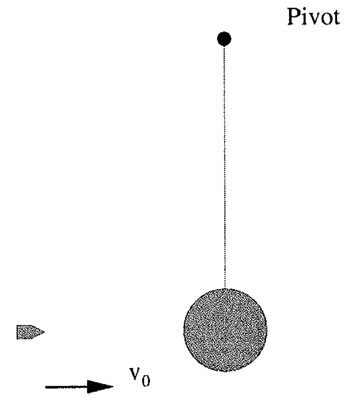
Classical Mechanics

1. **The ballistic pendulum:** Consider a pendulum with a bob of mass m connected to a frictionless pivot by an ideal massless rigid rod of length ℓ. A projectile of mass ϵm ($0 < \epsilon << 1$) moving horizontally at speed v_0 hits the center of the bob, as shown. When it strikes, it becomes imbedded in the bob.



   (a) What is the minimum initial speed of the projectile such that the pendulum will make a full rotation? (2 points)

   (b) The rod is replaced by an ideal massless non-rigid string. What is the minimum initial speed of the projectile such that the pendulum will make a full revolution without the string going slack? (3 points)

   (c) Now assume that projectile rebounds elastically from the bob in the horizontal direction. What is the minimum initial speed of the projectile such that the pendulum will make a full revolution without the string going slack? (2 points)

   (d) Finally, assume that the projectile passes completely through the pendulum bob, in a time $t << \sqrt{\ell/g}$. After it exits, it carries with it some of the original mass of the bob, such that the exiting projectile now has a mass $2\epsilon m$ and moves at a speed $3v_0/4$. What is the minimum initial speed of the projectile such that the pendulum will make a full revolution without the string going slack? (3 points)

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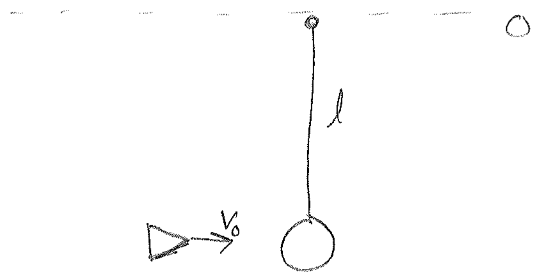
Classical #1



$m_b = \epsilon m$

$m_p = m$

a) * Collision (Inelastic)

$$\cancel{\tfrac{1}{2} m\epsilon v_0^2 = \tfrac{1}{2} m(1+\epsilon)v^2} \qquad \epsilon m v_0 = (1+\epsilon)mv$$

$$\cancel{\frac{\epsilon}{1+\epsilon} v_0^2 = v^2} \qquad \frac{\epsilon}{1+\epsilon} v_0 = v$$

* Conservation of energy

$$\frac{1}{2} m(1+\epsilon)\left(\frac{\epsilon}{1+\epsilon} v_0\right)^2 - mg\ell = mg\ell + \frac{1}{2} m\cancelto{0}{(1+\epsilon)} v^2$$

$$\frac{1}{2} m \frac{\epsilon^2}{1+\epsilon} v_0^2 = 2mg\ell + \frac{1}{2} m\cancelto{0}{(1+\epsilon)} v^2$$

$$\frac{\epsilon^2}{1+\epsilon} v_0^2 = 4mg\ell$$