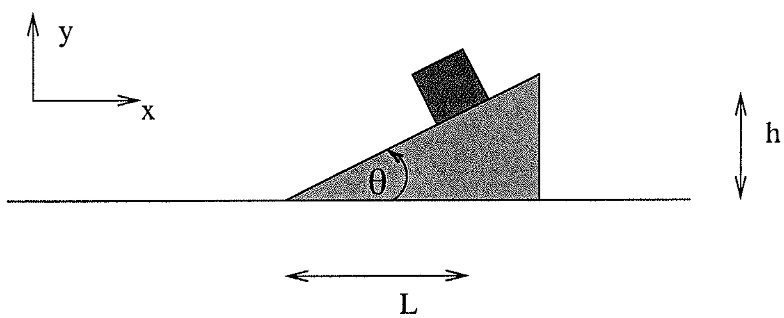
Classical Mechanics

1. A block of mass m_1 sits atop a triangular wedge of mass m_2, which is itself on a frictionless plane, as shown. The two are initially at rest, and the block is a height h above the surface of the plane, a horizontal distance L from the bottom edge of the wedge. The wedge has an opening angle θ, as shown.



   (a) Assume that there is no friction between the block and the wedge. The block slides down the wedge. What are the velocities (measured with respect to the fixed inertial reference frame denoted by the x and y axes shown) of the block and wedge just as the block reaches the lower edge of the wedge? (3 points).

   (b) Now replace the block by a ball of radius R (and mass m_1). The ball rolls down the wedge without slipping. What are the velocities of the ball and wedge just as the ball reaches the lower edge of the wedge? (3 points).

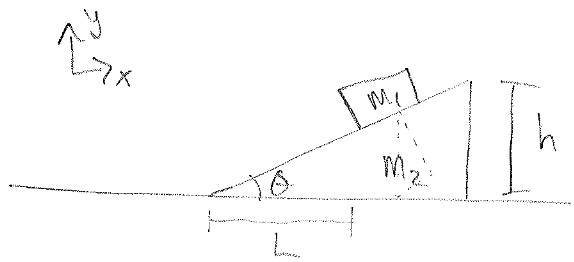
   (c) Return to the block problem, but now assume that the coefficients of static and kinetic friction between the block and the wedge are μ (they have the same value). What is μ_{min}, the minimum value of μ for which the system is stable? (1 point).

   (d) If $\mu < \mu_{\text{min}}$, calculate the minimum **horizontal** force that can be applied to the wedge such that the block will not accelerate down the wedge. (3 points).

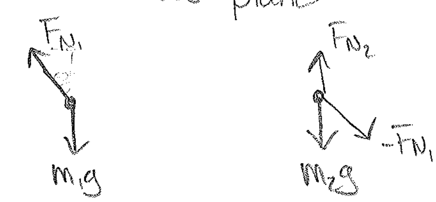
Note: you can neglect the finite size of the block in your calculation, and you are asked for the velocities before the block or ball make contact with the frictionless plane.

Jan 2008

Classical #1



* Frictionless plane



a) * Assume no friction b/w block + wedge

* Find velocities when block reaches bottom of wedge

- Forces

  * Block $= \langle -F_{N_1}\sin\theta, -m_1 g + F_{N_1}\cos\theta \rangle$

  * Wedge $= \langle F_N \sin\theta, -m_2 g - F_{N_1}\cos\theta + F_{N_2} \rangle$

- Energy

  $\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = mgh$

- Kinematics

  $r_{1,f} = r_{1,i} + \frac{1}{2}a_i t^2$

  $v_{1,f}^2 = 2a\Delta r \qquad v_1 = a_1 t$

$r_{1,f} = \langle x_{f,1}, 0 \rangle$ * Note: $-m_2 g - F_{N_1}\cos\theta + F_{N_2} = 0$

$\Rightarrow \cancel{y_{1,f}}^{0} = y_{1,i} + \cancel{v_{1,i}t}^{0} + \frac{1}{2}a_{1,y}t^2$

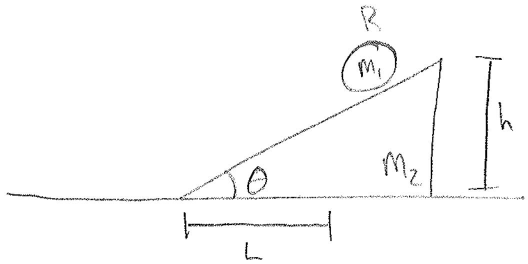
$0 = h + \frac{1}{2}a_{1,y}t^2$

$\left(\frac{-2hm_1}{F_{N_1}\cos\theta - m_1 g}\right)^{1/2} = t$

$\Rightarrow v_1 = at$

$v_1 = \left(\frac{-2h}{F_{N_1}\cos\theta - m_1 g}\right)\langle -F_{N_1}\sin\theta, F_{N_1}\cos\theta - m_1 g \rangle, \quad v_2 = \left(\frac{-2hm_1}{m_2(F_{N_1}\cos\theta - m_1 g)}\right)\langle F_{N_1}\sin\theta, 0 \rangle$

b) * Block is now ball rolling w/o slipping (ie. $V_{tangent} = V_{cm}$)



* Frictionless Plane

- Forces

* Ball $= \langle -F_{N_1} \sin\theta,\ -m_1 g + F_{N_1} \cos\theta \rangle$

* Wedge $= \langle F_{N_1} \sin\theta,\ -m_2 g - F_{N_1} \cos\theta + F_{N_2} \rangle$

- Energy

$$m_1 g h = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} I_1 \omega_1^2 + \tfrac{1}{2} m_2 v_2^2 \quad , \quad I = \tfrac{2}{5} m_1 R^2$$

- Kinematics

$$\vec{r}_f = \vec{r}_i + \tfrac{1}{2} \vec{a} t^2$$
$$\vec{\theta}_f = \vec{\theta}_i + \tfrac{1}{2} \vec{\alpha} t^2$$

$>$ $v = \omega R$, $a = \alpha R$

$$\vec{v}_f^{\,2} = 2 \vec{a}\, \Delta \vec{r}$$
$$\vec{v}_f = \vec{a} t$$