Astro Formolas

Basics: $m_2 - m_1 = -2.5 \log\left(\frac{F_2}{F_1}\right)$

$M = m - 5 \log\left(\frac{d}{10\,pc}\right)$

$F = \sigma T_{eff}^4 = \frac{L}{4\pi r^2} \Rightarrow L = 4\pi r^2 \sigma T_{eff}^4$

$L_{Edd} = \frac{4\pi G c}{\kappa} m$

Stellar Evolution: P-P Chain: ${}^1H + {}^1H \rightarrow {}^2H + e^+ + \nu$

${}^2H + {}^1H \rightarrow {}^3He + \gamma \Big\} \times 2$ ← limiting rxn

${}^3He + {}^3He \rightarrow {}^4He + 2\,{}^1H$

CNO Cycle: $C^{12} + H^1 \rightarrow N^{13} + \gamma$

$N^{13} \rightarrow C^{13} + e^+ + \nu$

$C^{13} + H^1 \rightarrow N^{14} + \gamma$

$N^{14} + H^1 \rightarrow O^{15} + \gamma$ ← limiting rxn

$O^{15} \rightarrow N^{15} + e^+ + \nu$

$H^1 + N^{15} \rightarrow C^{12} + He^4$

Stellar Structure: $\frac{dP}{dr} = \frac{-Gm}{r^2}\rho$

$\frac{dr}{dm} = 4\pi r^2 \rho$

$\frac{dl}{dm} = \epsilon_{nuc}$

$\frac{dT}{dm} = \frac{-Gm}{4\pi r^4}\frac{T}{P}\nabla$

$\nabla_{rad} = \frac{3}{16\pi a c G} \cdot \frac{P}{T^4}\frac{\kappa l}{m}$

$\nabla_{ad} = \frac{\partial(\log T)}{\partial(\log P)}$

Virial Thm: $E_{int} = \frac{-\varphi}{3} E_G$, $\varphi = \begin{cases} 3/2 & \text{for ideal gas/non-relativistic} \\ 3 & \text{for relativistic} \end{cases}$

$= \frac{-1}{2} E_G$

Redshift: $z \equiv \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}} = \frac{\Delta\lambda}{\lambda_{rest}}$

$z + 1 = \frac{\Delta t_{obs}}{\Delta t_{rest}}$

Binary/Orbit Problems: $P^2 = \frac{4\pi^2}{G(M+m)} a^3$

* Remember, for bodies orbiting a mutual center of mass:

$\frac{m_1}{m_2} = \frac{r_2}{r_1} = \frac{a_2}{a_1}$; $\alpha = \frac{a}{d}$, where α = angle subtended, d = distance to system

$\Rightarrow \frac{m_1}{m_2} = \frac{\alpha_2}{\alpha_1}$

$\Rightarrow m_1 + m_2 = \frac{4\pi^2}{G}\frac{(\alpha_1 + \alpha_2)^3 d^3}{P^2}$

Astro Formulas

Orbital Mechanics

$$P^2 = \frac{4\pi^2}{G(m_1+m_2)} a^3 \qquad \mu = \frac{m_1 m_2}{m_1+m_2} \qquad x_{cm} = \frac{\sum_i r_i m_i}{\sum_i m_i}$$

$$\frac{m_1}{m_2} = \frac{a_2}{a_1} = \frac{v_2}{v_1} = \frac{v_{2,r}\sin(i)}{v_{1,r}\sin(i)} \qquad v_r = v\sin(i)$$

$$a = \frac{P}{2\pi}(v_{1,r}+v_{2,r})$$

$$d\alpha = a,$$

d = distance

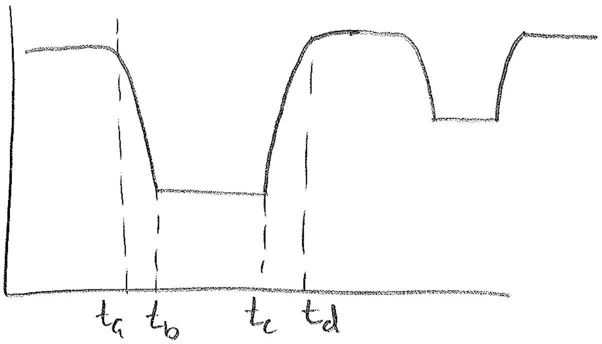
α = separation in radians

a = semi-major axis

$$m_1+m_2 = \frac{P}{2\pi G}(v_{1,r}+v_{2,r})$$

$$\frac{m_2^3}{(m_1+m_2)^2} = \frac{P v_{1,r}}{2\pi G}$$

* For an eclipsing binary system:



$$r_s = \frac{v_1+v_2}{2}(t_b-t_a)$$

$$r_L = \frac{v_1+v_2}{2}(t_c-t_a)$$

$$\frac{T_s}{T_L} = \left(\frac{B_0-B_P}{B_0-B_S}\right)^{1/4}$$

* where absolute magnitudes given

* To find temperature of a planet

$$T_p = T_s(1-a)^{1/4}\sqrt{\frac{R_s}{2D}}$$

a = albedo

D = distance

R_s = radius of star

T_s = temp of star

Miscellaneous Basics

$$F = \sigma T^4 \qquad m_1 - m_2 = -2.5\log\left(\frac{F_1}{F_2}\right) \qquad z = \frac{\Delta\lambda}{\lambda_{rest}}$$

$$L = 4\pi r^2 F \qquad \underline{M} = m - 5\log\left(\frac{d}{pc}\right) \qquad z+1 = \frac{\Delta t_{obs}}{\Delta t_{rest}}$$

$$L_{edd} = \frac{4\pi G c M}{\kappa} \qquad F_G = \frac{G m_1 m_2}{r^2} \qquad r_{\substack{light\\cyl.}} = \frac{cP}{2\pi}$$

$$\left(\text{from } \left|\frac{dP_{rad}}{dr}\right| < \left|\frac{dP}{dr}\right|\right) \qquad v_{esc} = \sqrt{\frac{2GM}{r}}$$

$$\left.\begin{aligned} E &= \tfrac{1}{2}U \\ T &= -\tfrac{1}{2}U \end{aligned}\right\rangle \text{Virial Thm}$$

$$R_s = \left(\frac{3\Phi}{4\alpha n^2\pi}\right)^{1/3} \qquad \frac{L_s}{L_\odot} = 100^{(M_\odot - M_s)/5}$$

$$U_G = -\frac{G M_1 M_2}{r} = \overset{\text{selfgravity}}{\downarrow}\ -\frac{G M^2}{r}$$

Astro Formulas (cont.)

Stellar Evolution

$$\tau_{KH} = \frac{\Delta E_a}{L_s}$$

P-PI chain:

$$\left.\begin{aligned} 2H^1_1 &\rightarrow H^2_1 + e^+ + \nu + \gamma \\ H^2_1 + H^1_1 &\rightarrow He^3_2 + \gamma \quad (L.R.) \end{aligned}\right\rangle \times 2$$

$$2He^3_2 \rightarrow He^4_2 + 2H^1_1$$

CNO Cycle!

$$C^{12}_6 + H^1_1 \rightarrow N^{13}_7 + \gamma$$

$$N^{13}_7 \rightarrow C^{13}_6 + e^+ + \nu$$

$$C^{13}_6 + H^1_1 \rightarrow N^{14}_7 + \gamma$$

$$N^{14}_7 + H^1_1 \rightarrow O^{15}_8 + \gamma \qquad (L.R)$$

$$O^{15}_8 \rightarrow N^{15}_7 + e^+ + \gamma$$

$$N^{15}_7 + H^1_1 \rightarrow C^{12}_6 + He^4_2$$

Stellar Structure

$$\frac{dr}{dm} = 4\pi r^2 \rho \qquad \frac{dL}{dm} = \varepsilon_{nuc} \qquad \nabla_{rad} = \frac{-3\kappa L P}{16\pi acGmT^4}$$

$$\frac{dP}{dr} = \frac{-Gm\rho}{r^2} \qquad \frac{dT}{dm} = \frac{-GMT}{4\pi r^4 P} \nabla_{rad}$$

Radiative Transport

Pulsars: ⇒ Light cylinder

$$r = \frac{cP}{2\pi}$$

Galaxies/AGN's!

Radiative Transfer: ⇒ Plane Parallel Approx RTE:

$$\mu \frac{dI_\nu}{dz} = \eta_\nu - \kappa_\nu I_\nu$$

Astro Qualifier Study Guide

Basics

Angles / Solid Angles - Useful for measuring shifts in position / area of object on sky

⇒ Angle is 1-D, [radians]

Solid Angle is 2-D; [steradians]

* Distance formula using paralax: $d = 1/p$, where d is distance in pc
p is angle in arcsec

Flux - aka apparent brightness of a star; $\frac{dE}{dA \cdot dt}$ ⇒ $[\mathrm{W/m^2}]$ or $[\mathrm{erg/cm^2 s}]$

$F = \sigma T_{eff}^4 = \frac{L}{4\pi r^2}$

*flux can be integrated over all λ / ν or determined monochromatically

$F_\nu = 2\pi \int_{-1}^{1} I_\nu(z,u)\, u\, du$

Luminosity / Eddington Luminosity - aka intrinsic brightness of a star; $\frac{dE}{dt}$ ⇒ $[\mathrm{W}]$ or $[\mathrm{erg/s}]$

$L = 4\pi r^2 F = 4\pi r^2 \sigma T_{eff}^4$

* Eddington luminosity is maximum luminosity of a star in hydro-static equilibrium

$L_E \approx 3.8 \cdot 10^4 \left(\frac{M}{M_\odot}\right)\left(\frac{0.34\ \mathrm{cm^2/g}}{\kappa}\right) L_\odot$ (for star dominated by κ_{es})

Derivation: $\frac{dT}{dr} = \frac{-3\kappa\rho l}{16\pi a c T^4 r^2}$; $P_{rad} = \frac{1}{3} a T^4$

$$\frac{dP_{rad}}{dr} = \frac{4}{3} a T^3 \frac{dT}{dr}$$

$$= \frac{-\kappa\rho l}{4\pi e r^2}$$

⇒ for stars in H.S.E

$$\left|\frac{dP_{rad}}{dr}\right| < \left|\frac{dP}{dr}\right|$$

$$\frac{\kappa\rho l}{4\pi c r^2} < \frac{GM\rho}{r^2}$$

$$\Rightarrow l < \frac{4\pi c G M}{\kappa}$$

Magnitudes - 2 Types: ① Apparent $m_2 - m_1 = -2.5 \log\left(\frac{F_2}{F_1}\right)$

② Absolute $M = m - 5 \log(d/10\,\mathrm{pc})$

> m - M is distance modulus

Binary Systems

* Remember Keplers Laws: ① Planets have elliptical orbits w/ star at 1 focus

② Equal areas in equal time (Conservation of Angular Momentum

③ $P^2/a^3 = k$ for all planets $\Rightarrow P^2 = \frac{4\pi^2}{G(M+m)} a^3$

* Binary Systems allow us to determine mass of stars from orbital dynamics

ex. Two stars orbiting mutual center of mass

* knowing $\frac{m_1}{m_2} = \frac{r_2}{r_1} = \frac{a_2}{a_1}$ and that angle subtended by major axes is

$\alpha = \frac{a}{d}$, we see that: $\frac{m_1}{m_2} = \frac{\alpha_2}{\alpha_1}$

$\Rightarrow M_1 + M_2 = \frac{4\pi^2}{G} \frac{(\tilde{\alpha} d)^3}{P^2}$ where $\tilde{\alpha} = \alpha_1 + \alpha_2$

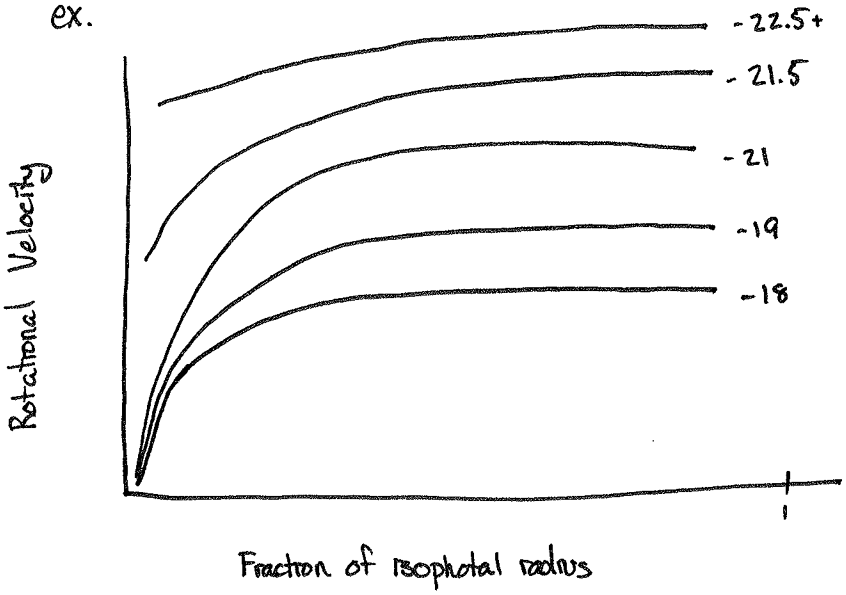
* We can then determine masses by ratio of semi-major axes of individual ellipses

Galaxies

*3 Types: ① Spiral ⇒ Both barred and unbarred
② Elliptical
③ Irregular

* Light profile gives distribution of luminous matter in galaxy but need rotation curves to measure dark matter / total matter distribution

ex.



* Isophotal radius is estimation of size of galaxy based off a defined minimum brightness level

* These rotation curves illustrate a matter distribution that has dark matter at the edge of disk to increase rotation speed as amount of visible matter decreases.
⇒ Implies spherical distribution of matter, $\rho \approx$ constant in center, $\rho \propto r^{-2}$ on edges

* Tulley-Fisher relation implies relation b/w luminosity + max rotation velocity of galaxy (from 21 cm H

* Parts of a galaxy include:
① Thin disk - composed of young stars, dust, + gas; active star formation
② Thick disk - older stellar population, little to no star formation
③ Bulge -
④ Halo - Globular clusters + field stars
⑤ Dark Matter Halo
⑥ Spiral Arms/Bar
⑦ Magnetic Field
⑧ SMBH

Nuclear Processes

* b/c stars are in equilibrium (thermal), they require an internal energy source to shine

$\Rightarrow E_{lost} = E_{produced}$; E produced via nuclear fusion

- but particles must overcome Coloumb potential in order to begin fusion

$\Rightarrow$ @ $T \approx 10^7$ K, $\langle E \rangle = 1.3$ keV, $E_{fusion} \approx 1.44$ MeV

* However, Gamow discovered there is a finite probability of tunnelling

$$P = P_o e^{-b/\sqrt{E}} \, , \quad b = \frac{2\pi z_i z_j e^2}{\hbar}\left(\frac{M}{2}\right)^{1/2}$$

$\Rightarrow P \uparrow$ as $E \uparrow$, $P \downarrow$ as $z \uparrow$

* H-fusion occurs via 2 processes

① P-P Chain

$$\left.\begin{array}{l} {}^1H + {}^1H \rightarrow {}^2H + e^+ + \nu \\ {}^2H + {}^1H \rightarrow {}^3He + \gamma \end{array}\right\} \times 2$$

$$2He^3 \rightarrow He^4 + 2H^1$$

* releases 26.2 MeV

* $E_{pp} \propto T^4$

② CN cycle

* utilizes C, N, O as catalysts if large enough quantities are present

$C^{12} + H^1 \rightarrow N^{13} + \gamma$

$N^{13} \rightarrow C^{13} + e^+ + \nu$

$C^{13} + H^1 \rightarrow N^{14} + \gamma$

$N^{14} + H^1 \rightarrow O^{15} + \gamma$ (limiting rxn)

$O^{15} \rightarrow N^{15} + e^+ + \nu$

$N^{15} + H^1 \rightarrow C^{12} + He^4$

* CN cycle dominates if $T \gtrsim 1.5 \cdot 10^7$ K

* $E_{CN} \propto T^{18}$

* He-fusion occurs at $T > 10^8$ K

Net rxn: $3\,He^4 \rightarrow C^{12} + \gamma$

* Note: $2\,He^4 \leftrightarrow Be^8$

$Be^8 + He^4 \rightarrow C^{12} + \gamma$

* $E_{3\alpha} \propto T^{40}$

* $E_{3\alpha} = 7.275$ MeV

* C-burning occurs @ $T > 5 \cdot 10^8$ K; competes w/ He-burning if enough C is initially present

$$2C^{12} \rightarrow Mg^{24*} \rightarrow Ne^{20} + \alpha \quad (50\%)$$
$$\rightarrow Na^{23} + p \quad (50\%)$$

* indicates neutron rich isotope

Note: α and p will react w/ other nuclei to yield: O^{16}, Ne^{20}, Mg^{24}

⇒ 95% by mass fraction is O^{16}, Ne^{20}, Mg^{24}

* Ne-burning occurs @ $T > 1.5$ billion K

⇒ photodisintegration now possible b/c γ have $E \sim 1$ MeV

Net rxn: $\underline{2Ne^{20} \rightarrow O^{16} + Mg^{24}}$

Really: $Ne^{20} + \gamma \rightarrow O^{16} + \alpha$

$Ne^{20} + \alpha \rightarrow Mg^{24} + \gamma$

* O-burning occurs @ $T > 2 \cdot 10^9$ K

$$2O^{16} \rightarrow S^{32} \rightarrow \left.\begin{matrix} Si^{28} + \alpha \\ P^{31} + p \end{matrix}\right\} \text{neutron excess increases}$$

⇒ 90% Si^{28} and S^{32} by mass

* Si-burning occurs at $T > 3 \cdot 10^9$ K

* Coloumb potential is too high for 2 Si nuclei to merge

$${}^{28}Si(\gamma,\alpha){}^{24}Mg(\gamma,\alpha){}^{20}Ne(\gamma,\alpha){}^{16}O(\gamma,\alpha){}^{12}C(\gamma,\alpha)2\alpha$$

⇒ photodisintegration chain produces extra α particles

$${}^{28}Si(\alpha,\gamma){}^{32}S(\alpha,\gamma){}^{36}Ar \ldots\ldots {}^{56}Ni$$

⇒ Nickel decays to Fe^{56}, results in mostly Fe^{56} by mass

* Iron is heaviest element that can be fused in core of star

* Elements heavier than Fe formed by neutron capture + subsequent β-decay via s-process and r-process (slow/rapid)

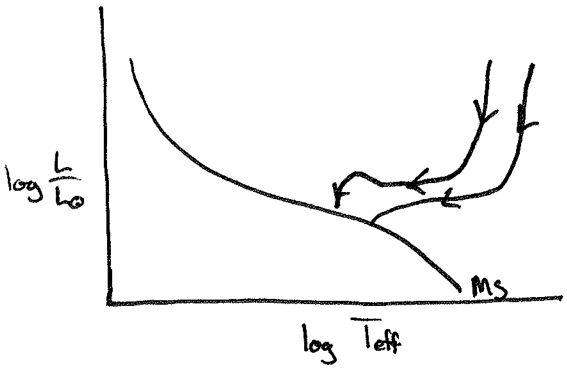
⇒ Heaviest elements form thru r-process b/c of larger neutron flux that allows multiple neutron captures during ½-life of unstable particles.

Stellar Evolution

* Pre-Main Sequence Evolution
    - Starts as cool interstellar gas cloud that begins contraction + heating

$$L = -\frac{1}{2} E_g$$

- proto-star is a cool object w/ large opacity; fully convective
    - ⇒ At turning point on Hyashi track, develops radiative core + "falls" onto MS



- H^2 burning begins @ $T \approx 10^6$ K while star is still on Hyashi track
    - all H^2 in star used up
    - contraction halts for $\sim 10^5$ yr
- Li burning begins @ higher temps; contraction again stops temporarily
- $\tau_{pms} \approx 10^7 \left(\frac{M}{M_\odot}\right)^{-2.5}$ yr
- When star reaches MS, star is in both H.S / Thermo equilibrium; nearly homogeneous composition
    - metal poor stars are hotter & smaller than metal rich stars

* H-burning phase (similar for stars of all masses)
    * since stars remain in equilibrium, changes occur due to changing composition of core

        ex. as H → He, $\mu \uparrow$ and $L \uparrow$ b/c $L \propto \mu^4 m^3$

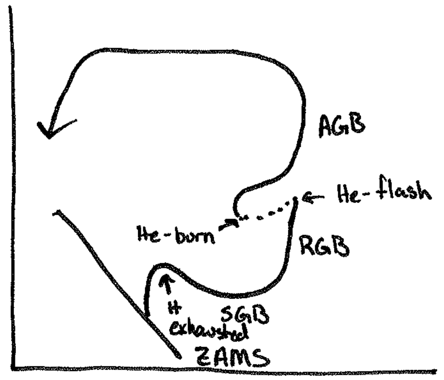
    - during central H-burning: $L \uparrow$, $X \downarrow$, $T_c \uparrow$ (still ~constant), $\mu \uparrow$
        - ⇒ $\rho_c \uparrow$ or $P_c \downarrow$ due to $\frac{P_c}{\rho_c} \propto \frac{T_c}{\mu}$ from E.O.S for ideal gas ∴ ENVELOPE EXPANDS
        - if $M > 1.3 M_\odot$ (CNO stars): $P_c \downarrow$ b/c $E \propto \rho T^{18}$, larger envelope expansion

            convective core results in contraction + $T_c \uparrow$ in late stages of MS life

        - if $M < 1.3 M_\odot$ (P-P stars): $E \propto \rho T^4$ ⇒ smaller expansion

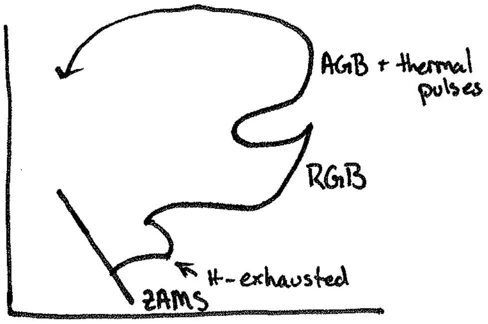
            radiative core results in gradual depletion of X; smoothly transitions to H-shell burning

* for star of $M < 2M_\odot$



- After H exhausted in core, star transitions to H-shell borning ($L\uparrow$, $T_{eff}\downarrow$), which dumps He-ash onto an increasingly degenerate core. When degenerate core mass becomes large enough, through contraction it reaches a temperature high enough to start fusion via He-flash. After several flashes, the core degenerecy is lifted and the core expands, the envelope contracts, and $T\uparrow$ as the star is now constantly borning its He-core. Eventually, an inert C-O core is built up and the star begins He-shell borning (still w/ outer H-shell borning). If the convective part of outer envelope is large enough, C-O, from core is dredged up to surface where it forms dust that is blown away by stars radiation. This radiation induced mass loss causes the star to become a C-O white dwarf at the center of a planetary nebula.

* for a star of $M \sim 5M_\odot$



Similar evolotion to above, but star starts w/ convective core + radiative envelope. Once H is exhausted in core, star transitions to H-shell borning. Inert He-shell contracts + envelope expands until He-borning starts (Note: Core never becomes degenerate). As star begins to build up C/O in core, it becomes an AGB star and undergoes thermal pulses. These thermal pulses cause mass loss and begin forming a planetary nebula w/ C-O white dwarf at center.

Stellar Structure

* 4 equations of stellar structure

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho} \qquad \frac{d\ell}{dm} = E_{nuc}$$

$$\frac{dP}{dm} = \frac{-GM}{4\pi r^4} \qquad \frac{dT}{dm} = \frac{-GM}{4\pi r^4}\frac{T}{P}\nabla \; ; \quad \nabla = \begin{cases} \nabla_{rad} = \dfrac{-3K\ell P}{16\pi a c G m T^4} \\ \nabla_{ad} \end{cases}$$

Radiative Transport

$$\frac{\upsilon}{\rho}\frac{dI_\nu}{dz} = j_\nu - k_\nu I_\nu \qquad \text{or} \qquad \upsilon\frac{dI_\nu}{d\tau} = I_\nu - S_\nu$$

* to find a general solution

$$\left(\upsilon\frac{dI_\nu}{d\tau} = I_\nu - S_\nu\right)e^{-\tau/\upsilon}$$

$$\upsilon\frac{dI_\nu}{d\tau}e^{-\tau/\upsilon} = I_\nu e^{-\tau/\upsilon} - S_\nu e^{-\tau/\upsilon}$$

$$\frac{dI_\nu}{d\tau}e^{-\tau/\upsilon} - \frac{I_\nu}{\upsilon}e^{-\tau/\upsilon} = -\frac{S_\nu}{\upsilon}e^{-\tau/\upsilon}$$

$$\Rightarrow \int_{\tau_1}^{\tau_2}\frac{d\left(I_\nu e^{-\tau/\upsilon}\right)}{d\tau} = \int_{\tau_1}^{\tau_2}\frac{-S_\nu}{\upsilon}e^{-\tau/\upsilon}d\tau$$

$$I_\nu e^{-\tau/\upsilon}\Big|_{\tau_1}^{\tau_2} = \int\frac{-S_\nu}{\upsilon}e^{-\tau/\upsilon}d\tau$$

$$I_\nu(\tau_1) = I_\nu(\tau_2)e^{(\tau_1-\tau_2)/\upsilon} + \int_{\tau_1}^{\tau_2}\frac{-S_\nu}{\upsilon}e^{-\tau/\upsilon}d\tau$$

* moment of radiative field

Astro Qualifier Topic List

*This topic list covers the Spring 2012 to Spring 2015 Qualifiers

- Astrophysics Basics
  - ✓ Angles/Solid Angles
  - ✓ Flux
  - ✓ Luminosity, Eddington Luminosity
  - ✓ Magnitudes
  - Power Law
  - Standard Candles
    - Cepheid Variable Stars
    - Type Ia SNe
- Binary Systems
  - Habitable Zones
  - ✓ Masses of bodies in system
  - Orbits/Semi-major axis calculations (from parallax)
  - Radial Velocity Curves
  - Separation Distance
  - Surface gravity calculations
  - Transits (both star and planet)
    - Transit depth
    - Transit duration
    - Luminosity calculations during transit
- Cosmology
  - Composition
    - Effects of relative abundances
    - Equations of state for each component
  - Constants and their meanings
  - Cosmological redshift and how to determine age from it
  - Distances
    - Angular diameter
    - Co-moving line of sight
    - Co-moving transverse
    - Luminosity
  - Friedman Equation
  - Inflation
    - Equation of state
    - Impact on energy/momentum
  - Scale factor (including derivation)
  - Surface brightness calculations
  - Type Ia SNe
    - Calculations
    - Uncertainties
- Galaxies
  - Age-metallicity relation
  - Components (include evidence)
    - Dark matter
    - Bulge
    - Halo
    - Cold/hot ISM

        - Central black hole
        - Population I/II stars
    - G-dwarf problem
    - ✓ Isophotal radius
    - Luminosity functions
        - Schechter luminosity function
    - Mass-light ratio
    - Mergers
    - ✓ Rotational velocity curves
    - Tully-Fisher Relation
    - Thin v thick disk
- Interstellar gas clouds
    - Stromgren Sphere
    - Wind speed of expanding gas clouds
- Lorentz Force
- Kepler's Laws & Mechanics
- Neutron Stars
    - GR effects
    - Magnetic field strength
- Nuclear Fusion
    - ✓ PP Chain (including rxn's and tunneling)
    - ✓ CNO cycle (including rxn's)
    - Impact of fusion on elemental abundances outside of star
    - ✓ S-process and r-process
    - ✓ He burning
    - ✓ Heavy element (C, O, Ne, Si) burning
- Planetary systems
    - Derive temperature
    - Derive period
- Pulsars
    - Types
    - Light Cylinder
    - Rotational velocity/Energy Loss
    - Magnetic Fields
    - PP Diagram
    - Period-distance relation
    - Period variability
- Quasars
    - Damped Lyman Alpha systems
    - Calculations using cosmology
- Stellar Evolution
    - Timescales
    - ✓ Evolutionary Tracks (including pre-MS)
    - ✓ Formation
        - Initial Mass Function
    - Virial Theorem & Gravitational Energy
    - Lifetime estimates
- Stellar Structure
    - ✓ 4 equations
    - Polytropes

    - Hydro-static equilibrium calculations
    - Equations of State
- Radiative Transport
    - Plane-parallel approximation
        - Derive zeroth and first moments
    - Grey Atmosphere approximation
    - Optically thin v optically thick
    - Rosseland Mean Opacity
    - Source Functions, etc.
    - Semi-infinite gas clouds
- Synchrotron Radiation
- Telescope Optics
    - Focal Length
    - Diffraction Limit
    - Quantum efficiency
    - CCD’s
- Virial Theorem
    - Derive Jeans mass
    - Derive central temp of star
- 21 cm H-I Line
    - Temperature of Interstellar Medium
    - Optical Density
    - Radiative Transport