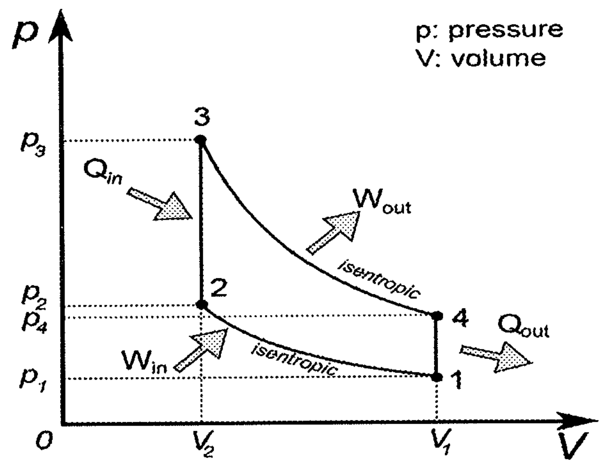
Problem 4 (10 Points):

The diesel engine uses the Otto cycle. Below is the P-V diagram for this process. Assume a monatomic ideal gas.



a. Find the work done during each cycle. **(3 Points)**

b. Find the heat exchanged each cycle. **(3 Points)**

c. What is the efficiency of this engine? **(3 Points)**

d. To produce work, which way does the cycle operate? Clockwise or counter clockwise in the diagram. **(1 Points)**

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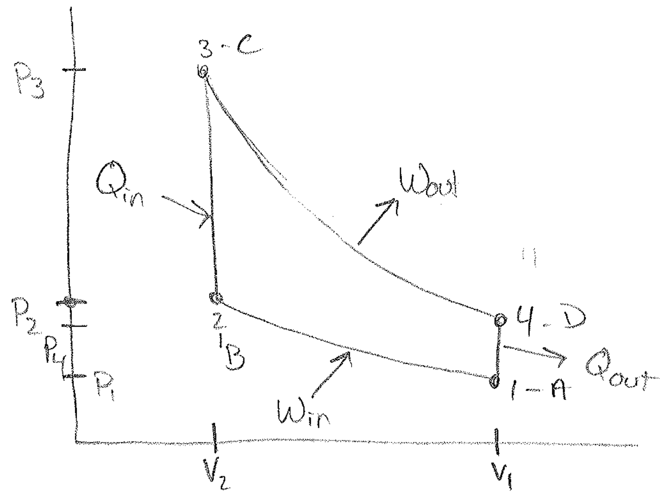
Stat Mech #1



* Assume ideal monatomic gas

$\rightarrow C_V = \frac{3}{2}R$
$C_P = \frac{5}{2}R$ $\rangle$ $\gamma = 5/3$

$P_A = P_1$ $\qquad$ $P_B = P_2$ $\qquad$ $P_C = P_3$ $\qquad$ $P_D = P_4$

$V_A = V_1$ $\qquad$ $V_B = V_2$ $\qquad$ $V_C = V_2$ $\qquad$ $V_D = V_1$

$T_A = \frac{P_1 V_1}{nR}$ $\qquad$ $T_B = \frac{P_1 V_1^{5/3}}{nR V_2^{2/3}}$ $\qquad$ $T_C = \frac{P_1 P_3 V_1^{5/3}}{nR P_2 V_2^{2/3}}$ $\qquad$ $T_D = \frac{P_4 V_1}{nR}$

$$T_B = T_A\left(\frac{V_A}{V_B}\right)^{\gamma-1} = \frac{P_1V_1}{nR}\left(\frac{V_1}{V_2}\right)^{2/3} = \frac{P_1 V_1^{5/3}}{nR V_2^{2/3}}$$

$$T_C = T_B\left(\frac{P_C}{P_B}\right) = \frac{P_1 V_1^{5/3}}{nR V_2^{2/3}}\left(\frac{P_3}{P_2}\right)$$

$$T_D = T_A\left(\frac{P_D}{P_A}\right) = \frac{P_1V_1}{nR}\left(\frac{P_4}{P_1}\right)$$

a) Find the work done during the cycle.

$$W_{A\to B} = \frac{P_BV_B - P_AV_A}{1-\gamma} = \frac{P_2V_2 - P_1V_1}{1-5/3} = \frac{-3}{2}(P_2V_2 - P_1V_1)$$

$$W_{C\to D} = \frac{P_DV_D - P_CV_C}{1-\gamma} = \frac{P_4V_1 - P_3V_2}{1-5/3} = \frac{-3}{2}(P_4V_1 - P_3V_2)$$

$W_{B\to C} = 0$ b/c isochoric

$W_{D\to A} = 0$ b/c isochoric

$$\Rightarrow W_{TOT} = \frac{3}{2}(P_1V_1 - P_2V_2) + \frac{3}{2}(P_3V_2 - P_4V_1)$$
$$= \frac{3}{2}\left(V_1[P_1 - P_4] + V_2[P_3 - P_2]\right)$$

b) Find the heat exchanged each cycle

$Q_{A\to B} = 0$ b/c adiabatic

$$Q_{B\to C} = nC_v\Delta T$$
$$= n\left(\tfrac{3}{2}R\right)\left(\frac{P_1P_3V_1^{5/3}}{nR\,P_2V_2^{2/3}} - \frac{P_1V_1^{5/3}}{nR\,V_2^{2/3}}\right)$$
$$= \frac{3}{2}\,\frac{P_1V_1^{5/3}}{V_2^{2/3}}\left(\frac{P_3}{P_2} - 1\right)$$

$Q_{C\to D} = 0$ b/c adiabatic

$$Q_{D\to A} = nC_v\Delta T$$
$$= n\left(\tfrac{3}{2}R\right)\left(\frac{P_1V_1}{nR} - \frac{P_4V_1}{nR}\right)$$
$$= \frac{3}{2}(P_1 - P_4)$$

$$Q_{TOT} = \frac{3}{2}\left[\frac{P_1V_1^{5/3}}{V_2^{2/3}}\left(P_3/P_2 - 1\right) + P_1 - P_4\right]$$

c) What is the efficiency of the engine?

$$\eta = 1 - \left|\frac{Q_{out}}{Q_{in}}\right|$$
$$= 1 - \left|\frac{\frac{3}{2}(P_1 - P_4)}{\frac{3}{2}\frac{P_1V_1^{5/3}}{V_2^{2/3}}\left(\frac{P_3}{P_2} - 1\right)}\right|$$
$$= 1 - \left|\frac{P_1 - P_4}{\frac{P_1V_1^{5/3}}{V_2^{2/3}}\left(\frac{P_3}{P_2} - 1\right)}\right|$$

d) Which direction does the engine operate?

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