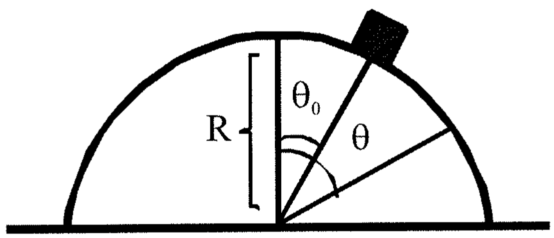
Lagrangian Mechanics

why constraint?

⇒ RADIAL + ANGULAR MOTION

Problem 2: (10 Points)

A hemisphere of radius R rests on the ground. A particle of mass m starts from rest on the sphere at an angle of θ_0 from the vertical that passes through the center of the sphere. Express answers in terms of R, θ_0 and the acceleration due to gravity near the surface of the earth, g.



a. The particle is released and slides without friction. At what angle, θ, measured relative to the vertical, does the particle leave the surface of the sphere? **(4 Points)**

b. What is the angle θ when $\theta_0 = 0$? **(1 Points)**

c. Assume the particle was released with $\theta_0 = 0$. Once the particle leaves the sphere, how long does it take it to hit the ground? **(3 Points)**

d. How far from the center of the sphere is the particle when it hits the ground? **(2 Points)**

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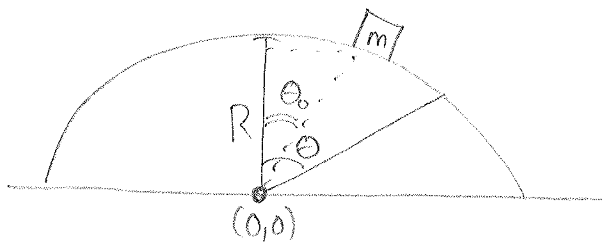
Classical #2

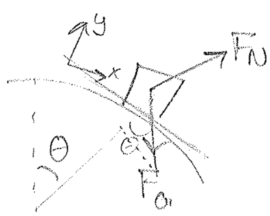


* Normal force 0 when block leaves surface of hemisphere

a) Block is released + slides w/o friction. At what θ (relative to vertical) does block leave the surface of the hemisphere?

$x = R\sin\theta$

$y = R\cos\theta$



$\langle \parallel, \perp \rangle$

$\Rightarrow F_N = \langle 0, F_N \rangle$

$F_G = mg\langle \sin\theta, -\cos\theta \rangle$

$\Rightarrow ma_x = mg\sin\theta$

$ma_y = F_N + mg\cos\theta$

* but $a_y = \frac{v^2}{R}$

$-m\frac{v^2}{R} = -mg\cos\theta_c$

$\frac{v^2}{gR} = \cos\theta_c$

or

$v = \sqrt{gr\cos\theta_c}$

$mgy = \frac{1}{2}mv^2 + mgy$

$mg\cos\theta_0 = \frac{1}{2}mv^2 + mg\cos\theta_c$

$\cancel{m}gR\cos\theta_0 = \frac{1}{2}\cancel{m}gR\cos\theta_c + \cancel{m}gR\cos\theta_c$

$\cos\theta_0 = \frac{3}{2}\cos\theta_c$

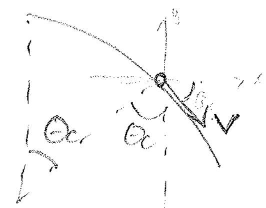
$\Rightarrow \theta_c = \cos^{-1}\left(\frac{2}{3}\cos\theta_0\right)$ ✓

b) What is θ_c if $\theta_0 = 0$?

$$\theta_c = \cos^{-1}\left(\tfrac{2}{3}\cos\theta_0\right)$$

$$\theta_c = \cos^{-1}\left(\tfrac{2}{3}\right)$$

c) If particle released at $\theta = 0$, how long does it take to hit the ground after it leaves the surface of the hemisphere?



$y_i = R\cos\theta_c \qquad v_i = v\sin\theta_c \qquad a = -g$

$y_f = 0 \qquad v_f = ?$

$$0 = -\tfrac{1}{2}gt^2 - v\sin\theta_c + R\cos\theta_c$$

$$\Rightarrow \quad t = \frac{v\sin\theta_c \pm \sqrt{v^2\sin^2\theta_c - 2gR\cos\theta_c}}{g}$$

$$t = \frac{v\sin\theta_c \pm \sqrt{v^2\sin^2\theta_c - 2v^2}}{g}$$

$$t = \frac{v\sin\theta_c \pm v\sqrt{\sin^2\theta_c - 2}}{g}$$