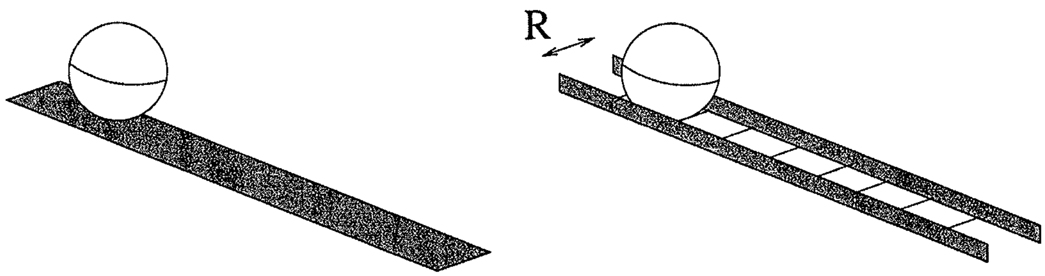
Classical Mechanics

1. Rolling spheres: Given that the moment of inertia of a sphere of mass m and radius R is $(2/5)mR^2$, please answer the following.



   (a) A sphere of radius R and mass m rolls without slipping down an inclined plane on to a horizontal table (left figure). The condition of "rolling without slipping" forces a relationship between v, the speed of the center of mass of the sphere, and ω, the angular velocity of the sphere about its center of mass. What is this relationship? (0.5 pt.)

   (b) Calculate the speed of the sphere at the bottom of the ramp if the center of mass of the sphere has dropped a distance h when it just touches the table. Assume that $h >> R$. (1.5 pt.)

   (c) The ramp is now replaced by two narrow rails separated by a distance R (right figure). Again the ball rolls downward without slipping, supported by the two rails. In this case, what the relationship between v and ω? (1 pt.)

   (d) In this second case, calculate the speed of the sphere at the bottom if the center of mass has dropped a distance h. (2 pts).

   (e) After the ball reaches the bottom of the rails (part b) it continues to move on the horizontal table. It will either be rolling too fast or too slow to roll without slipping. Which will it be? You must prove your result. (1 pt.)

   (f) Friction between the sphere and the plane will adjust the speed of the sphere until it can again roll without slipping. If the magnitude of the force of friction between the sphere and the plane is

μmg, determine the speed of the ball when it again rolls without slipping. (If you did not solve part (b) above, assume the sphere is moving at speed v_0 without rolling and determine its speed when it rolls without slipping). (4 pts.)

Aug 2009

Classical #1

a) $v = \omega r$

b) $E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{5}mv^2$$

$$gh = \frac{7}{10}v^2$$

$$\Rightarrow v = \sqrt{\frac{10gh}{7}}$$

c)