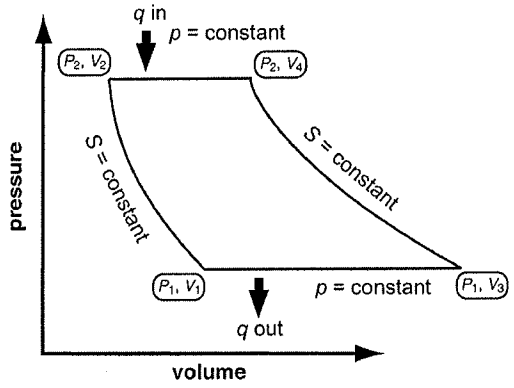
Statistical Mechanics

4. The gas turbine (jet engine) can be modeled as a Brayton cycle. Below is the P-V diagram for this process.



Assume that the working fluid is an ideal monatomic gas.

(a) Calculate the work done by the gas on each step in the cycle. (3 pts.)

(b) Find the heat for each step in the cycle. (3 pts.)

(c) Find the efficiency of this engine. Your answer should be in terms of the pressures (P_1 and P_2) and the volumes (V_1, V_2, V_3, and V_4). (3 pts.)

(d) To produce work, which way does the cycle operate? Clockwise or counter clockwise? (1 pt.)

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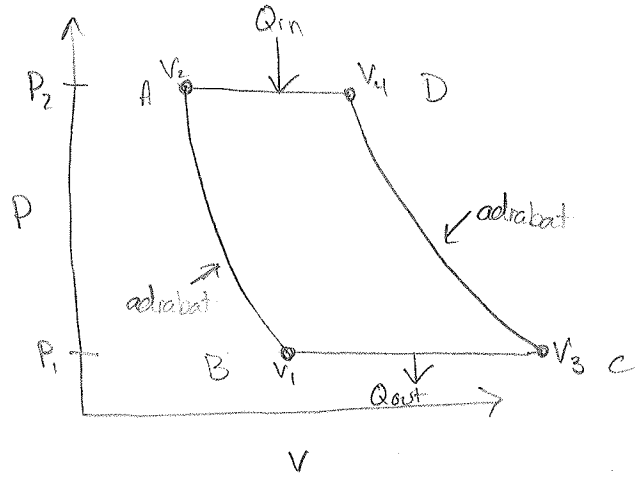
Stat Mech #1



* Ideal monatomic gas

$\Rightarrow C_V = \frac{3}{2}R$, $C_P = \frac{5}{2}R$, $\gamma = \frac{5}{3}$

$P_A = P_2$ $\quad P_B = P_1$ $\quad P_C = P_1$ $\quad P_D = P_2$

$V_A = V_2$ $\quad V_B = V_1$ $\quad V_C = V_3$ $\quad V_D = V_4$

$T_A = \frac{nR}{P_2 V_2}$ $\quad T_B = \frac{nR}{P_1 V_1}$ $\quad T_C = \frac{nR}{P_1 V_3}$ $\quad T_D = \frac{nR}{P_2 V_4}$

a) Calculate the work done in each step.

$$W_{A\to D} = P\Delta V = P(V_4 - V_2)$$

$$W_{D\to C} = \frac{P_C V_C - P_D V_D}{1-\gamma} = \frac{P_1 V_3 - P_2 V_4}{1-5/3} = -\frac{3}{2}(P_1 V_3 - P_2 V_4)$$

$$W_{C\to B} = P\Delta V = P_1(V_1 - V_3)$$

$$W_{B\to A} = \frac{P_A V_A - P_B V_B}{1-\gamma} = \frac{P_2 V_2 - P_1 V_1}{1-5/3} = -\frac{3}{2}(P_2 V_2 - P_1 V_1)$$

b) Find the heat for each step

$$Q_{A\to D} = nC_P\Delta T = n\frac{5}{2}R\left(\frac{P_2 V_4}{nR} - \frac{P_2 V_2}{nR}\right) = \frac{5}{2}P_2(V_4 - V_2)$$

$Q_{D\to C} = 0$ b/c adiabatic

$$Q_{C\to B} = nC_P\Delta T = n\frac{5}{2}R\left(\frac{P_1 V_3}{nR} - \frac{P_1 V_1}{nR}\right) = \frac{5}{2}P_1(V_3 - V_1)$$

$Q_{B\to A} = 0$ b/c adiabatic

c) Find the efficiency of the engine

$$\eta = 1 - \frac{Q_{in}}{Q_{out}} \uparrow \downarrow$$

$$= 1 - \left(\frac{P_2 (V_4 - V_2)}{P_1 (V_3 - V_1)} \right)^{-1}$$

d) Which way does the cycle operate?

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