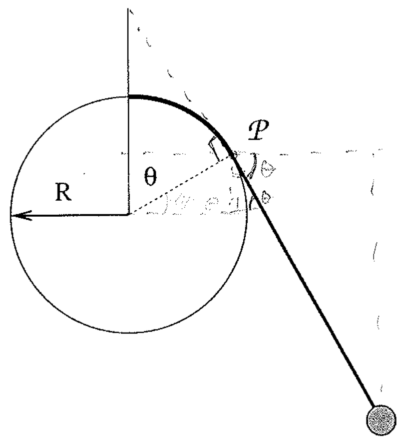
2. A stationary disk of radius R is aligned vertically so that its axis is parallel to the ground. The disk is fixed and does not rotate. A string of length ℓ is attached to the top of the disk, and $\ell > \pi R$. A point mass m is attached to the end of the string and can swing in a vertical plane (left to right in the figure below). As the mass m swings, the point $\mathcal{P}$ where the string just contacts the disk will move. Assume that the string is always taut. The angle between $\mathcal{P}$ and the vertical is θ; it will be the generalized coordinate in this problem.



(a) Determine the x and y position of the point mass as a function of θ, R and ℓ. Use the center of the disk as the origin of your coordinate system. (Hint: Knowing the value of θ determines both the amount of string wrapped on the disk and the angle the straight length of string makes with the vertical.) (1 point)

(b) Treating $\theta(t)$ as the generalized coordinate, determine the kinetic energy of the point mass as a function of m, θ, $\dot{\theta}$, R, and ℓ. (2 points)

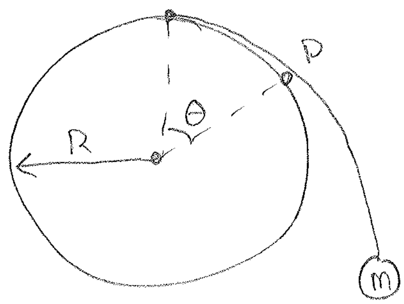
(c) What is the Lagrangian for the system in terms of this generalized coordinate? (2 points)

(d) What are the equations of motion? (1 point)

(e) Assume that the point mass makes small oscillations about some angle θ_0 (which might not be zero). Determine θ_0 and the angular frequency of these oscillations. (4 points).

Spring 2016

Classical #2



* Sting of length l

a) $\vec{r}_m = \langle R\cos(90-\theta),\ R\sin(90-\theta)\rangle$

$= \langle R\sin\theta,\ R\cos\theta\rangle + \langle (l-R\theta)\cos\theta, -(l-R\theta)\sin\theta\rangle$

$= \langle R\sin\theta + (l-R\theta)\cos\theta;\ R\cos\theta - (l-R\theta)\sin\theta\rangle$

b) $T = \frac{1}{2}mv^2$

$= \frac{1}{2}m\dot{\vec{r}}^2$

$\dot{\vec{r}} = \langle R\cos\theta\,\dot{\theta} - l\sin\theta\,\dot{\theta} - [R\dot{\theta}\cos\theta - R\theta\sin\theta\dot{\theta}],\ -R\sin\theta\dot{\theta} - l\cos\theta\dot{\theta} + R\dot{\theta}\sin\theta + R\theta\cos\theta\dot{\theta}\rangle$

$\dot{\vec{r}} = \dot{\theta}\langle R\cos\theta - l\sin\theta - R\cos\theta + R\theta\sin\theta,\ -R\sin\theta - l\cos\theta + R\sin\theta + R\theta\cos\theta\rangle$

$= \dot{\theta}\langle -l\sin\theta + R\theta\sin\theta,\ R\theta\cos\theta - l\cos\theta\rangle$

$\dot{\vec{r}}^2 = \dot{\theta}^2[l^2\sin^2\theta + R^2\theta^2\sin^2\theta - 2lR\theta\sin^2\theta + R^2\theta^2\cos^2\theta + l^2\cos^2\theta - 2R\theta l\cos^2\theta]$

$= \dot{\theta}^2[l^2 + R^2\theta^2 - 2lR\theta] \Rightarrow T = \frac{1}{2}m\dot{\theta}^2[l^2 + R^2\theta^2 - 2lR\theta]$

c) $\mathcal{L} = T - U$

$= \frac{1}{2}m\dot{\theta}^2[l^2 + R^2\theta^2 - 2lR\theta] - mg(R\cos\theta - (l-R\theta)\sin\theta)$

d) $\frac{\partial \mathcal{L}}{\partial\theta} = \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial\dot{\theta}}$

$\cancel{\frac{1}{2}}m\dot{\theta}^2R^2\theta - 2lR + mgR\sin\theta + mgl\cos\theta - R\sin\theta - R\theta\cos\theta = \frac{\partial}{\partial t}[m\dot{\theta}(l^2 + R^2\theta^2 - 2lR\theta)]$

$m\dot{\theta}^2R^2\theta - 2lR + R\sin\theta(mg - 1) + \cos\theta(mgl - R\theta) = ml^2\ddot{\theta} + R^2m\ddot{\theta}\theta^2 + 2R^2m\dot{\theta}^2\theta - 2lR(\dot{\theta}+\theta$