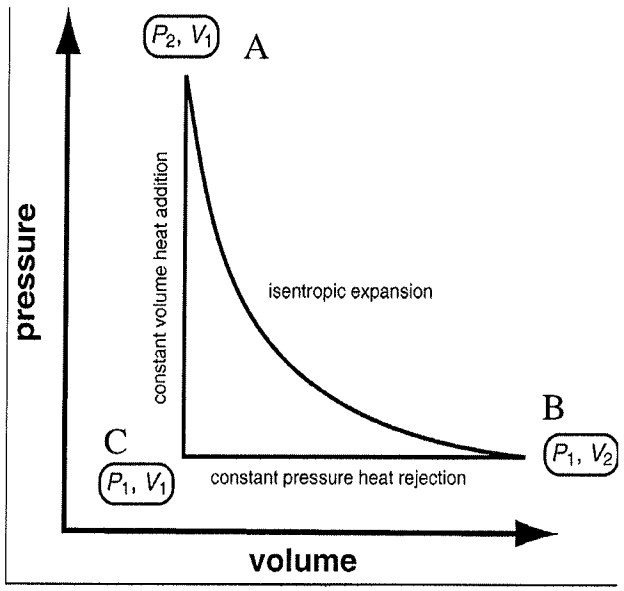
Statistical Mechanics

4. **Heat Engines:** A pulse jet operates under a Lenoir cycle. This consists of an adiabat, an isobar, and an isochore, as shown.



Assuming that the working fluid is an ideal 3D monoatomic gas of N particles:

(a) Find the work done in one complete cycle. (3 points)

(b) Find the heat exchanged in each step in the cycle. (3 points)

(c) Find the efficiency of the engine. Express your answer in terms of pressures and volutmes. (3 points)

(d) To produce work, should the engine cycle operate clockwise ($A \to B \to C \to A$) or counterclockwise ($A \to C \to B \to A$) ? (1 point)

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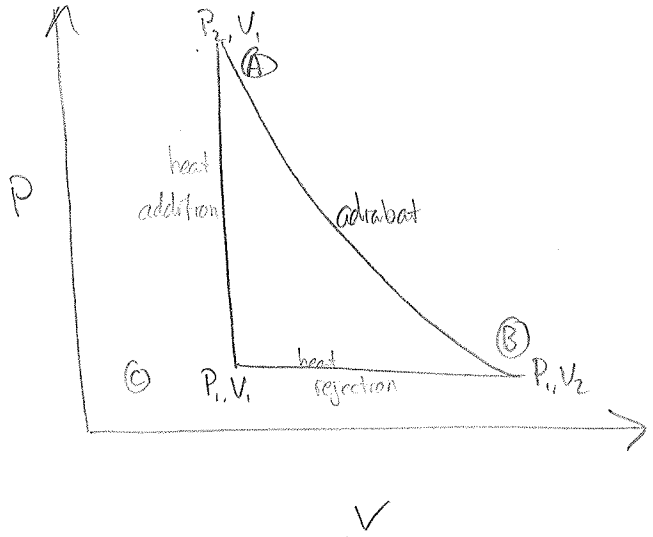
Stat Mech #1



* Assume 3-D monatomic gas of N particles

$\Rightarrow PV = Nk_BT$

a) Find work done in one complete cycle

$P_A = P_2$ $\quad P_B = P_1$ $\quad P_C = P_1$

$V_A = V_1$ $\quad V_B = V_2$ $\quad V_C = V_1$

$T_A = \frac{Nk_B}{P_2V_1}$ $\quad T_B = \frac{Nk_B}{P_1V_2} = \frac{Nk_B}{P_2V_1}$ $\quad T_C = \frac{Nk_B}{V_2P_2}$

$\frac{V_B}{T_B} = \frac{V_C}{T_C}$

$\Rightarrow T_C = \frac{V_C T_B}{V_B}$

$T_C = \frac{V_1}{V_2}\frac{Nk_B}{P_2V_1}$

$= \frac{Nk_B}{V_2P_2}$

$W = \int P dV$

$W_{A\to C} = 0, \quad dV = 0$

$W_{C\to B} = P_1(V_2 - V_1)$

$W_{B\to A} = \frac{P_AV_A - P_BV_B}{1-\gamma}$

$= \frac{P_2V_1 - P_1V_2}{1 - 5/3}$

$= -\frac{3}{2}(P_2V_1 - P_1V_2)$

$W_{TOT} = 0 + P_1V_2 - P_1V_1 - \frac{3}{2}(P_2V_1 - P_1V_2)$

$= \frac{5}{2}P_1V_2 - P_1V_1 - \frac{3}{2}P_2V_1$

$= \frac{5}{2}P_1V_2 - V_1(P_1 + \frac{3}{2}P_2)$

b) Find the heat exchanged in each step of the cycle.

$$Q_{A\to C} = nC_V\Delta T$$
$$= \frac{N}{6.02\cdot10^{23}}\cdot\frac{3}{2}R\cdot\left(\frac{nR}{V_2P_2}-\frac{nR}{P_2V_1}\right)$$
$$= \frac{3nR}{2}\cdot\frac{nR}{P_2}\left(\frac{1}{V_2}-\frac{1}{V_1}\right)$$
$$= \frac{3n^2R^2}{2P_2}\left(\frac{1}{V_2}-\frac{1}{V_1}\right)$$

$$Q_{C\to B} = nC_P\Delta T$$
$$= \frac{5nR}{2}\left(\frac{nR}{P_2V_1}-\frac{nR}{V_2P_2}\right)$$
$$= \frac{5n^2R^2}{2P_2}\left(\frac{1}{V_1}-\frac{1}{V_2}\right)$$

$Q_{B\to A} = 0$ b/c adiabatic

$$Q_{TOT} = \frac{3n^2R^2}{2P_2}\left(\frac{1}{V_2}-\frac{1}{V_1}\right)+\frac{5n^2R^2}{2P_2}\left(\frac{1}{V_1}-\frac{1}{V_2}\right)$$
$$= \frac{n^2R^2}{P_2}\left(\frac{1}{V_1}-\frac{1}{V_2}\right)$$

c) Find the efficiency of the engine

$$\eta = \frac{W_{out}}{Q_{in}}$$

* Note: Q_{in} occurs in $A\to C$

$$\eta = \frac{\frac{5}{2}P_1V_2 - V_1\left(P_1+\frac{3}{2}P_2\right)}{\frac{3n^2R^2}{2P_2}\left(\frac{1}{V_2}-\frac{1}{V_1}\right)}$$
$$= \frac{5P_1V_2 - 2V_1P_1 - 3V_1P_2}{\frac{3n^2R^2}{P_2}\left(\frac{1}{V_2}-\frac{1}{V_1}\right)}$$
$$= \frac{5P_1P_2V_2 - 2P_1P_2V_1 - 3P_2^2V_1}{3n^2R^2\left(\frac{1}{V_2}-\frac{1}{V_1}\right)}$$

d) To produce work, does engine operate clockwise or counter-clockwise?

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