Problem 1: (10 Points)

A child of mass m is playing on a swing hanging from a support by a uniform chain of length L and negligible mass. In this question, you will explore the behavior of the swing in a number of situations.

a. Determine the equation of motion for the system in polar coordinates. **(2 Points)**

b. Consider small oscillations about the equilibrium position. What is the equation of motion for these conditions? **(1 Points)**

c. What is the oscillation frequency for the conditions described in part (b.)? **(2 Points)**

d. By starting at a sufficiently large speed at the bottom of the swing ($\theta = 0°$) the child can go 'over the top' ($\theta = 180°$). If the chain remains maximally extended at the top of the loop, what is the minimum velocity the child must have at the bottom of the loop ($\theta = 0$)? θ is the angle that the chain forms with the vertical. **(2 Points)**

e. What is the minimum force applied to the child by the swing, that the child experiences at the bottom of the loop in part (d.)? **(1 Point)**

f. If the chain is replaced by a rigid rod of negligible mass, what is the minimum velocity of the child at the bottom required to go over the top? **(1 Point)**

g. What is the minimum force applied to the child by the swing, that the child experiences at the bottom of the loop in part (f.)? **(1 Point)**

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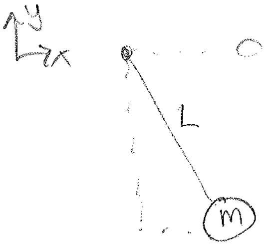
Classical #1



a) Determine equation of motion in polar coordinates

$$\mathcal{L} = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - mgy$$

$$= \frac{1}{2}mL^2\dot{\varphi}^2 - mgl\sin\varphi$$

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \dot{r}}$$

$$\frac{\partial \mathcal{L}}{\partial r} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \varphi} = \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \dot{\varphi}}$$

$$-mgL\cos\varphi = \frac{\partial}{\partial t}(mL^2\dot{\varphi})$$

$$-mgL\cos\varphi = mL^2\ddot{\varphi}$$

$$\Rightarrow \ddot{\varphi} = -\frac{g}{L}\cos\varphi$$

b) Consider small oscillations about equilibrium. What is the equation of motion?

* Make small angle approximation

$$\Rightarrow \mathcal{L} = \frac{1}{2}mL^2\dot{\varphi}^2 - mgl\varphi$$

$$\frac{\partial \mathcal{L}}{\partial \varphi} = \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \dot{\varphi}}$$

$$-mg\cancel{L} = \frac{\partial}{\partial t}[mL^2\dot{\varphi}]$$

$$-\cancel{m}g\cancel{L} = \cancel{m}L^{\cancel{2}}\ddot{\varphi}$$

$$\ddot{\varphi} = -\frac{g}{L}$$