3. Consider the Lagrangian for a 1D system with generalized coordinate q:

$$L(q, \dot{q}, t) = e^{\lambda t/m}\left[\frac{m}{2}\dot{q}^2 - \frac{m\omega_0^2}{2}q^2\right] \tag{1}$$

In the above expression, m is a mass, ω_0 is a frequency, and λ is a positive and dimensionless constant.

(a) Derive the equation of motion for the system. (1 pt.)

(b) What is the canonical momentum, p? (1 pt.)

(c) Calculate the Hamiltonian. (3 pts.)

(d) We wish to make a canonical transformation $(q, p) \to (Q, P)$ using the generating function

$$F_2(q, P, t) = e^{\lambda t/2m} q\, P$$

What is the new coordinate and canonical momentum in terms of the old? (2 pts.)

(e) Show that the canonically transformed Hamiltonian is not time dependent. (3 pts.)

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Classical #3

$$\mathcal{L}(q, \dot{q}, t) = e^{\lambda t/m}\left[\frac{m}{2}\dot{q}^2 - \frac{m\omega_0^2}{2}q^2\right]$$

m is mass
ω_0 is a frequency
λ is positive, dimensionless const.

a) Derive equation of motion

$$\frac{\partial \mathcal{L}}{\partial q} = \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \dot{q}}$$

$$-e^{\lambda t/m} m\omega_0^2 q = \frac{\partial}{\partial t}\left[e^{\lambda t/m} m\dot{q}\right]$$

$$-m\omega_0^2 q \cancel{e^{\lambda t/m}} = m\ddot{q}e^{\lambda t/m} + \cancel{m}\dot{q}\frac{\lambda}{\cancel{m}}e^{\lambda t/m}$$

$$-m\omega_0^2 q = m\ddot{q} + \lambda\dot{q}$$

$$\Rightarrow 0 = m\omega_0^2 q + \lambda\dot{q} + m\ddot{q}$$

b) Find the canonical momentum

$$P_q = \frac{\partial \mathcal{L}}{\partial \dot{q}}$$

$$= m\dot{q}e^{\lambda t/m}$$

c) Find the Hamiltonian

$$H = \Sigma_i p\dot{q} - \mathcal{L}$$

$$= m\dot{q}^2 e^{\lambda t/m} - e^{\lambda t/m}\left[\frac{m}{2}\dot{q}^2 - \frac{m\omega_0^2}{2}q^2\right]$$

$$= \frac{m}{2}\dot{q}^2 e^{\lambda t/m} + \frac{m\omega_0^2}{2}q^2 e^{\lambda t/m}$$

d)