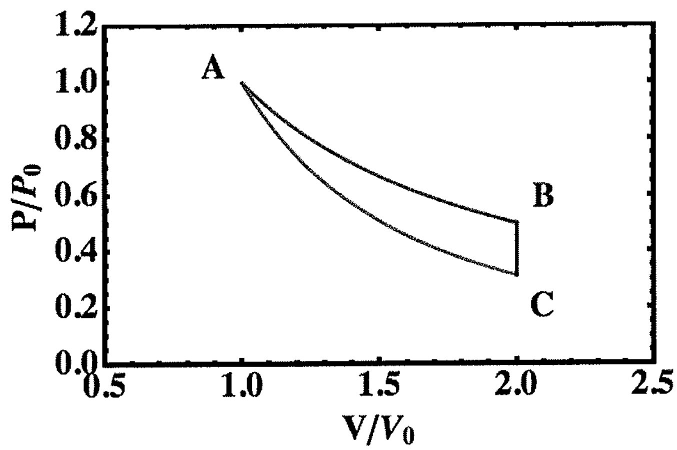
Statistical Mechanics

4. Consider an ideal monatomic gas used as the working fluid in a thermodynamic cycle. The number of particles is n_0. It follows a cycle consisting of one adiabat, one isochore and one isotherm, as shown below.



(a) Calculate the pressure, temperature, and volume at each corner of the cycle, A, B, and C, expressing your answer in terms of P_0, V_0, n_0 and perhaps R, the ideal gas constant. Note that point A the pressure is P_0 and the volume is V_0. (3pts)

(b) Calculate the work done on the system, the heat into the system and the change in the internal energy of the system for each process step. (4.5pts)

(c) What direction around the ycle must the system follow to be used as a functional heat engine? (1/2pt)

(d) What is the efficiency of the cycle, run as an engine? (1pt)

(e) What is the efficiency of an ideal Carnot engine run between reservoirs B and C? (1pt)

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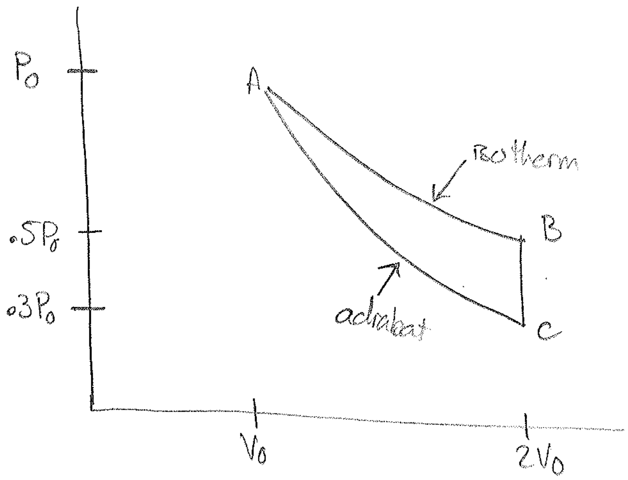
Stat Mech #1



* cycle consists of adiabat, isochore, and isotherm

* gas is ideal + monatomic

$$\Rightarrow \begin{matrix} C_V = \frac{3}{2}R \\ C_P = \frac{5}{2}R \end{matrix} \rangle \gamma = 5/3$$

a) Find P, V, and T at each corner of the cycle in terms of P_0, V_0, n_0, and R

$P_A = P_0$	$P_B = .5P_0$	$P_C = .3P_0$
$V_A = V_0$	$V_B = 2V_0$	$V_C = 2V_0$
$T_A = \frac{P_0V_0}{nk_B}$	$T_B = \frac{P_0V_0}{nk_B}$	$T_C = \frac{.6P_0V_0}{nk_B}$

$PV = nk_BT$

$\Rightarrow T = \frac{PV}{nk_B}$

* For $A \to C$ (adiabat)

$$P_AV_A^\gamma = P_CV_C^\gamma$$

$$T_AV_A^{\gamma-1} = T_CV_C^{\gamma-1}$$

$$\frac{P_AV_A}{T_A} = \frac{P_CV_C}{T_C}$$

$$\frac{P_AV_A}{T_A} = \frac{\frac{P_AV_A^\gamma}{V_C^\gamma}V_C}{\frac{T_AV_A^{\gamma-1}}{V_C^{\gamma-1}}}$$

$$V_A = \frac{V_A^\gamma V_C^{1-\gamma}}{V_A^{\gamma-1}V_C^{1-\gamma}}$$

b) Find the work done on the system, the heat into the system, and the change in internal energy during each step of the cycle

$$W_{B\to C} = 0$$

$$Q_{B\to C} = nC_V\Delta T = \frac{A}{N_{av}}\left(\frac{3}{2}R\right)\left(\frac{2}{5}\frac{P_0V_0}{nk_B}\right) = \frac{3}{5}P_0V_0$$

$$\Delta E = Q = \frac{3}{5}P_0V_0$$

$$W_{C\to A} = \frac{P_fV_f - P_iV_i}{1-\gamma} = \frac{.6P_0V_0 - P_0V_0}{1-5/3} = \frac{-2/5}{-2/3}P_0V_0 = -3/5\,P_0V_0$$

$$Q_{C\to A} = 0$$

$$\Delta E = -W = 3/5\,P_0V_0$$

b) $W_{A\to B} = n k_B T \ln\left(\frac{V_B}{V_A}\right)$

$= n k_B \frac{P_0 V_0}{n k_B} \ln\left(\frac{2V_0}{V_0}\right)$

$= P_0 V_0 \ln(2)$

$Q = W = P_0 V_0 \ln(2)$

$\Delta E = 0$ b/c isotherm

c) Which direction does the heat engine flow?

CW

d) What is the efficiency of the heat engine?

$\eta = 1 - \left|\frac{Q_{out}}{Q_{in}}\right|$

$= 1 - \frac{.6 P_0 V_0}{P_0 V_0 \ln(2)}$

$= 1 - \frac{.6}{\ln(2)}$