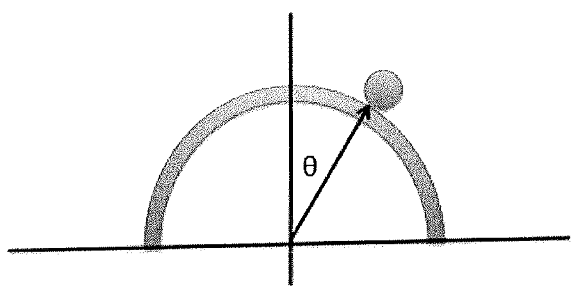
Classical Mechanics

1. A solid uniform marble with mass m and radius r starts from rest on top of a hemisphere with radius R. It will start to roll to the right, and eventually fly off the hemisphere.

   (a) Assume that the marble rolls without slipping at all times. Calculate θ_1, the angle with respect to the vertical at which the marble loses contact with the hemisphere. (3pts).

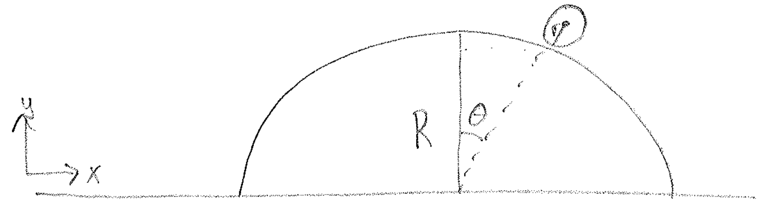
   (b) Where will the marble hit the ground, as measured from the center of the hemisphere? You may use the variable θ_1 in your answer. (If you do not solve part (a), you can still attempt this problem by writing your answer in terms of this variable.) (3pts).

   (c) Now assume that the force of friction between marble and the hemisphere is μN, where N is the normal force between the marble and the hemisphere. Calculate the angle θ_2 at which the marble will no longer roll without slipping. (4pts).



Aug 2011

Classical #1



a) Assume the marble rolls w/o slipping. Find θ_1, where marble looses contact w/ the hemisphere

* Normal force is 0 when marble leaves surface

$$mg(R+r) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgy$$

$x = (R+r)\sin\theta$

$y = (R+r)\cos\theta$

$$mg(R+r) = \frac{1}{2}mv^2 + \frac{1}{5}mr^2\left(\frac{V}{r}\right)^2 + mg(R+r)\cos\theta_1$$

$$mg(R+r) = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 + mg(R+r)\cos\theta_1$$

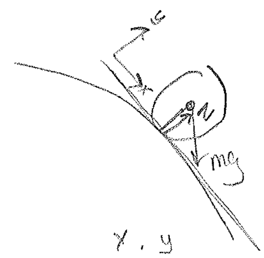
$$g(R+r) = \frac{3}{5}v^2 + g(R+r)\cos\theta_1$$

$$g(R+r) = \frac{3}{5}gR\cos\theta_1 + g(R+r)\cos\theta_1$$

$$\frac{g(R+r)}{\frac{3}{5}g + g(R+r)} = \cos\theta_1$$

$$\frac{R+r}{\frac{3}{5} + R+r} = \cos\theta_1$$

$$\Rightarrow \theta_1 = \cos^{-1}\left(\frac{R+r}{R+r+3/5}\right)$$



x, y

$\langle \parallel, \perp \rangle$

$F_N = \langle 0, N \rangle$

$F_G = mg\langle \sin\theta, -\cos\theta \rangle$

$\Rightarrow ma_x = mg\sin\theta$

$ma_y = N + mg\cos\theta$

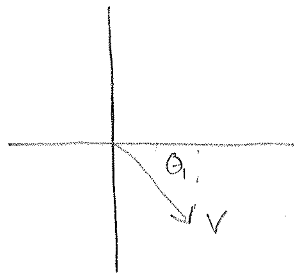
* at θ_1, $N = 0$, $a_y = \frac{v^2}{R}$

$ma_y = mg\cos\theta_1$

$\frac{v^2}{R} = g\cos\theta_1$

$\Rightarrow v = \sqrt{gR\cos\theta_1}$

b) Where will the marble hit the ground, measured from the center of the hemisphere?



$V_x = V\cos\theta$

$V_y = V\sin\theta$

$y_i = (R+r)\cos\theta_1 \qquad V_i = V$

$y_f = 0 \qquad a = g$

$$0 = -\tfrac{1}{2}gt^2 + vt + (R+r)\cos\theta_1$$

$$0 = -\tfrac{1}{2}gt^2 - \sqrt{gR\cos\theta_1}\,\sin\theta_1\, t + (R+r)\cos\theta_1$$

$$\Rightarrow \quad t = \frac{+\sqrt{gR\cos\theta_1}\sin\theta_1 \pm \sqrt{gR\cos\theta_1\sin^2\theta_1 - 4(-\tfrac{1}{2}g)(R+r)\cos\theta_1}}{-g}$$

$$= \frac{\sqrt{gR\cos\theta_1}\sin\theta_1 \pm \sqrt{gR\cos\theta_1(\sin^2\theta_1 + 2) + 2rg\cos\theta_1}}{-g}$$

$$= \frac{\sqrt{gR\cos\theta_1}\sin\theta_1 - \sqrt{gR\cos\theta_1(\sin^2\theta_1 + 2) + 2rg\cos\theta_1}}{-g}$$

(need (−) root to make overall time positive)

$x_i = (R+r)\sin\theta_1 \qquad v = \sqrt{gR\cos\theta_1}\cos\theta_1$

$x_f = ??$

$$x_f = vt + x_i$$

$$= gR\cos^{3/2}\theta_1 t + (R+r)\sin\theta_1$$

$$= gR\cos^{3/2}\theta_1\left(\tfrac{1}{-g}\left[\sqrt{gR\cos\theta_1}\sin\theta_1 - \sqrt{gR\cos\theta_1(\sin^2\theta_1+2) + 2rg\cos\theta_1}\right]\right) + (R+r)\sin\theta_1$$