \frac{1}{2}\tilde{V}q^2 = \frac{1}{2}\frac{\partial^2 V_{eff}}{\partial q^2}|_{qmin}q^2 \tag{17}$$
4. Write the T matrix as:
$$T = \frac{1}{2}\tilde{T}\dot{q}^2 \tag{18}$$
5. Solve for the frequency using $\tilde{V}$ and $\tilde{T}$:
$$\tilde{V} - \omega^2\tilde{T} = 0 \tag{19}$$

Quick way to get frequency: Make the Lagrangian look like: $L = \frac{1}{2}m'\dot{\eta}^2 - \frac{1}{2}k'\eta^2$. Then $\omega = \sqrt{\frac{k'}{m'}}$

3.6 Variational Calculus

The Euler-Lagrange equation can also solve other physics of path minimization, such as the brachistone problem of minimizing time for a particle in a force field to travel between two points. To use the E-L for this type of problem:

1. Write an equation that describes the motion and the element to minimize, such as $dt = \frac{ds}{v}$. The element to minimize should be alone on the LHS.
2. Add an integration symbol on both sides: $t = \int \frac{ds}{v}$
3. Write the RHS differential in terms of path variables, such as dx and dy, in order to evaluate the integral, such as: $t = \int \frac{\sqrt{1+x'^2}}{\sqrt{2gy}}dy$
4. Use the E-L equation on the integrand, using the appropriate variables, such as: $\frac{\partial F}{\partial x} - \frac{d}{dy}\frac{\partial F}{\partial x'} = 0$
5. Solve the resulting equation by separation of variables, such as $x(y) = \int \sqrt{\frac{y}{(c^2/2g)-y}}dy$

4 Vector Potentials

Remember that the vector potential due to a particle in a magnetic field is:

$$\vec{A} = -\frac{1}{2}B_0(y\hat{x} - x\hat{y}) \tag{20}$$

And to find the potential, use:

$$U = q\phi - q\vec{A}\cdot\vec{v} \tag{21}$$

where ϕ represents the electric potential.

5 Small Oscillations

Standard coordinates define how the blocks are displaced *relative to each other*, while small coordinates (usually η) define how the blocks are displaced *relative to their original equilibrium position.* Start by writing the Lagrangian in standard coordinates, then transform to small coordinates. Then use these notations:

$$L = \frac{1}{2}\mathbf{T}\dot{\eta}_i\dot{\eta}_j - \frac{1}{2}\mathbf{V}\eta_i\eta_j \tag{22}$$

Use $\frac{\partial V}{\partial q_i}|_{q0i} = 0$ to find the minimum point q_{0i}, and $\mathbf{V} = \frac{\partial^2 V_{eff}}{\partial q_i^2}|_{q0i} = \frac{\partial^2 V_{eff}}{\partial \eta_i \eta_j}|_0$ to find $\mathbf{V}$.

Then use $\mathbf{T}$ and $\mathbf{V}$ to solve for the frequency(s):

$$|\mathbf{V} - \lambda\mathbf{T}| = 0 \tag{23}$$

where $\lambda = \omega^2$, to solve for the frequencies ω_i. To find the eigenvectors:

$$(\mathbf{V} - \lambda_i\mathbf{T})\,\vec{c}_i = 0 \tag{24}$$

these $\vec{c}_i$ also make up the amplitude ratios for λ_i, $\frac{A_1}{A_2}$:

$$(\mathbf{V} - \lambda_i\mathbf{T})\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0 \tag{25}$$

To normalize the eigenvectors:

$$\vec{C}_i = N_i\vec{c}_i \rightarrow \vec{C}_i^T\mathbf{T}\vec{C}_i = 1 \tag{26}$$

Solve for N_i. Finally, to write the displacement of the system as a function of time:

$$A_i = \vec{C}_i^T\mathbf{T}\eta(0) \tag{27}$$

$$\omega_i^2 > 0 \rightarrow \omega_i B_i = \vec{C}_i^T\mathbf{T}\dot{\eta}(0) \tag{28}$$

$$\omega_i = 0 \rightarrow B_i = \vec{C}_i^T\mathbf{T}\dot{\eta}(0) \tag{29}$$

The general solution can now be written as:

$$\vec{\eta}(t) = \sum_{\omega_i^2>0} \vec{C}_i(A_i cos\omega_i t + B_i \sin\omega_i t) + \sum_{\omega_i^2=0} \vec{C}_i(A_i + B_i t) \tag{30}$$

Smaller ω's correspond to more symmetry in the oscillation mode.

6 Central Forces & the Hamilton Jacobi Equation

Whenever we have two masses exerting a force on each other, we can move into the center of mass reference frame and consider the reduced mass combination acted on by a central force, since the center of mass of the system does not move.

Orbits & Stability

- A circular orbit is stable if $\frac{\partial^2 V_{eff}}{\partial r^2} > 0$
- To find the radius for circular orbit, set $\frac{\partial V}{\partial r} = 0$ and solve for r (can also use Hamilton's equations)
- To find the condition on the radius for circular orbit, find $\frac{\partial^2 V_{eff}}{\partial r^2} > 0$ and substitute in the radius for circular orbit

Steps for Solving Motion with the Hamilton-Jacobi

1. *Background*: We can transform H without loss of generality to $K = H + \frac{\partial S}{\partial t} = 0$. Assuming then that S, Hamilton's principle/generating function is separable ($S(q,t) = S_1(t) + S_2(q)$) and $p = \frac{\partial S}{\partial q}$, we can rearrange K to be:

$$\frac{1}{2m}\left(\frac{\partial S_2}{\partial q}\right)^2 + V(q) = -\frac{\partial S_1}{\partial t} \tag{31}$$

   Now the variables are separated, and we can set both sides equal to a constant, E. This makes solving for S_1 and S_2 a matter of maths.

2. Write Hamilton's equation, and substitute $\frac{\partial S_2}{\partial q}$ for each p_q term. (S_2 is sometimes referred to as W)

3. Separate variables - this usually entails writing everything not dependent on r inside a bracket, and setting that bracket equal to α_3. (This is usually the total angular momentum, which we can see is a constant of the motion by finding $[L, H] = 0$). Or solve so that r is on one side of the equation, and θ and ϕ are on the other side, then set both sides equal to α_3.

4. Assuming W is separable (example $W(r,\theta,\phi) = W_r + W_\theta + W_\phi$), find integrals defining each component of W.

5. Use $p_q = \frac{\partial W}{\partial q}$ to find the meaning of α_2 and α_3.

6. Use the form $\frac{\partial W}{\partial E} = t + \beta$ to solve for the motion of r depending on E and α's.

7. *Additional*: It may be useful to also remember that $Q = \frac{\partial S_2}{\partial P} = \frac{\partial S_2}{\partial E}$ and $\dot{Q} = \frac{\partial H}{\partial P} = \frac{\partial H}{\partial E}$.

The "action", J is equivalent to $S_2(q)$ as long as $S(q,t)$ is separable:

$$J = \int p dq = \int P dQ \tag{32}$$

Given this J, the frequency of motion is:

$$\nu_i = \frac{\partial E}{\partial J_i} \tag{33}$$

where E came from integrating the action J and solving for $E(J)$.

7 The Poisson Bracket

The poisson bracket is a good method of determining which elements associated with a Hamiltonian are constants of motion:

$$\frac{du}{dt} = [u, H]_{qi,pi} + \frac{\partial u}{\partial t} \tag{34}$$

$$[u, H]_{qi,pi} = \sum_i^n \left(\frac{\partial u}{\partial q_i}\frac{\partial H}{\partial p_i} - \frac{\partial u}{\partial p_i}\frac{\partial H}{\partial q_i}\right) \tag{35}$$

For example, given angular momentum $J = q_1 p_2 - q_2 p_1$, the poisson bracket of J with H quickly shows that the angular momentum is conserved:

$$\frac{dJ}{dt} = [J, H]_{qi,pi} = 0 \tag{36}$$

In general, to find whether an element is a constant of motion:

1. Write the element A in terms of q_i and p_i
2. Write the Hamiltonian according to the physical description
3. Find $\frac{dA}{dt} = [A, H]_{qi,pi} + \frac{\partial A}{\partial t}$

For canonical variables:

$$[q_i, q_j] = 0 \quad [q_i, p_j] = \delta_{ij} \quad [p_i, p_j] = 0 \tag{37}$$

The poisson bracket also helps verify that transformations are properly canonical:

$$[Q, P]_{q,p} = 1 \tag{38}$$

8 Extra

8.1 Conservative Forces

A force is conservative if $\vec{\nabla} \times \vec{F} = 0$. In Cartesian coordinates, can find this as: $\frac{\partial F_i}{\partial j} = \frac{\partial F_j}{\partial i}$

8.2 Nonhomogeneous Equations

Solving a non-homogeneous equation requires the combination of a *particular* and a *complementary* solution:

$$\dot{y} + ay = b \quad \rightarrow \quad y(t) = y_p(t) + y_c(t) \tag{39}$$

1. The particular solution should be of the form $y_p(t) = At^2 + Bt + C$, keeping only the terms so that $y_p(t)$ is a polynomial of the same order as the right hand side of the original equation. So in this example, $y_p(t) = C$.
2. The complementary solution solves $y(t)$ for the right hand side equalling zero: $\dot{y} + ay = 0$. Solve this the usual way, including the constant of integration.
3. Write $y(t) = y_p(t) + y_c(t)$, and substitute these results back into the original equation. Use the original equation and initial conditions to solve for the constants of integration.

Remember that a second derivative equation of motion can be handled as a first derivative equation by writing it in terms of velocity instead of position: $\ddot{y} + a\dot{y} = b \rightarrow \dot{v}_y + av_y = b$

9 Coordinate Systems

9.1 Cartesian

Convert to spherical: $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$

Convert to cylindrical: $x = \rho\cos\phi$, $y = \rho\sin\phi$, $z = z$

9.2 Spherical

$$\hat{r} = \sin\theta\cos\phi\hat{x} + \sin\theta\sin\phi\hat{y} + \cos\theta\hat{z} \tag{40}$$

$$\hat{\theta} = \frac{\partial\hat{r}}{\partial\theta} \;\&\; \hat{\phi} = \frac{\partial\hat{r}}{\partial\phi} \tag{41}$$

Derivation of a small chunk of circular area (such as in Kepler's law for orbits):

$$S = r\theta \rightarrow dS = rd\theta \rightarrow dA = R^2 d\theta \tag{42}$$

9.3 Cylindrical

$$\hat{r} = \cos\theta\hat{x} + \sin\theta\hat{y} \tag{43}$$

$$\hat{\theta} = \frac{\partial\hat{r}}{\partial\theta} \tag{44}$$

Stat Mech Main Points

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1 Basic Thermo

Definitions:

- Adiabatic: No heat is exchanged in or out of system. $dQ = 0$
- Quasi-static: Uniform pressure throughout system
- Isoentropic: Adiabatic, quasi-static, and constant entropy
- Reversible: constant entropy

Equations:

- Most all basic thermo relations can be derived from these two equations/derivations:

$$F = E - TS \rightarrow dF = dE - TdS - SdT \tag{1}$$

$$dU = TdS - PdV + \Sigma \mu dN \tag{2}$$

- Internal energy:

$$\Delta U = Q - W \tag{3}$$

- Work done **by** the system is calculated as:

$$W = \int PdV \tag{4}$$

- Relationship between pressure, force and area:

$$P = \frac{F}{A} \tag{5}$$

- **Ideal Gases**:
  - Ideal gas law equation: $PV = NkT = nRT$. $N = \#$ of particles, $n = \#$ of moles of particles
  - Energy of an ideal gas $= \frac{N}{2}kT$, where N represents degrees of freedom.
    Energy of a **monatomic** ideal gas $= \frac{3}{2}NkT$
    Energy of a **diatomic** ideal gas $= \frac{5}{2}NkT$
- Clausius-Clapeyron Equation:

$$\frac{dP}{dT} = \frac{L}{T(V_G - V_L)} \tag{6}$$

- We can also see from the free energy equation that we can relate the pressure derivative to the entropy at equilibrium:

$$\frac{dP}{dT} = \frac{S_G - S_L}{V_G - V_L} \tag{7}$$

- Enthalpy:

$$H = E - pV \tag{8}$$

1.1 Extensivity

A thermo property such as entropy is properly **extensive** if: $S(\lambda N, \lambda E, \lambda V) = \lambda S$. Equations of state can be given as **intensive** instead. That means they are given as a per-particle measurement - such as:

$$e = \frac{E}{N} \quad v = \frac{V}{N} \quad s = \frac{S}{N} \tag{9}$$

All the same derivative relations still hold for these intensive properties.

1.2 Basic Temperature & Entropy Relationships

- Heat to raise/lower temp: $Q = C_p m \int dT$
- Heat to melt ice: $Q = mL$
- $\Delta S = \frac{Q}{T}$; plug in Q and then integrate (the integral in definition of Q)
- C_v and C_p for a liquid are basically the same
- At high temperatures, we expect the macrostate of a system to be in its most random state. That means that every microstate should have equal probability of occurring.

1.3 Maxwell Relations

Knowing the dU and dF equations listed previously, we can find additional physics by taking mixed partial derivatives. Since partial derivatives can switch order without changing the result, doing so can lead to additional physics relationships. For example:

$$\frac{\partial}{\partial E}\frac{\partial S}{\partial V} = \frac{\partial}{\partial V}\frac{\partial S}{\partial E} \rightarrow \frac{\partial}{\partial E}\frac{P}{T} = \frac{\partial}{\partial V}\frac{1}{T} \rightarrow P = \frac{\partial E}{\partial V} \tag{10}$$

1.4 Engines

- Work done by an engine is positive if the **area under the curve of a P-V diagram** is **positive**.
- A **Carnot cycle** consists of **two isotherms** and **two adiabats**.
- An adiabat line goes from a lower isotherm to a higher isotherm line (so an adiabat line is **steeper** than an isotherm line).
- Adiabat: $dQ = 0$; and since $dS = \frac{dQ}{t}$, we find that $dS = 0$.
  Also for an adiabat:
  $$PV^\gamma = constant \quad \& \quad TV^{\gamma-1} = constant \tag{11}$$
  Where $\gamma = \frac{f+2}{f} = \frac{C_P}{C_V}$ and f represents the degrees of freedom (3 for monatomic ideal gas).
- Isotherm: $dT = 0$
- Isochore: $dV = 0$, and $Q = C_v m \Delta T$

Efficiency:

$$\eta = \frac{benefit}{cost} = \frac{W}{Q_h} = 1 - \frac{Q_c}{Q_h} \tag{12}$$

2 Classical Statistical Mechanics

Classical stat mech systems are usually **distinguishable** and energy levels can be **either discrete or continuous**. Usually the question must give information about the particles' energy levels and distinguishability.

Given an average measurement per time $\bar{n}_s$, we find the **variance** as:

$$\overline{(\Delta n_s)^2} = < (n_s - < n_s >)^2 > = < n_s^2 > - \bar{n}_s^2 \tag{13}$$

2.1 Ensembles

2.1.1 Microcanonical Ensemble (E, N, V fixed)

Entropy:

$$S = k \ln \Omega \tag{14}$$

Where Ω represents multiplicity, which is the number of possible states present.

Partition function is $\frac{1}{\Omega}$

Binomial distribution:

$$\binom{N}{n} = \frac{N!}{n!(N-n)!} \tag{15}$$

If asked to calculate the multiplicity Ω or other derivations in the MCE for a more complicated system, such as a classical harmonic oscillator ($H = \frac{p^2}{2m} + \frac{m}{2}\omega^2 x^2$), remember that Ω represents the total number of states present - so integrate with a step function in order to include all energies up to the given energy:

$$\Omega = \left[\int \frac{d^3x d^3p}{h^3} \Theta \left(E - \sum_i \left(\frac{p_i^2}{2m} + \frac{m}{2}\omega^2 x^2 \right) \right) \right]^N \tag{16}$$

We can write this as a 6-dimensional sphere of radius $\sqrt{E}$ in phase space. (Probably won't encounter it.)

2.1.2 Canonical Ensemble (N, V, T fixed)

The canonical ensemble represents a system in equilibrium with a reservoir.

Use this partition function for a single particle:

$$z = \sum_{n=0}^{\infty} e^{-\beta n} \tag{17}$$

and for N particles (add an $\frac{1}{N!}$ if indistinguishable):

$$Z = z^N \tag{18}$$

Along with the relations derived from the dF and dU equations, we also have:

$$E = -\frac{\partial}{\partial \beta} \ln Z \qquad F = -\frac{1}{\beta} \ln Z \qquad C_V = \frac{\partial E}{\partial T} \tag{19}$$

More CE details:

- If given energy in term of p or k (in CE or GCE), as long as it is not bosons or fermions, integrate over all phase space to find the partition function. This means the partition function in 3D looks like:

$$z = \frac{1}{h^3} \int e^{-\beta E(x,p)} d^3x d^3p \tag{20}$$

  Remember that if integrating over momentum space, p, divide by a factor of h for each dimension in the integration. If integrating over k space instead, only divide by a factor of 2π for each dimension.

- If a system has multiple sources of energy, the partition function is the multiplicative combination of each energy source. For example, for a system of indistinguishable particles with internal energy as well as translational energy (if they are point particles):

$$Z = Z_{trans} Z_{internal} = \frac{1}{N!} (z_{trans} z_{internal})^N \tag{21}$$

  These z's can be calculated independently. For example, if given point particles with two internal energy states, 0 and Δ:

$$Z = \frac{1}{N!} \left[e^0 + e^{-\beta\Delta} \right]^N \left[\int \frac{d^3x d^3p}{h^3} e^{-\beta p^2/2m)} \right]^N \tag{22}$$

- Similarly, if we have a gas made up of two or more particles, the combined partition function is the multiplicative combination of each kind of particle. For example, the partition function for a gas made up of two particles, A and B, looks like:
$$Z = Z_A Z_B \tag{23}$$
- If two or more gases/liquids are in equilibrium in the same volume, their chemical potentials must be equal.
- If asked for less traditional calculations (such as the average height of atoms in a gravitational field), look at the partition function and see what derivative to take to bring that quantity down from the exponent (and divide by any extraneous terms that would come down). For example, for the average height of atoms in a gravitational field:
$$Z = \frac{1}{N!}\left(\int \frac{d^3x d^3p}{h^3} e^{-\beta(p^2/2m - mgz)}\right)^N \tag{24}$$
$$\langle z \rangle = -\frac{1}{N\beta m}\frac{\partial}{\partial g}\ln Z \tag{25}$$

2.1.3 Grand Canonical Ensemble (V, T, μ fixed)

Along with the dU and dF equations, use the grand potential $\mathcal{G}$ to derive relations for the GCE:
$$\mathcal{G} = F - \mu N = E - TS - \mu N = -pV = -kT\ln Q = -\frac{1}{\beta}\ln Q \tag{26}$$

Use the grand partition function with this ensemble:
$$Q = \Sigma e^{\beta\mu N}\Sigma e^{-\beta E_n N} \tag{27}$$

Another entity used in GCE is the **grand canonical potential**, ψ (is actually derivable from $d\mathcal{G}$ as $\psi = \frac{PV}{kT}$):
$$\psi = \ln Q \tag{28}$$

As with CE, if given a composite substance (such as electrons and positrons), it's easiest to write the partition function as a product of the two partition functions:
$$Q_{tot} = Q_+ Q_- \tag{29}$$

This also works if given an energy that varies based on spin, such as $H = \frac{\hbar^2 k^2}{2m} + m_z B$, where $m_z = \pm 1$:
$$Q_{tot} = Q(m_z = 1) \times Q(m_z = -1) \tag{30}$$

3 Quantum Statistical Mechanics

Quantum statistical mechanics particles usually have **discrete energy levels** and are usually **indistinguishable**, but in the case of *bosons* or *fermions*, we take care of indistinguishability when we calculate the partition function according to **distribution functions** rather than just using the $\frac{1}{N!}$ that we used for classical particles.

Usually we use the Grand Canonical Ensemble to analyze quantum systems, but we can no longer sum over N in the partition function (Z), since we need to sum over *states* rather than *particles*.

For example, for a Fermi gas with a particular defined energy:
$$Q = \sum_{N=0}^{\infty}\sum_{\{\sigma\}} e^{\beta\mu N} e^{-\beta E_\sigma} \tag{31}$$

We must convert our sum over N to a sum over states, so $N = \sum_{i=0}^{\infty} n_i$:

$$Q = \sum_{n_i=0}^{\infty} e^{\sum_i n_i \beta(\mu - E_i)} = \prod_i \left(\sum_{n_i=0}^{\infty} e^{-n_i \beta(E_i - \mu)} \right) \tag{32}$$

It's easiest to calculate this for spin zero particles, in which case $n_i = 0, 1$:

$$Q = \prod_i \left(1 + e^{-\beta(E_i - \mu)}\right) \tag{33}$$

For nonzero spin, the n_i should be multiplied by a factor of $2S + 1$. So for example, for a particle with spin $\frac{5}{2}$, $2S + 1 = 6$ and the grand partition function becomes:

$$Q = \prod_i \left(1 + e^{-\beta(E_i - \mu)}\right)^6 \tag{34}$$

As a general rule, quantum effects should dominate at low temperatures; classical effects should dominate at high temperatures.

3.1 Photons

Remember that **N is not conserved for photons**, and likewise, $\mu = 0$.

3.2 Condensates

How to tell if we have a condensate:
Set $\mu = 0$ and $T = 0$. If $N \to \infty$, no condensate. If $N \to 0$ or $N \to$ a number, condensate.

Or, solve for μ, set $T = 0$ and see if we have any restrictions on N to make μ equal the ground state energy. No restrictions on N = no condensate. Essentially if you can even find μ by itself, there's no condensate.

How to find critical temperature:
Set $\mu = 0$ and solve for T as a function of N. This is the critical temperature.

3.3 Distribution Functions

We can use the probability distribution $f(\epsilon_i)$ for bosons and fermions to calculate various useful things. (We can derive these distribution functions from the canonical ensemble, where Z has an added $\frac{1}{N!}$ for bosons.) In these formulas:

- The distribution function $f(\epsilon_i)$ represents the **number of particles with energy** ϵ_i. When called a "probability function", it does not include g_s in the numerator; when used to calculate "population", it does. Be careful with wording and note which case you are assuming.
- Spin degeneracy g_s represents the **spin degeneracy of state i** being calculated. This is $2S + 1$ except in the case of massless particles, which have spin degeneracy $2S$. If the energy ϵ_i depends on the spin, this is just 1.
- ϵ_i represents an **energy** that the state can have.
- $g(k)$ and $g(\epsilon)$ represent the **density of states**. These must be calculated depending on the energy and dimensions of the question. The easiest way to do that is:

$$g(\epsilon) = \int \left(\frac{L}{2\pi}\right)^n d\vec{k}\, \delta(\epsilon - \epsilon_k) \tag{35}$$

In this definition for density of states, $d\vec{k}$ should be written according to the number of dimensions, and ϵ_k is the reference energy relating ϵ and k (for example, $\epsilon_k = pc = \hbar kc$). Write $d\vec{k}$ in terms of

energy and the integral becomes trivial.

Remember this definition does not include **spin degeneracy** g_s, which some people do include in their definition of density of states.

The following calculations assume that g_s is **not** included in the distribution function $f(\epsilon)$ or the density of states $g(\epsilon)$.

Bosons:

$$f(\epsilon) = \frac{1}{e^{\beta(\epsilon-\mu)} - 1} \tag{36}$$

Fermions:

$$f(\epsilon) = \frac{1}{e^{\beta(\epsilon-\mu)} + 1} \tag{37}$$

Boltzmann Distribution:

$$f(\epsilon) = \frac{1}{e^{\beta(\epsilon-\mu)}} \tag{38}$$

The following are particularly helpful for quantum systems:

Average internal energy:

$$U = \int \epsilon \, g_s \, g(\epsilon) f(\epsilon) d\epsilon \tag{39}$$

Number of particles:

$$N = \int g_s \, g(\epsilon) f(\epsilon) d\epsilon \tag{40}$$

Specific heat capacity:

$$C = \frac{\partial}{\partial T} \int \epsilon \, g_s \, g(\epsilon) f(\epsilon) d\epsilon \tag{41}$$

The average of any random quantity n, if we know that quantity in terms of ϵ, $n(\epsilon)$:

$$\langle n \rangle = \int n(\epsilon) \, f(\epsilon) \, g(\epsilon) \, d\epsilon \tag{42}$$

We can also find the average of a quantity by integrating over phase space:

$$\langle \epsilon \rangle = \int \epsilon_k \, f(k) \, g(k) \, d^3k d^3x \tag{43}$$

At $T = 0K$, we can find a few more properties:

$$N = \int_0^{\epsilon_f} g(\epsilon) d\epsilon \tag{44}$$

$$\mu = \epsilon_f \ \ at \ T = 0K \tag{45}$$

4 Phase Transitions & Mean Field Theory

Magnetization:

$$M = -\frac{\partial F}{\partial B} = \mu_0 (N_+ - N_-) \tag{46}$$

Magnetic Susceptibility :

$$\chi = \frac{\partial M}{\partial B} \tag{47}$$

Evidence of **Phase Transition**:

- Divergence of E or M (or divergence of their derivatives)
- Multiple states for a fixed temperature (example, high and low density states simultaneously present)

To find the solution(s) of a transcendental equation, set the *slope* of each side equal to each other.

For example, when looking for spontaneous magnetization, remember that the slope of $\tanh(\alpha x)$ at $x = 0$ is α (or just take the derivative of the function to get the slope! evaluate result at M=0 for most linear case). So, to find solutions for $\beta x = \tanh(\alpha x)$, set the slopes equal: $\beta = \alpha$ in this case.

5 Helpful Maths

Taylor expansion:

$$F(x_0 + dx) = F(x_0) + dx \frac{dF}{dx}|_{x=x_0} + dx^2 \frac{d^2F}{dx^2}|_{x=x_0} + ... \tag{48}$$

Don't forget the chain rule for derivatives:

$$\frac{dT}{dz} = \frac{\partial T}{\partial P}\frac{\partial P}{\partial z} \tag{49}$$

Stirling's approximation:

$$\ln N! = N \ln N - N \tag{50}$$

To take the derivative of a function that has multiple variables, such as $S(T, N, V)$:

$$dS(T, N, V) = \frac{\partial S}{\partial T} dT + \frac{\partial S}{\partial N} dN + \frac{\partial S}{\partial V} dV \tag{51}$$

We can think of this from taking the whole time derivative and "canceling out" the dt from each term:

$$\frac{d}{dt} S(T, N, V) = \frac{\partial S}{\partial T}\frac{dT}{dt} + \frac{\partial S}{\partial N}\frac{dN}{dt} + \frac{\partial S}{\partial V}\frac{dV}{dt} \tag{52}$$

Expansions for small x (useful for temperature limits!):

$$(1 + x)^n \simeq 1 + nx \tag{53}$$

$$e^x \simeq 1 + x + ... \tag{54}$$

Helpful summation tricks

$$\sum_{N=0}^{\infty} a^N = \frac{1}{1-a} \quad a \ll 1 \tag{55}$$

$$\sum_{N=0}^{\infty} \frac{1}{N!} x^N = e^x \tag{56}$$

Sometimes a partition function (grand partition functions especially) cannot be easily calculated. Then it helps to look at the high and low temperature limits. For example:

$$Z \sim \sum e^{-\beta E n^2} \tag{57}$$

For $T \to \infty$, $\beta \to 0$, so exponent gets small. **Integrate.**

For $T \to 0$, $\beta \to \infty$, so $e^{-\beta E}$ gets small. **Sum**, keeping only the first couple terms.

Remember this integral for help solving N in grand canonical ensemble problems:

$$\int_0^\infty \frac{x^{n-1}}{z^{-1}e^x - 1} = \Gamma(n) g_n(z) \tag{58}$$

Standard deviation:

$$\Delta n^2 = \langle n^2 \rangle - \langle n \rangle^2 \tag{59}$$

Gamma function:

$$\Gamma(n+1) = n\Gamma(n) \qquad \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} \tag{60}$$

Table 19.1 summarizes the results for specific gas processes. This table shows W_s, the work done *by* the system, so the signs are opposite those in Chapter 17.

TABLE 19.1 Summary of ideal-gas processes

Process	Gas law	Work W_s	Heat Q	Thermal energy
Isochoric	$p_i/T_i = p_f/T_f$	0	$nC_V\Delta T$	$\Delta E_{th} = Q$
Isobaric	$V_i/T_i = V_f/T_f$	$p\Delta V$	$nC_P\Delta T$	$\Delta E_{th} = Q - W_s$
Isothermal	$p_iV_i = p_fV_f$	$nRT\ln(V_f/V_i)$ $pV\ln(V_f/V_i)$	$Q = W_s$	$\Delta E_{th} = 0$
Adiabatic	$p_iV_i^\gamma = p_fV_f^\gamma$ $T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$	$(p_fV_f - p_iV_i)/(1-\gamma)$ $-nC_V\Delta T$	0	$\Delta E_{th} = -W_s$
Any	$p_iV_i/T_i = p_fV_f/T_f$	area under curve		$\Delta E_{th} = nC_V\Delta T$

There is one entry in this table that you haven't seen before. The expression

$$W_s = \frac{p_fV_f - p_iV_i}{1-\gamma} \qquad \text{(work in an adiabatic process)} \qquad (19.12)$$

for the work done in an adiabatic process follows from writing $W_s = -\Delta E_{th} = -nC_V\Delta T$, which you learned in Chapter 17, then using $\Delta T = \Delta(pV)/nR$ and the definition of γ. The proof will be left for a homework problem.

You learned in Chapter 18 that the thermal energy of an ideal gas depends only on its temperature. Table 19.2 lists the thermal energy, molar specific heats, and specific heat ratio $\gamma = C_P/C_V$ for monatomic and diatomic gases.

TABLE 19.2 Properties of monatomic and diatomic gases

	Monatomic	Diatomic
E_{th}	$\frac{3}{2}nRT$	$\frac{5}{2}nRT$
C_V	$\frac{3}{2}R$	$\frac{5}{2}R$
C_P	$\frac{5}{2}R$	$\frac{7}{2}R$
γ	$\frac{5}{3} = 1.67$	$\frac{7}{5} = 1.40$

8.12, 8.13 Activ Online Physics

A Strategy for Heat-Engine Problems

The engine of Example 19.1 was not a realistic heat engine, but it did illustrate the kinds of reasoning and computations involved in the analysis of a heat engine. A basic strategy for analyzing a heat engine follows.

PROBLEM-SOLVING STRATEGY 19.1 Heat-engine problems (MP)

MODEL Identify each process in the cycle.

VISUALIZE Draw the pV diagram of the cycle.

SOLVE There are several steps in the mathematical analysis.

- Use the ideal-gas law to complete your knowledge of n, p, V, and T at one point in the cycle.
- Use the ideal-gas law and equations for specific gas processes to determine p, V, and T at the beginning and end of each process.
- Calculate Q, W_s, and ΔE_{th} for each process.
- Find W_{out} by adding W_s for each process in the cycle. If the geometry is simple, you can confirm this value by finding the area enclosed within the pV curve.
- Add just the *positive* values of Q to find Q_H.
- Verify that $(\Delta E_{th})_{net} = 0$. This is a self-consistency check to verify that you haven't made any mistakes.
- Calculate the thermal efficiency η and any other quantities you need to complete the solution.

ASSESS Is $(\Delta E_{th})_{net} = 0$? Do all the signs of W_s and Q make sense? Does η have a reasonable value? Have you answered the question?

Thermodynamics Review

Ch 16 - Macroscopic Description of Matter

- Thermal equilibrium is defined by non-changing state variables (ex. P, V, T)
- At phase equilibrium, any amount of each state possible at those points can exist
  - critical point is limit where gas + liquid can be distinguished

* Ideal gas systems present good systems to develop thermodynamic basics on

$PV = nRT$ (Ideal Gas Law) alternatively, $PV = Nk_BT$; n = # of moles, N = # of Particles

- Manipulations of the Ideal Gas Law yield:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$ * # of particles constant

- Processes that occur in thermal equilibrium are known as <u>quasi-static</u>. They are also <u>reversible</u>

- Constant volume processes are <u>isochoric</u>
- Constant pressure processes are <u>isobaric</u>
- Constant temperature processes are <u>isothermic</u>

> Typically represented on a P-V diagram

Ch 17 - Work, Heat, + the First Law of Thermodynamics

- Redefining the work-energy theorem

$W_{ext} = \Delta K + \Delta U + \Delta E_{th}$ where $E_{th} = K_{micro} + U_{micro}$

* However, not all changes in energy go towards work, some is lost to heat

$\Rightarrow \Delta E = W + Q$

- Note: Work is not a state variable;

$$W = \int \vec{F} \cdot d\vec{s}$$

$$= -\int P\,dV$$ (for gases) * P is typically a function of V *

* Work is not path-independent quantity *

- <u>Heat</u> is energy transferred b/w a system + the environment due to a difference in temperature.
  * no motion of the system is required

$\Delta E_{th} = W + Q$ (First Law of Thermodynamics)

* simply energy conservation

- A process is adiabatic if $Q = 0$
  ⇒ This does not imply that $\Delta T = 0$

- The specific heat of a substance is the amount of energy necessary to raise 1kg of it by 1K
  * Assuming $W = 0$,

$$Q = Mc\Delta T = nC\Delta T$$

(C: molar specific heat)

- The heat of transformation is the amount of energy necessary to transform 1kg of a substance from one phase to another

$$Q = \pm ML, \text{ where } L \text{ is heat of transformation}$$

- Gases have two types of specific heat; C_p and C_v
  ⇒ $C_p = C_v + R$
  * for adiabatic processes, $PV^\gamma = \text{constant}$; $\gamma = C_p/C_v$

Ch 18: The Macro/Micro Connection

- Kinetic theory states that the macroscopic properties of a system are related to the avg. behavior of the molecules/atoms that compose the system

- The mean free path of a molecule/atom is:

$$\lambda = \frac{1}{4\sqrt{2}\pi(N/V)r^2}, \text{ where } \frac{N}{V} \text{ is \# density of particles}$$

- The pressure of a gas is:

$$P = \frac{F}{A} = \frac{1}{3}nmv_{rms}^2, \quad v_{rms} = \sqrt{\langle v^2 \rangle}, \quad A = \text{Area}, \quad n = \text{\# density}$$

$$\epsilon_{avg} = \frac{3}{2}k_B T \Rightarrow T = \frac{2}{3k_B}\epsilon_{avg}$$

→ Temperature measures avg translational kinetic energy

- For ideal, monatomic gases, $C_v = \frac{3}{2}R$; ideal diatomic gases $C_v = \frac{5}{2}R$
- The equipartition theorem states, that the thermal energy of a system is equally divided
  ex. rotational energy = kinetic energy
  $v_x = v_y$
- Heat will be exchanged by two systems in contact until they reach thermal equilibrium, i.e the avg energy of a particle in each system is the same

$$\Rightarrow \epsilon_{avg,1} = \epsilon_{avg,2}, \quad \text{Not } E_1 = E_2 \text{ (only if } N_1 = N_2)$$

- Equilibrium is the most probable state of a system

- The Second Law of Thermodynamics states that entropy (the measure of disorder in a system) never decreases

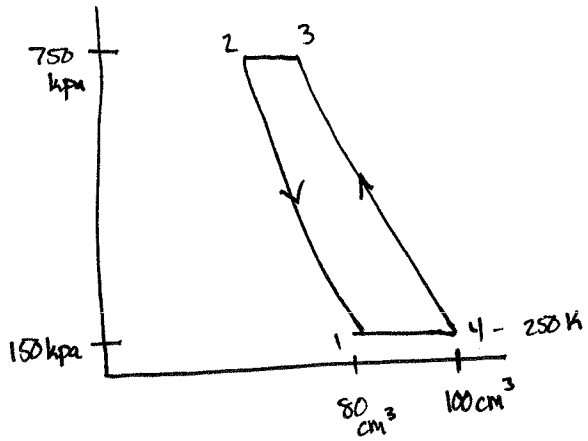
  ⇒ Implies that heat is always transferred from hot to cold

  ⇒ Irreversible processes imply that entropy only increases in one direction

Ch 19 - Heat Engines + Refrigerators

- Heat engine refers to a cyclical process that transforms heat into work
- Refrigerators is a device that uses work to move heat from a cold object to a hot object
- Devices that turn heat into work (ie heat engines) must return to their initial state at the end of the process + be able to repeat the process

ex. Brayton-cycle



* He gas

$P_1 = 150$ kPa	$V_1 = 80\ cm^3$	$T_1 = 200$ K
$P_2 = 750$ kPa	$V_2 =$	$T_2 = 381$ K
$P_3 = 750$ kPa	$V_3 =$	$T_3 = 476$ K
$P_4 = 150$ kPa	$V_4 = 100\ cm^3$	$T_4 = 250$ K

$$\frac{V_1}{T_1} = \frac{V_4}{T_4}$$

$$\Rightarrow T_1 = T_4 V_1 / V_4$$

$$= 200 K$$

$$P_4 V_4^{\gamma} = P_3 V_3^{\gamma}$$

* but $V = \frac{nRT}{P}$, $\gamma = \frac{C_p}{C_v}$

$$\Rightarrow P_4^{\gamma-1} T_4^{\gamma} = P_3^{\gamma-1} T_3^{\gamma}$$

$$\Rightarrow T^3 = T^4 \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 476\ K$$

$$P_2^{\gamma-1} T_2^{\gamma} = P_1^{\gamma-1} T_1^{\gamma}$$

$$P_2^{\frac{\gamma-1}{\gamma}} T_2 = P_1^{\frac{\gamma-1}{\gamma}} T_1$$

$$T_2 = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} T_1$$

$$= 381\ K$$

$$Q_{3\to 2} = nC_p \Delta T$$

$$= 14.3\ J$$

$$Q_{1\to 4} = nC_p \Delta T$$

$$= 7.5\ J$$

$$\Rightarrow W = 14.3 - 7.5 = 6.8\ J$$

$$K = \frac{Q_c}{W_{in}} = \frac{7.5}{6.8} = 1.1$$

Classical / Stat Mech Qualifier Topics

Exam: Spring 2008

Question #1 - Forces/Newtons Laws, Friction (all types), Kinematics, Rotational motion
Question #2 - Lagrangian Mechanics, Oscillations, Constraint forces, moments of motion
Question #3 - Hamiltonian Mechanics, Poisson Brackets, Orbital motion
Question #4 - Heat Engine (Work, efficiency, direction of cycle)
Question #5 - Partition Function + related quantities
Question #6 - Grand Partition Function, Quantum stat mech (occupation #'s)

Exam: Fall 2008

Question #1 - Pendulum, Collisions, Kinematics
Question #2 - Harmonic Oscillator, Lagrangian mechanics,
Question #3 - Hamilton-Jacobi Theory, Canonical Variables, Characteristic Functions
Question #4 - Helmholtz Free Energy, Pistons, Equilibrium, Heat Reservoir
Question #5 - One-dimensional gas, quantum limit, classical limit
Question #6 - Fermionic Gas, massless gas, thermodynamic quantities

Exam: Spring 2009

Question #1: Newtons Laws, Lagrangian Mechanics, Oscillations, Equations of motion
Question #2: Central Force, Kepler's Laws, Orbits, Oscillations
Question #3: Power, Maximization, Equations of motion
Question #4: Heat Engine (Work, Efficiency, direction of cycle)
Question #5: Free Energy, Ideal Gas, thermodynamic quantities,
Question #6: Boson Gas, Grand Canonical Free Energy

Exam: Fall 2009

Question #1: Rotational Motion, Friction
Question #2: Lagrangian Mechanics, Equations of motion
Question #3: Lagrangian Mechanics, Equations of motion, Hamiltonian Mechanics, Canonical variables
Question #4: Helmholtz Free Energy, Carnot Cycle, Heat Engine
Question #5: Free Energy, Thermodynamic quantities
Question #6: —

Exam: Spring 2010

Question #1: Pendulum, Oscillations, Rotational Motion, Forces/Newton's Laws
Question #2: Rotational Motion, Kinematics, Newton's Laws/Forces
Question #3: Lagrangian Mechanics, Hamiltonian Mechanics, Forces/Newtons Laws
Question #4: Hydrostatic Equilibrium, Adiabatic Processes
Question #5: Boson gas, Fermionic Gas, Boltzmann Gas, Thermodynamic Quantities
Question #6: Lattice Gas, Grand Canonical Ensemble, Thermodynamic Quantities, Phase Transition

Exam: Fall 2010

Question #1: Oscillations, Eigenvalue/Eigenvectors, Forces/Newtons Laws, Equations of Motion
Question #2: Rotational Motion
Question #3: Lagrangian Mechanics, Hamiltonian Mechanics, Canonical Transform, Equations of Motion
Question #4: Cooling, Entropy
Question #5: Electron gas, Density states, Fermi energy, Thermodynamic quantities
Question #6: Statistics (Avg, variance), Clump/Anti-clump, Instrumentation

Exam: Fall 2011

Question #1: Rotational Motion, Kinematics, Forces/Newton's Laws, Friction
Question #2: Central Force, Angular momentum, Orbits
Question #3: Lagrangian Mechanics, Hamiltonian Mechanics, Equations of motion
Question #4: Heat Engine (Work, efficiency, direction of cycle)
Question #5: Partition Function, Thermodynamic quantities, Equipartition Thm, quantum limit
Question #6: Dispersion relationship, Grand Canonical Ensemble, Fermi energy, Entropy

Exam: Spring 2012

Question #1: Rotational Motion, Kinematics, Forces/Newton's Laws, Friction
Question #2: Pendulum, Lagrangian Mechanics, Equations of Motion
Question #3: Lagrangian Mechanics, Canonical variables, Action principles, Calculus of variations
Question #4: Maxwell relations, Thermodynamic variables (P, V, T, S, μ, N, etc)
Question #5: D-dimensional boson gas, Condensation
Question #6: Lattice gas, partition function, Helmholtz Free Energy, Thermodynamic quantities

Exam: Fall 2012

Question #1: Rotational Motion, Kinematics, Forces/Newton's Laws, Friction, Equations of motion
Question #2: Rotational Motion, Lagrangian Mechanics, Equations of motion, Oscillations
Question #3: Hamiltonian Mechanics, Characteristic Function, Residue Thm, Action Principle, Orbits
Question #4: Helmholtz Free Energy, thermodynamic variables, thermodynamic quantities, Thermo Laws
Question #5: Lattice gas, Grand Potential, canonical ensemble, microcanonical ensemble
Question #6: D-dimensional Fermi gas, Fermi energy, Grand Potential, Ideal Gas Law

Exam: Spring 2013

Question #1: Rotational Motion, Kinematics, Forces/Newtons Law, Work-Energy Thm
Question #2: Collisions, Rotational Motion, Kinematics, Torque?
Question #3: Lagrangian Mechanics, Lagrange multipliers, Equations of motion
Question #4: Thermodynamic Laws, Maxwell Relations, Free Energy
Question #5: Partition Function, microcanonical ensemble, canonical ensemble, free energy, thermo quant
Question #6: Thermodynamic quantities, equation of state, partition function, Helmholtz Free Energy

Exam: Fall 2013

Question #1: Rotational Motion, Forces/Newton's Laws, Kinematics
Question #2: Lagrangian Mechanics, Rotational Motion, Equations of motion, Oscillations, Forces/Newton
Question #3: Pendulum, Lagrangian Mechanics, Hamiltonian Mechanics, Harmonic motion, Equations of Motion
Question #4: Heat Engine (Work, efficiency, direction of cycle)
Question #5: Partition function, high/low temp limits,
Question #6: Thermodynamic Equilibrium, temp limits, Entropy, thermodynamic quantities

Exam: Spring 2014

Question #1: Rotational Motion, Forces/Newtons Laws, Kinematics, Energy
Question #2: Pendulum, Lagrangian Mechanics, Equations of motion, Oscillations
Question #3: Poisson Brackets, Angular Momentum, generalized coordinates
Question #4: Pistons, Thermodynamic Equilibrium, Thermodynamic quantities
Question #5: Entropy, canonical ensemble, partition function, adiabatic processes, Thermo equilibrium
Question #6: Density of states, Fermi energy, temp limits, thermodynamic quantities

Exam: Fall 2014

Question #1: Rotational Motion, Forces/Newton's Laws, Oscillations
Question #2: Corrolis force, Rotational Motion, Forces/Newton's Law, Moving coordinate systems
Question #3: Hamiltonian Mechanics, Hamilton-Jacobi eqn, Hamilton principle function, eqn of motion
Question #4: Phase transition, Entropy, Gibbs Energy
Question #5: Lattice gas, Grand Canonical Partition Function, Mean field, Thermodynamic quantities
Question #6: Photon gas, Thermodynamic Functions, Thermodynamic quantities

Exam: Spring 2015

Question #1: Collisions, Forces/Newtons Laws, Kinematics
Question #2: Rotational Motion, Eqn of Motion, Lagrangian Mechanics, Pendulum
Question #3: Poisson Bracket, Orbits, Hamiltonian Mechanics, Runge-Lenz vector
Question #4: Ideal gas, Thermodynamic eqns, Thermodynamic quantities
Question #5: Lattice, Thermodynamic eqns, Entropy
Question #6: Boson gas, dispersion relations, Density of states, Thermodynamic eqns, Condensation

Exam: Fall 2015

Question #1: Rotational Motion, Kinematics, Forces/Newton's Laws
Question #2: Forces/Newtons Laws, Kinematics, Eqn of motion
Question #3: Hamiltonian Mechanics, Poisson Brackets, Canonical coordinates
Question #4: Thermodynamic Equilibrium, Entropy
Question #5: Partition function, Thermodynamic quantities, Thermodynamic eqns
Question #6: Fermion gas, Density of States, Fermi energy, Thermodynamic eqns

Exam: Spring 2016

Question #1: Rotational Motion, Kinematics, Forces/Newton's Laws, Friction
Question #2: Lagrangian Mechanics, Eqn of motion, Oscillations, Rotational Motion
Question #3: Double pendulum, Lagrangian Mechanics, eigenvalue/eigenvector
Question #4: Heat engine (efficiency, work)
Question #5: Partition function, temp limits, thermodynamic quantities
Question #6: Boson gas, density of states, thermodynamic eqns, thermodynamic quantities