Astro Formolas

Basics:

$$m_2 - m_1 = -2.5 \log\left(\frac{F_2}{F_1}\right)$$

$$M = m - 5\log\left(\frac{d}{10pc}\right)$$

$$F = \sigma T_{eff}^4 = \frac{L}{4\pi r^2} \Rightarrow L = 4\pi r^2 \sigma T_{eff}^4$$

$$L_{Edd} = \frac{4\pi G c}{\kappa} m$$

Stellar Evolution: P-P Chain:

$$\left.\begin{array}{l} {}^1H + {}^1H \rightarrow {}^2H + e^+ + \gamma \\ {}^2H + {}^1H \rightarrow {}^3He + \gamma \end{array}\right\} \times 2 \quad \leftarrow \text{limiting rxn}$$

$${}^3He + {}^3He \rightarrow {}^4He + 2\,{}^1H$$

CNO Cycle:

$$C^{12} + H^1 \rightarrow N^{13} + \gamma$$

$$N^{13} \rightarrow C^{13} + e^+ + \nu$$

$$C^{13} + H^1 \rightarrow N^{14} + \gamma$$

$$N^{14} + H^1 \rightarrow O^{15} + \gamma \quad \leftarrow \text{limiting rxn}$$

$$O^{15} \rightarrow N^{15} + e^+ + \nu$$

$$H^1 + N^{15} \rightarrow C^{12} + He^4$$

Stellar Structure:

$$\frac{dP}{dr} = \frac{-Gm}{r^2}\rho$$

$$\frac{dr}{dm} = 4\pi r^2 \rho$$

$$\frac{dl}{dm} = \epsilon_{nuc}$$

$$\frac{dT}{dm} = \frac{-Gm}{4\pi r^4}\frac{T}{P}\nabla$$

$$\nabla_{rad} = \frac{3}{16\pi a c G} \cdot \frac{P}{T^4}\frac{\kappa l}{m}$$

$$\nabla_{ad} = \frac{\partial(\log T)}{\partial(\log P)}$$

Virial Thm: $E_{int} = \frac{-\varphi}{3} E_G$, $\varphi = \begin{cases} 3/2 \text{ for ideal gas/non-relativistic} \\ 3 \text{ for relativistic} \end{cases}$

$$= \frac{-1}{2} E_G$$

Redshift: $z \equiv \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}} = \frac{\Delta\lambda}{\lambda_{rest}}$

$$z + 1 = \frac{\Delta t_{obs}}{\Delta t_{rest}}$$

Binary/Orbit Problems: $P^2 = \frac{4\pi^2}{G(M+m)} a^3$

* Remember, for bodies orbiting a mutual center of mass:

$\frac{m_1}{m_2} = \frac{r_2}{r_1} = \frac{a_2}{a_1}$; $\alpha = \frac{a}{d}$, where α = angle subtended, d = distance to system

$$\Rightarrow \frac{m_1}{m_2} = \frac{\alpha_2}{\alpha_1}$$

$$\Rightarrow m_1 + m_2 = \frac{4\pi^2}{G}\frac{(\alpha_1 + \alpha_2)^3 d^3}{P^2}$$

Astro Formulas

Orbital Mechanics

$$P^2 = \frac{4\pi^2}{G(m_1+m_2)} a^3 \qquad \mu = \frac{m_1 m_2}{m_1+m_2} \qquad X_{cm} = \frac{\sum_i r_i m_i}{\sum_i m_i}$$

$$\frac{m_1}{m_2} = \frac{a_2}{a_1} = \frac{v_2}{v_1} = \frac{v_{2,r}\sin(i)}{v_{1,r}\sin(i)} \qquad v_r = v\sin(i)$$

$$a = \frac{P}{2\pi}(v_{1,r} + v_{2,r})$$

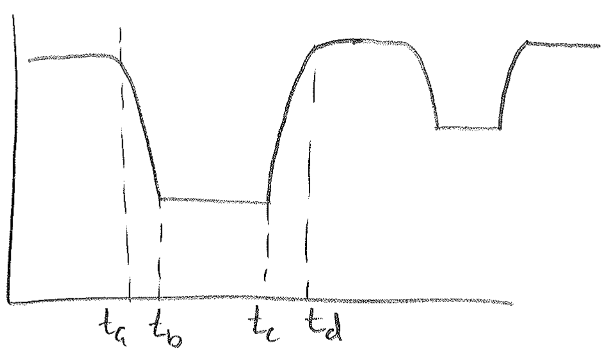
$$d\alpha = a,$$

d = distance
α = separation in radians
a = semi-major axis

$$m_1 + m_2 = \frac{P}{2\pi G}(v_{1,r} + v_{2,r})$$

$$\frac{m_2^3}{(m_1+m_2)^2} = \frac{P v_{1,r}}{2\pi G}$$

* For an eclipsing binary system:



$$r_s = \frac{v_1+v_2}{2}(t_b - t_a)$$

$$r_L = \frac{v_1+v_2}{2}(t_c - t_a)$$

$$\frac{T_s}{T_L} = \left(\frac{B_0 - B_p}{B_0 - B_s}\right)^{1/4}$$

* where absolute magnitudes given

* To find temperature of a planet

$$T_p = T_s(1-a)^{1/4}\sqrt{\frac{R_s}{2D}}$$

a = albedo
D = distance
R_s = radius of star
T_s = temp of star

Miscellaneous Basics

$$F = \sigma T^4 \qquad m_1 - m_2 = -2.5\log\left(\frac{F_1}{F_2}\right) \qquad z = \frac{\Delta\lambda}{\lambda_{rest}}$$

$$L = 4\pi r^2 F \qquad M = m - 5\log\left(\frac{d}{pc}\right) \qquad z+1 = \frac{\Delta t_{obs}}{\Delta t_{rest}}$$

$$L_{edd} = \frac{4\pi G c M}{\kappa} \qquad F_G = \frac{G m_1 m_2}{r^2} \qquad r_{\substack{light\\cyl.}} = \frac{cP}{2\pi}$$

$$\left(\text{from } \left|\frac{dP_{rad}}{dr}\right| < \left|\frac{dP}{dr}\right|\right) \qquad v_{esc} = \sqrt{\frac{2GM}{r}}$$

$$E = \frac{1}{2}U \qquad T = -\frac{1}{2}U \qquad \text{Virial Thm}$$

$$R_s = \left(\frac{3\Phi}{4\alpha n^2\pi}\right)^{1/3} \qquad \frac{L_s}{L_\odot} = 100^{(M_\odot - M_s)/5}$$

$$U_G = -\frac{G M_1 M_2}{r} = \underbrace{-\frac{GM^2}{r}}_{\text{self gravity}}$$

Astro Formulas (cont.)

Stellar Evolution

$$\tau_{KH} = \frac{\Delta E_a}{L_s}$$

P-P I chain:

$$2H_1^1 \rightarrow H_1^2 + e^+ + \nu + \gamma$$
$$H_1^2 + H_1^1 \rightarrow He_2^3 + \gamma \quad \text{(L.R.)}$$
$$\left.\right\rangle \times 2$$
$$2He_2^3 \rightarrow He_2^4 + 2H_1^1$$

CNO Cycle:

$$C_6^{12} + H_1^1 \rightarrow N_7^{13} + \gamma$$
$$N_7^{13} \rightarrow C_6^{13} + e^+ + \nu$$
$$C_6^{13} + H_1^1 \rightarrow N_7^{14} + \gamma$$
$$N_7^{14} + H_1^1 \rightarrow O_8^{15} + \gamma \quad \text{(L.R.)}$$
$$O_8^{15} \rightarrow N_7^{15} + e^+ + \gamma$$
$$N_7^{15} + H_1^1 \rightarrow C_6^{12} + He_2^4$$

Stellar Structure

$$\frac{dr}{dm} = 4\pi r^2 \rho \qquad \frac{dL}{dm} = \varepsilon_{nuc} \qquad \nabla_{rad} = \frac{-3\kappa L P}{16\pi a c G m T^4}$$

$$\frac{dP}{dr} = \frac{-Gm\rho}{r^2} \qquad \frac{dT}{dm} = \frac{-GmT}{4\pi r^4 P} \nabla_{rad}$$

Radiative Transport

Pulsars: $\Rightarrow$ Light cylinder

$$r = \frac{cP}{2\pi}$$

Galaxies / AGN's!

Radiative Transfer: $\Rightarrow$ Plane Parallel Approx RTE:

$$\mu \frac{dI_\nu}{dz} = \eta_\nu - \kappa_\nu I_\nu$$

Astro Qualifier Study Guide

Basics

Angles / Solid Angles - Useful for measuring shifts in position / area of object on sky

$\Rightarrow$ Angle is 1-D, [radians]

Solid Angle is 2-D; [steradians]

* Distance formula using paralax: $d = 1/p$, where d is distance in pc
p is angle in arcsec

Flux - aka apparent brightness of a star; $\frac{dE}{dA \cdot dt} \Rightarrow [W/m^2]$ or $[erg/cm^2 s]$

$$F = \sigma T_{eff}^4 = \frac{L}{4\pi r^2}$$

*flux can be integrated over all λ / ν or determined monochromatically

$$F_\nu = 2\pi \int_{-1}^{1} I_\nu(z, \upsilon)\, \upsilon\, d\upsilon$$

Luminosity / Eddington Luminosity - aka intrinsic brightness of a star; $\frac{dE}{dt} \Rightarrow [W]$ or $[erg/s]$

$$L = 4\pi r^2 F = 4\pi r^2 \sigma T_{eff}^4$$

* Eddington luminosity is maximum luminosity of a star in hydro-static equilibrium

$$L_E \approx 3.8 \cdot 10^4 \left(\frac{M}{M_\odot}\right)\left(\frac{0.34\ cm^2/g}{\kappa}\right) L_\odot \quad \text{(for star dominated by } \kappa_{es})$$

Derivation: $\frac{dT}{dr} = \frac{-3\kappa\rho l}{16\pi a c T^4 r^2}$; $P_{rad} = \frac{1}{3} a T^4$

$$\frac{dP_{rad}}{dr} = \frac{4}{3} a T^3 \frac{dT}{dr}$$

$$= \frac{-\kappa \rho l}{4\pi c r^2}$$

$\Rightarrow$ for stars in H.S.E

$$\left|\frac{dP_{rad}}{dr}\right| < \left|\frac{dP}{dr}\right|$$

$$\frac{\kappa \rho l}{4\pi c r^2} < \frac{GM\rho}{r^2}$$

$$\Rightarrow l < \frac{4\pi c G M}{\kappa}$$

Magnitudes - 2 Types: ① Apparent $m_2 - m_1 = -2.5 \log\left(\frac{F_2}{F_1}\right)$

② Absolute $M = m - 5 \log(d/10\,pc)$

> m - M is distance modulus

Binary Systems

* Remember Keplers Laws: ① Planets have elliptical orbits w/ star at 1 focus

② Equal areas in equal time (Conservation of Angular Momentum

③ $P^2/a^3 = k$ for all planets $\Rightarrow P^2 = \frac{4\pi^2}{G(M+m)} a^3$

* Binary Systems allow us to determine mass of stars from orbital dynamics

ex. Two stars orbiting mutual center of mass

* Knowing $\frac{m_1}{m_2} = \frac{r_2}{r_1} = \frac{a_2}{a_1}$ and that angle subtended by major axes is

$\alpha = \frac{a}{d}$, we see that: $\frac{m_1}{m_2} = \frac{\alpha_2}{\alpha_1}$

$\Rightarrow m_1 + m_2 = \frac{4\pi^2}{G} \frac{(\tilde{\alpha} d)^3}{P^2}$ where $\tilde{\alpha} = \alpha_1 + \alpha_2$

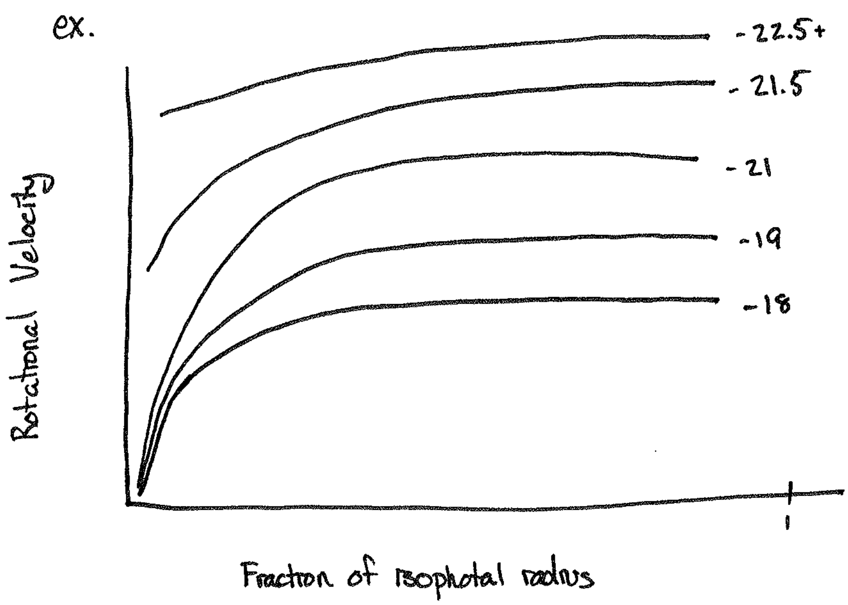
* We can then determine masses by ratio of semi-major axes of individual ellipses

Galaxies

* 3 Types: ① Spiral ⇒ Both barred and unbarred
  ② Elliptical
  ③ Irregular

* Light profile gives distribution of luminous matter in galaxy but need rotation curves to measure dark matter / total matter distribution

ex.



* Isophotal radius is estimation of size of galaxy based off a defined minimum brightness level

* These rotation curves illustrate a matter distribution that has dark matter at the edge of disk to increase rotation speed as amount of visible matter decreases.
  ⇒ Implies spherical distribution of matter, $\rho \approx$ constant in center, $\rho \propto r^{-2}$ on edges

* Tulley-Fisher relation implies relation b/w luminosity + max rotation velocity of galaxy (from 21 cm H)

* Parts of a galaxy include:
  ① Thin disk - composed of young stars, dust, + gas; active star formation
  ② Thick disk - older stellar population, little to no star formation
  ③ Bulge -
  ④ Halo - Globular clusters + field stars
  ⑤ Dark Matter Halo
  ⑥ Spiral Arms/Bar
  ⑦ Magnetic Field
  ⑧ SMBH

Nuclear Processes

* b/c stars are in equilibrium (thermal), they require an internal energy source to shine

$\Rightarrow E_{lost} = E_{produced}$; E produced via nuclear fusion

- but particles must overcome Coloumb potential in order to begin fusion

$\Rightarrow$ @ $T \approx 10^7$ K, $\langle E \rangle = 1.3$ keV, $E_{fusion} \approx 1.44$ MeV

* However, Gamow discovered there is a finite probability of tunnelling

$$\underline{P} = P_0 e^{-b/\sqrt{E}}, \quad b = \frac{2\pi z_i z_j e^2}{\hbar}\left(\frac{M}{2}\right)^{1/2}$$

$\Rightarrow \underline{P}\uparrow$ as $E\uparrow$, $\underline{P}\downarrow$ as $z\uparrow$

* H-fusion occurs via 2 processes

① P-P chain

$$\left.\begin{array}{l} {}^1H + {}^1H \rightarrow {}^2H + e^+ + \nu \\ {}^2H + {}^1H \rightarrow {}^3He + \gamma \end{array}\right\} \times 2$$

$$2He^3 \rightarrow He^4 + 2H^1$$

* releases 26.2 MeV
* $E_{pp} \propto T^4$

② CN cycle

* utilizes C, N, O as catalysts if large enough quantities are present

$C^{12} + H^1 \rightarrow N^{13} + \gamma$

$N^{13} \rightarrow C^{13} + e^+ + \nu$

$C^{13} + H^1 \rightarrow N^{14} + \gamma$

$N^{14} + H^1 \rightarrow O^{15} + \gamma$ (limiting rxn)

$O^{15} \rightarrow N^{15} + e^+ + \nu$

$N^{15} + H^1 \rightarrow C^{12} + He^4$

* CN cycle dominates if $T \gtrsim 1.5 \cdot 10^7$ K
* $E_{CN} \propto T^{18}$

* He-fusion occurs at $T > 10^8$ K

Net rxn: $3He^4 \rightarrow C^{12} + \gamma$

* Note: $2He^4 \leftrightarrow Be^8$

$Be^8 + He^4 \rightarrow C^{12} + \gamma$

* $E_{3\alpha} \propto T^{40}$
* $E_{3\alpha} = 7.275$ MeV

* C-burning occurs @ $T > 5 \cdot 10^8$ K; competes w/ He-burning if enough C is initially present

$$2C^{12} \rightarrow Mg^{24*} \rightarrow Ne^{20} + \alpha \quad (50\%)$$
$$\rightarrow Na^{23} + p \quad (50\%)$$

* indicates neutron rich isotope

Note: α and p will react w/ other nuclei to yield: O^{16}, Ne^{20}, Mg^{24}

$\Rightarrow$ 95% by mass fraction is O^{16}, Ne^{20}, Mg^{24}

* Ne-burning occurs @ $T >$ 1.5 billion K

$\Rightarrow$ photodisintigration now possible b/c γ have $E \sim 1$ MeV

Net rxn: $\underline{2Ne^{20} \rightarrow O^{16} + Mg^{24}}$

Really: $Ne^{20} + \gamma \rightarrow O^{16} + \alpha$

$Ne^{20} + \alpha \rightarrow Mg^{24} + \gamma$

* O-burning occurs @ $T > 2 \cdot 10^9$ K

$$2O^{16} \rightarrow S^{32} \rightarrow \left.\begin{matrix} Si^{28} + \alpha \\ P^{31} + p \end{matrix}\right\rangle \text{ neutron excess increases}$$

$\Rightarrow$ 90% Si^{28} and S^{32} by mass

* Si-burning occurs at $T > 3 \cdot 10^9$ K

* Coloumb potential is too high for 2 Si nuclei to merge

$$^{28}Si(\gamma,\alpha)^{24}Mg(\gamma,\alpha)^{20}Ne(\gamma,\alpha)^{16}O(\gamma,\alpha)^{12}C(\gamma,\alpha)2\alpha$$

$\Rightarrow$ photodisintegration chain produces extra α particles

$$^{28}Si(\alpha,\gamma)^{32}S(\alpha,\gamma)^{36}Ar \ldots\ ^{56}Ni$$

$\Rightarrow$ Nickel decays to Fe^{56}, results in mostly Fe^{56} by mass

* Iron is heaviest element that can be fused in core of star

* Elements heavier than Fe formed by neutron capture + subsequent β-decay via s-process and r-process (slow/rapid)

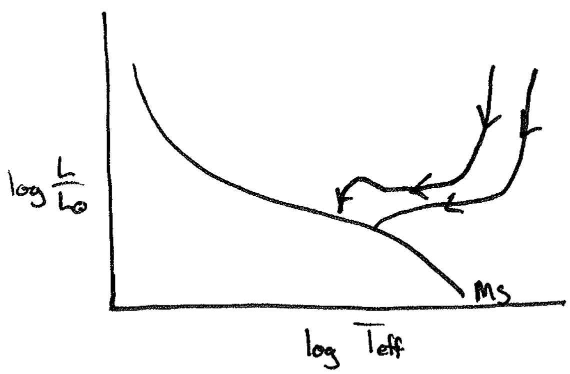
$\Rightarrow$ Heaviest elements form thru r-process b/c of larger neutron flux that allows multiple neutron captures during ½-life of onstable particles.

Stellar Evolution

* Pre-Main Sequence Evolution
  - Starts as cool interstellar gas cloud that begins contraction + heating

$$L = -\frac{1}{2}E_g$$

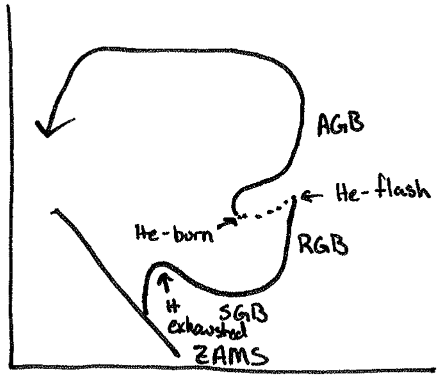
  - proto-star is a cool object w/ large opacity; fully convective
    ⇒ At turning point on Hyashi track, develops radiative core + "falls" onto MS



  - H^2 burning begins @ $T \approx 10^6$ K while star is still on Hyashi track
    - all H^2 in star used up
    - contraction halts for $\sim 10^5$ yr
  - Li burning begins @ higher temps; contraction again stops temporarily
  - $\tau_{pms} \approx 10^7 \left(\frac{M}{M_\odot}\right)^{-2.5}$ yr
  - When star reaches MS, star is in both H.S / Thermo equilibrium; nearly homogeneous composition
    - metal poor stars are hotter & smaller than metal rich stars

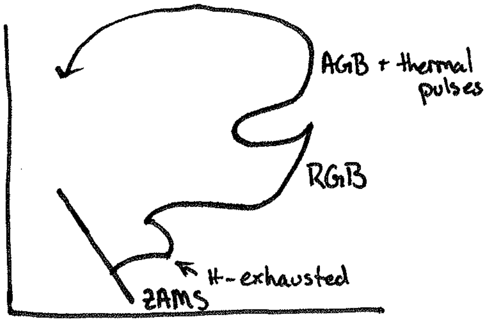
* H-burning phase (similar for stars of all masses)
  * since stars remain in equilibrium, changes occur due to changing composition of core
    ex. as H → He, $\mu\uparrow$ and $L\uparrow$ b/c $L \propto \mu^4 m^3$
  - during central H-burning: $L\uparrow$, $X\downarrow$, $T_c\uparrow$ (still ~constant), $\mu\uparrow$
    ⇒ $\rho_c\uparrow$ or $P_c\downarrow$ due to $\frac{P_c}{\rho_c} \propto \frac{T_c}{\mu}$ from E.O.S for ideal gas ∴ ENVELOPE EXPANDS
    - if $M > 1.3 M_\odot$ (CNO stars): $P_c\downarrow$ b/c $\varepsilon \propto \rho T^{18}$, larger envelope expansion
      convective core results in contraction + $T_c\uparrow$ in late stages of MS life
    - if $M < 1.3 M_\odot$ (P-P stars): $\varepsilon \propto \rho T^4$ ⇒ smaller expansion
      radiative core results in gradual depletion of X; smoothly transitions to H-shell burning

* for star of $M < 2M_\odot$



- After H exhausted in core, star transitions to H-shell burning ($L\uparrow$, $T_{eff}\downarrow$), which dumps He-ash onto an increasingly degenerate core. When degenerate core mass becomes large enough, through contraction it reaches a temperature high enough to start fusion via He-flash. After several flashes, the core degeneracy is lifted and the core expands, the envelope contracts, and $T\uparrow$ as the star is now constantly burning its He-core. Eventually, an inert C-O core is built up and the star begins He-shell burning (still w/ outer H-shell burning). If the convective part of outer envelope is large enough, C-O, from core is dredged up to surface where it forms dust that is blown away by stars radiation. This radiation induced mass loss causes the star to become a C-O white dwarf at the center of a planetary nebula.

* for a star of $M \sim 5M_\odot$



Similar evolution to above, but star starts w/ convective core + radiative envelope. Once H is exhausted in core, star transitions to H-shell burning. Inert He-shell contracts + envelope expands until He-burning starts (Note: Core never becomes degenerate). As star begins to build up C/O in core, it becomes an AGB star and undergoes thermal pulses. These thermal pulses cause mass loss and begin forming a planetary nebula w/ C-O white dwarf at center.

Stellar Structure

* 4 equations of stellar structure

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho} \qquad \frac{dl}{dm} = \epsilon_{nuc}$$

$$\frac{dP}{dm} = \frac{-GM}{4\pi r^4} \qquad \frac{dT}{dm} = \frac{-GM}{4\pi r^4}\frac{T}{P}\nabla \; ; \quad \nabla = \begin{cases} \nabla_{rad} = \dfrac{-3\kappa l P}{16\pi a c G m T^4} \\ \nabla_{ad} \end{cases}$$

Radiative Transport

$$\frac{\upsilon}{\rho}\frac{dI_\nu}{dz} = j_\nu - k_\nu I_\nu \qquad \text{or} \qquad \upsilon\frac{dI_\nu}{d\tau} = I_\nu - S_\nu$$

* to find a general solution

$$\left(\upsilon\frac{dI_\nu}{d\tau} = I_\nu - S_\nu\right)e^{-\tau/\upsilon}$$

$$\upsilon\frac{dI_\nu}{d\tau}e^{-\tau/\upsilon} = I_\nu e^{-\tau/\upsilon} - S_\nu e^{-\tau/\upsilon}$$

$$\frac{dI_\nu}{d\tau}e^{-\tau/\upsilon} - \frac{I_\nu}{\upsilon}e^{-\tau/\upsilon} = -\frac{S_\nu}{\upsilon}e^{-\tau/\upsilon}$$

$$\Rightarrow \int_{\tau_1}^{\tau_2}\frac{d(I_\nu e^{-\tau/\upsilon})}{d\tau} = \int_{\tau_1}^{\tau_2}-\frac{S_\nu}{\upsilon}e^{-\tau/\upsilon}d\tau$$

$$I_\nu e^{-\tau/\upsilon}\Big|_{\tau_1}^{\tau_2} = \int -\frac{S_\nu}{\upsilon}e^{-\tau/\upsilon}d\tau$$

$$I_\nu(\tau_1) = I_\nu(\tau_2)e^{(\tau_1-\tau_2)/\upsilon} + \int_{\tau_1}^{\tau_2}-\frac{S_\nu}{\upsilon}e^{-\tau/\upsilon}d\tau$$

* moment of radiative field

Astro Qualifier Topic List

*This topic list covers the Spring 2012 to Spring 2015 Qualifiers

- Astrophysics Basics
  - Angles/Solid Angles
  - Flux
  - Luminosity, Eddington Luminosity
  - Magnitudes
  - Power Law
  - Standard Candles
    - Cepheid Variable Stars
    - Type Ia SNe
- Binary Systems
  - Habitable Zones
  - Masses of bodies in system
  - Orbits/Semi-major axis calculations (from parallax)
  - Radial Velocity Curves
  - Separation Distance
  - Surface gravity calculations
  - Transits (both star and planet)
    - Transit depth
    - Transit duration
    - Luminosity calculations during transit
- Cosmology
  - Composition
    - Effects of relative abundances
    - Equations of state for each component
  - Constants and their meanings
  - Cosmological redshift and how to determine age from it
  - Distances
    - Angular diameter
    - Co-moving line of sight
    - Co-moving transverse
    - Luminosity
  - Friedman Equation
  - Inflation
    - Equation of state
    - Impact on energy/momentum
  - Scale factor (including derivation)
  - Surface brightness calculations
  - Type Ia SNe
    - Calculations
    - Uncertainties
- Galaxies
  - Age-metallicity relation
  - Components (include evidence)
    - Dark matter
    - Bulge
    - Halo
    - Cold/hot ISM

        - Central black hole
        - Population I/II stars
    - G-dwarf problem
    - Isophotal radius
    - Luminosity functions
        - Schechter luminosity function
    - Mass-light ratio
    - Mergers
    - Rotational velocity curves
    - Tully-Fisher Relation
    - Thin v thick disk
- Interstellar gas clouds
    - Stromgren Sphere
    - Wind speed of expanding gas clouds
- Lorentz Force
- Kepler's Laws & Mechanics
- Neutron Stars
    - GR effects
    - Magnetic field strength
- Nuclear Fusion
    - PP Chain (including rxn's and tunneling)
    - CNO cycle (including rxn's)
    - Impact of fusion on elemental abundances outside of star
    - S-process and r-process
    - He burning
    - Heavy element (C, O, Ne, Si) burning
- Planetary systems
    - Derive temperature
    - Derive period
- Pulsars
    - Types
    - Light Cylinder
    - Rotational velocity/Energy Loss
    - Magnetic Fields
    - PP Diagram
    - Period-distance relation
    - Period variability
- Quasars
    - Damped Lyman Alpha systems
    - Calculations using cosmology
- Stellar Evolution
    - Timescales
    - Evolutionary Tracks (including pre-MS)
    - Formation
        - Initial Mass Function
    - Virial Theorem & Gravitational Energy
    - Lifetime estimates
- Stellar Structure
    - 4 equations
    - Polytropes

    - Hydro-static equilibrium calculations
    - Equations of State
- Radiative Transport
    - Plane-parallel approximation
        - Derive zeroth and first moments
    - Grey Atmosphere approximation
    - Optically thin v optically thick
    - Rosseland Mean Opacity
    - Source Functions, etc.
    - Semi-infinite gas clouds
- Synchrotron Radiation
- Telescope Optics
    - Focal Length
    - Diffraction Limit
    - Quantum efficiency
    - CCD's
- Virial Theorem
    - Derive Jeans mass
    - Derive central temp of star
- 21 cm H-I Line
    - Temperature of Interstellar Medium
    - Optical Density
    - Radiative Transport

Astronomy Qualifier - August 2011

Lots of necessary (and some unnecessary) "constants" and possibly useful integrals at end.

Problem 1: Wang

The inflationay theory of the very early Universe solves the horizon problem of standard cosmology.

a) [2 pts] What is the horizon problem?

b) [2 pts] Show that inflation solves the horizon problem if $a(t) \propto t^{\alpha}$ during inflation, with $\alpha > 1$.

c) [4 pts] Derive the requirement from inflation on the equation of state of the matter-energy in the universe.

d) [2 pts] Does any matter-energy component that has been studied in cosmology satisfy this requirement?

Aug 2011

#1

a) What is the horizon problem?

The horizon problem is an issue in cosmology where parts of the universe are not causally connected yet share the same properties. For example, if we look at two points of the CMB separated by 180°, we have no reason to suspect that they share any common properties, as it takes information roughly 27.199 billion years to travel b/w the two points, Yet we know that both of these CMB points share the same temperature even though they are not causally connected.

Dai

Problem 2:

a) [5 pts] Assume that a model for the dark matter halo of the Galaxy is:

$$\rho(r) = \frac{C_0}{(a^2 + r^2)},$$

where ρ is density, r is distance from the galactic center, and $a = 2.8$ kpc. Show that the amount of dark matter interior to a radius r is given by the expression:

$$M_r = 4\pi C_0 \left[r - a\ tan^{-1}\left(\frac{r}{a}\right)\right]$$

b) [2 pts] If 5.5×10^{11} $M_\odot$ of dark matter is located within 100 kpc of the Galactic center, determine C_0 in units of $M_\odot$/kpc. Repeat your calculation if 1.3×10^{12} $M_\odot$ is located within 230 kpc of the Galactic center.

c) [3 pts] Estimate the amount of dark matter (in solar masses) within a radius of 50 kpc of the Galactic center. Compare your answer to the mass of the stellar halo (choose a reasonable value for the latter).

Aug 2011

Astro #2

a) Show the amount of dark matter interior to a radius r is $m(r) = 4\pi C_0[r - a\tan^{-1}(\frac{r}{a})]$ when $\rho(r) = \frac{C_0}{a^2+r^2}$

$$m(r) = \int_0^r \rho(r')dr'$$
$$= \int_0^r \frac{C_0}{a^2+r^2} dr^3$$
$$= 4\pi C_0 \int_0^r \frac{r^2}{a^2+r^2} dr$$
$$= 4\pi C_0 \left(r - \frac{1}{a}\tan^{-1}\left(\frac{r}{a}\right)\Big|_0^r\right)$$
$$= 4\pi C_0 \left(r - \frac{1}{a}\tan^{-1}\left(\frac{r}{a}\right) - \left(0 - \frac{1}{a}\tan^{-1}\left(\frac{0}{a}\right)\right)\right)$$
$$= 4\pi C_0 \left(r - \frac{1}{a}\tan^{-1}\left(\frac{r}{a}\right)\right) \checkmark$$

b) If $5.5 \cdot 10^{11} M_\odot$ of DM is located w/in 100 kpc of the Galactic center, find C_0 in terms of $M_\odot$/kpc. Repeat w/ $1.3 \cdot 10^{12} M_\odot$ w/in 230 kpc

$$m(r) = 4\pi C_0 \left(r - \frac{1}{a}\tan^{-1}\left(\frac{r}{a}\right)\right)$$
$$\Rightarrow C_0 = \frac{m(r)}{4\pi\left(r - \frac{1}{a}\tan^{-1}\left(\frac{r}{a}\right)\right)}$$

(1) $$C_0 = \frac{5.5 \cdot 10^{11} M_\odot}{4\pi\left(100 - \frac{1}{2.8}\tan^{-1}\left(\frac{100}{2.8}\right)\right)}$$
$$= 4.4 \cdot 10^8 \frac{M_\odot}{kpc}$$

(2) $$C_0 = \frac{1.3 \cdot 10^{12} M_\odot}{4\pi\left(230\,kpc - \frac{1}{2.8\,kpc}\tan^{-1}\left(\frac{230}{2.8}\right)\right)}$$
$$= 4.51 \cdot 10^8 \frac{M_\odot}{kpc}$$

c) Estimate the amount of DM w/in 50 kpc of galactic center. Compare to mass of stellar halo.

$\rightarrow C_0 \approx 4.45 \cdot 10^8$

$$m(r) = 4\pi\left(4.45 \cdot 10^8 \frac{M_\odot}{kpc}\right)\left(50\,kpc - \frac{1}{2.8\,kpc}\tan^{-1}\left(\frac{50\,kpc}{2.8\,kpc}\right)\right)$$
$$= 2.77 \cdot 10^{11} M_\odot$$

John

Problem 3: $\boxed{\text{B.O.B} - 7.4}$

Consider an eclipsing spectroscopic binary with the following properties:

- Orbital period is 6.31 yr.
- Maximum radial velocities of Star A and Star B are 5.4 $\mathrm{kms^{-1}}$ and 22.4 $\mathrm{km\ s^{-1}}$.
- Time period between first contact and minimum light is 0.58 d, and the length of the primary minimum is 0.64 d.
- The apparent bolometric magnitudes of the maximum, primary minimum, and secondary minimum are 5.40 magnitudes, 9.20 magnitudes, and 5.44 magnitudes, respectively.

Assuming circular orbits and that the plane of the system lies in our line of sight, find the following:

a) [2 pts] Ratio of the stellar masses.

b) [2 pts] Sum of the masses.

c) [2 pts] Individual masses.

d) [2 pts] Individual radii.

e) [2 pts] Ratio of the effective temperatures of the two stars.

Aug 2011

#3

$P = 6.31 \text{ yr}$

$V_{r,A} = 5.4 \frac{km}{s}$

$V_{r,B} = 22.4 \frac{km}{s}$

a) $$\frac{m_A}{m_B} = \frac{V_B}{V_A}$$

$$= \frac{22.4}{5.4}$$

$$= 4.15$$

b) $$P^2 = \frac{4\pi^2}{G(m_1+m_2)} a^3$$

$$(m_1+m_2) = \frac{4\pi^2}{GP^2} a^3$$

$$* \text{ but } \quad a = a_1 + a_2$$

$$= \frac{P}{2\pi} V_1 + V_2$$

$$= \frac{4\pi^2}{GP^2} \left(\frac{P}{2\pi} [V_1 + V_2] \right)^3$$

$$= \frac{4\pi^2 P}{G\,8\pi^3} (V_1+V_2)^3$$

$$= \frac{P}{2\pi G} (V_1+V_2)^3$$

$$= \frac{P}{2\pi G} \frac{(V_{1,R}+V_{2,R})^3}{\sin^3 i}$$

$$= 1.02 \cdot 10^{31} \text{ kg}$$

c) $$m_A = 4.15\, m_B$$

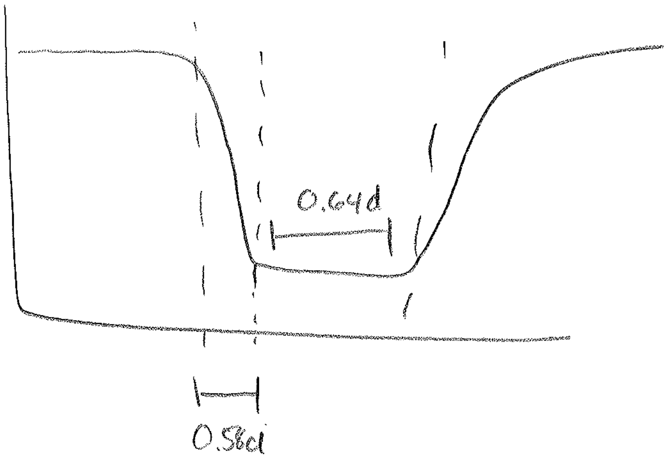
$$\Rightarrow \quad 5.15\, m_B = 1.02 \cdot 10^{31} \text{ kg}$$

$$m_B = 1.98 \cdot 10^{30} \text{ kg}$$

$$m_A = 8.23 \cdot 10^{30} \text{ kg}$$

#3 (cont.)

d)



$$r_B = \frac{v}{2}(0.58 \text{ days})$$

$$= \frac{\left(5.4 \frac{km}{s} + 22.4 \frac{km}{s}\right)}{2}\left(0.58 \cdot \cancel{365.25 \tfrac{day}{yr}} \cdot 24 \tfrac{hr}{day} \cdot 3600 \tfrac{s}{hr}\right)$$

$$= 6.965 \cdot 10^5 \text{ km}$$

$$= 6.97 \cdot 10^8 \text{ m}$$

$$r_A = \frac{5.4 \frac{km}{s} + 22.4 \frac{km}{s}}{2}\left(\left[0.58 + 0.64\right] \cdot 24 \cdot 3600\right)$$

$$= 1.47 \cdot 10^6 \text{ km}$$

$$= 1.47 \cdot 10^9 \text{ m}$$

e)

Kilic

Problem 4:

a) [4 pts] Compare the nucleosynthesis evolution of low-mass (stars like the sun) and high-mass (20 solar mass) stars. In particular, describe all of the hydrostatic and and/or explosive phases of element formation for each type of star. List the elements that are fused (or burned), the order that they happen during stellar evolution and the most likely products of those reactions.

b) [3 pts] What is the heaviest element that can be fused in low-mass and high-mass stars and why? What about iron fusion? When does it occur, or if not, why not? What about the heaviest elements such as precious metals? How are they formed? Describe the processes?

c) [3 pts] How do we know that nucleosynthesis occurs in stars? Give specific examples of observations that indicate element formation must occur in certain stars. What stage of evolution are these stars in, and how are the elements that we observe formed inside the star?

Aug 2011

Astro #4

a) In a low mass star, once it reaches the main sequence it will burn H to He via the P-P chain for billions of years. As He ash builds in the core, eventually the H-burning will occur in a shell instead of in the core. As more ash builds, the He core becomes degenerate and will spark in a He-flash up to several times to break the degenerecy and stably burn He to C and O. But, unlike the He ash, the C/O ash that builds up in the core is unable to break it's degenerecy + the star dies as a C/O white dwarf.

For the high mass star, it burn H to He via the CNO cycle. After that, it builds up a core of concentric shells of various elements (listed in order below) Due to the large mass of the star, it is not necessary for the core to become degenerate to transition from burning one element to another.

$$H \rightarrow He \rightarrow C \rightarrow O \rightarrow Ne \rightarrow Mg \rightarrow Si \rightarrow Fe$$

Once the core becomes filled w/ enough Si ash to begin Si burning, the Ni^{56} ash which subsequently decays to Fe^{56} no more fusion occurs, as fusion of iron into another element requires the addition of energy from its surroundings, instead of releasing energy during the process.

b) In a low mass star, the heaviest elements fused are C/O; heavier elements are not fused b/c the central temperatures of low mass are not high enough for more advanced fusion processes to occur.

In high mass stars, the heaviest element that can be fused is Fe. Fusing elements heavier than iron requires adding energy to the system instead of releasing energy.

The reason we find elements heavier than iron is because during core-collapse SN, the large amounts of neutrons present allow s and r process elements to form. Since neutrons are electrically neutral, they don't need to overcome the Coloumb potential to collide w/ the atomic nuclei. However, these neutron rich isotopes are unstable, and

b) when they decay they leave a proton in the nucleus and create heavier elements.

c)

??

Problem 5:

A telescopic survey to find nearby "space rocks" can find moving objects to a magnitude of 18.5. The relationship between magnitude and flux for the "visible" passband used is:

$$mag = -2.5 log(f/f_0)$$

where f is the flux from the target and f_0 is the flux from a mag = 0 object (assume f_0 = 1.0E−8 W m^{-2}).

An approximately spherical space rock, 50 meters in diameter, with an albedo of 0.2, comes near the Earth. The rock shines in the visible only by reflected sunlight.

a) [1 pts] Calculate the flux of sunlight in the visible at a distance of 1 AU from the Sun. Assume the "visible" pass band encompasses 1/3 of the bolometric power output of the Sun.

b) [2 pts] From the parameters given calculate the visible power of the rock (power of reflected sunlight) when approximately 1 AU from the Sun.

c) [4 pts] What is the maximum distance from Earth that the survey could detect the rock? (The rock will not emit isotropically, of course, but only from its illuminated side. Just assume it reflects uniformly from half its surface (the "day" side)). Don't worry about the changing solar flux with distance- just assume the rock is near 1 AU from Sun.

d) [3 pt] Assume the rock has a density of a typical rocky asteroid. Assume it hits the Earth with a speed equal to the escape speed of the Earth. How many megatons of energy would be released by the impact? (1 MT = 4.2E15 Joules).

Aug 2011

Astro #5

a) $mag = -2.5 \log(f/f_0)$

$\frac{-2}{5} m = \log(f/f_0)$

$e^{-.4m} = \frac{f}{f_0}$

$f_0 e^{-.4m} = f$

$1 \cdot 10^{-8} \frac{W}{m^2} e^{-.4(4.7)} = f_0$

$1.526 \cdot 10^{-9} \frac{W}{m^2} = f_0$

$\Rightarrow f_{VB} = 5.09 \cdot 10^{-10} \frac{W}{m^2}$

b) $f_{VB} = 5.09 \cdot 10^{-10} \frac{W}{m^2}$ $\Longleftarrow$ $I = \frac{P}{SA}$

$(0.2) \cdot f_{VB} \cdot \pi (25m)^2 = P_{rock}$

$P_{rock} = 1.99 \cdot 10^{-7} W \cdot 4$

c) $f_0 e^{-.4m} = f$

$\Rightarrow f_{18.5} = 6.11 \cdot 10^{-12} \frac{W}{m^2} = J/s$

$P_{18.5} = f_{18.5} \cdot \pi (25m)^2 \cdot 4$

$= 1.2 \cdot 10^{-8} W \cdot 4$

P

Aug 2011

#5 (cont.)

d) * Assume density of rocky asteroid is same as density of earth.

$$\rho = \frac{m}{V}$$
$$= \frac{5.97 \cdot 10^{27}\ g}{(6.37 \cdot 10^{8}\ cm)^3 \cdot \pi \cdot \frac{4}{3}}$$
$$= 5.51 \frac{g}{cm^3}$$

* Find escape velocity

$$\frac{1}{2} mv^2 = \frac{M m G}{R}$$
$$V = \sqrt{\frac{2MG}{R}}$$
$$V_{esc,\oplus} = \sqrt{\frac{2(5.97 \cdot 10^{27} g)(6.67 \cdot 10^{-8} \frac{cm^3}{g s^2})}{(6.37 \cdot 10^{8} cm)}}$$
$$= 1.12 \cdot 10^{6} \frac{cm}{s}$$

$$E_{asteroid} = \frac{1}{2} m V_{esc\oplus}^2$$
$$= \frac{1}{2}\left(\rho \cdot \frac{4}{3}\pi (2500\ cm)^3\right)\left(1.12 \cdot 10^{6} \frac{cm}{s}\right)^2$$
$$= 2.25 \cdot 10^{23} \frac{g\,cm^2}{s^2}$$
$$= 2.25 \cdot 10^{16}\ J$$
$$\boxed{= 5.36\ MT}$$

cm → m $(10^2)^2$
g → kg 10^3

Eddie

Problem 6:

The equation of radiative transfer in spherical coordinates is:

$$\mu\frac{\partial I_\nu}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial I_\nu}{\partial \mu} = -\chi_\nu I_\nu + \eta_\nu$$

a) [3pts] Show that the moment equations can be written:

$$\frac{1}{r^2}\frac{\partial}{\partial \tau_\nu}(r^2 H_\nu) = (J_\nu - S_\nu)$$

$$\frac{\partial K_\nu}{\partial \tau_\nu} + \frac{(J_\nu - 3K_\nu)}{(\chi_\nu r)} = H_\nu$$

b) [2pts] Introduce the Eddington factor $f_\nu = K_\nu/J_\nu$ and rewrite the moment equations in terms of it.

c) [2pts] Explain the problem with deriving a single second order equation for J_ν as is done in the plane-parallel case.

d) [3pts] Show that in fact with the sphericity factor:

$$ln(r^2 q_\nu) = \int_{r_c}^{r}[(3f_\nu - 1)/(r' f_\nu)]dr' + ln(r_c^2)$$

where r_c is the radius of the opaque core, the two moment equations can be combined to give:

$$\frac{\partial^2}{\partial X_\nu^2}(r^2 q_\nu f_\nu J_\nu) = q_\nu^{-1} r^2 (J_\nu - S_\nu)$$

where $dX_\nu = q_\nu d\tau_\nu$.

Aug 2011

Astro #6

RTE in spherical:

$$\mu \frac{\partial I_\nu}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial I_\nu}{\partial \mu} = -\chi_\nu I_\nu + \eta_\nu$$

a) $$\mu \frac{\partial I_\nu}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial I_\nu}{\partial \mu} = -\chi_\nu I_\nu + \eta_\nu$$

$$\frac{\mu}{-\chi_\nu}\frac{\partial I_\nu}{\partial r} + \frac{1-\mu^2}{-r\chi_\nu}\frac{\partial I_\nu}{\partial \mu} = I_\nu - S_\nu \qquad \text{where } S_\nu = \frac{\eta_\nu}{\chi_\nu}$$

$$? \quad \mu \frac{\partial I_\nu}{\partial \tau_\nu} + \frac{1-\mu^2}{\tau_\nu}\frac{\partial I_\nu}{\partial \mu} = I_\nu - S_\nu \qquad \text{where } d\tau_\nu = -\chi_\nu dr, \quad \tau_\nu = -\chi_\nu r$$

Moment equations:

$$J_\nu = \frac{1}{2}\int_{-1}^{1} I_\nu d\mu$$

$$K_\nu = \frac{1}{2}\int_{-1}^{1} \mu^2 I_\nu d\mu$$

$$H_\nu = \frac{1}{2}\int_{-1}^{1} \mu I_\nu d\mu$$

$$\Rightarrow I_\nu = S_\nu + \mu\frac{\partial I_\nu}{\partial \tau_\nu} + \frac{1-\mu^2}{\tau_\nu}\frac{\partial I_\nu}{\partial \mu}$$

$$\hookrightarrow H_\nu = \frac{1}{2}\int_{-1}^{1} \cancelto{0}{\mu S_\nu} + \mu^2 \frac{\partial I_\nu}{\partial \tau_\nu} + \frac{\mu-\mu^3}{\tau_\nu}\frac{\partial I_\nu}{\partial \mu}\, d\mu$$

$$H_\nu = \quad + \frac{\partial K_\nu}{\partial \tau_\nu} + \frac{1}{\chi_\nu r}\left(J_\nu - 3K_\nu\right) \checkmark$$

* Integrating RTE over all μ yields

$$J_\nu - S_\nu = \frac{\partial K_\nu}{\partial \tau_\nu} + \frac{1}{\tau_\nu}(I_\nu - H_\nu)$$

$$= \frac{\partial K_\nu}{\partial \tau_\nu} + \frac{1}{\tau_\nu}\left(I_\nu - \left[\frac{\partial K_\nu}{\partial \tau_\nu} + \frac{1}{\chi_\nu}(J_\nu - 3K_\nu)\right]\right)$$

$$= \frac{\partial K_\nu}{\partial \tau}\left(1 - \frac{1}{\tau_\nu}\right) + \frac{1}{\tau_\nu}\left(I_\nu - \frac{1}{\chi_\nu}\left[J_\nu - 3K_\nu\right]\right) \quad ???$$

Aug 2011

#6 (cont.)

b) * Assuming

$$\frac{1}{r^2}\frac{\partial}{\partial \tau_\nu}(r^2 H_\nu) = J_\nu - S_\nu$$

$$\frac{\partial K_\nu}{\partial \tau_\nu} + \frac{(J_\nu - 3K_\nu)}{\chi_\nu r} = H_\nu$$

* if $f_\nu = \frac{K_\nu}{J_\nu}$

$$\frac{1}{J_\nu}\left[\frac{\partial K_\nu}{\partial \tau_\nu} + \frac{J_\nu - 3K_\nu}{\chi_\nu r} = H_\nu\right]$$

$$\frac{\partial f_\nu}{\partial \tau_\nu} + \frac{1 - 3f_\nu}{\chi_\nu r} = \frac{H_\nu}{J_\nu}$$

$$= \frac{H_\nu K_\nu}{f_\nu}$$

CONSTANTS

$\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}; $c = 3.00 \times 10^{10}$ cm s^{-1}; $T_\odot = 5,800$K

$G = 6.67 \times 10^{-8}$ g^{-1} cm^3 s^{-2}; $k = 1.38 \times 10^{-16}$ erg K^{-1}

$m_H = 1.67 \times 10^{-24}$ g; $m_e = 9.11 \times 10^{-28}$ g; $M_\odot = 1.99 \times 10^{33}$ g

$M_{\rm earth} = 5.97 \times 10^{27}$ g; $M_G = 4.0 \times 10^{11} M_\odot$

$h = 6.63 \times 10^{-27}$ erg s; $a = 7.56 \times 10^{-15}$ erg cm^{-3} K^{-4}

$R_\odot = 6.96 \times 10^{10}$ cm; $R_{\rm earth} = 6.37 \times 10^8$ cm

1 AU $= 1.496 \times 10^{13}$ cm

1 parsec $= 3.09 \times 10^{18}$ cm; $1\mathring{A} = 10^{-8}$ cm

$M_V(\odot) = 4.8$; $M_{bol}(\odot) = 4.7$; $L_\odot = 3.9 \times 10^{33}$ ergs s^{-1}

1 year $= 3.16 \times 10^7$ s

ASTRONOMY QUALIFYING EXAM

January 2012

Possibly Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \times 10^{10}$ cm
1 AU = 1.5 $\times 10^{13}$ cm
1 pc = 3.26 Ly. = 3.1 $\times 10^{18}$ cm
a = 7.56 $\times 10^{-15}$ erg cm^{-3} K^{-4}
c = 3 $\times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
k = 1.38 $\times 10^{-16}$ erg K^{-1}
e = 4.8 $\times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
G = 6.67 $\times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
h = 6.63 $\times 10^{-27}$ erg s
1 amu = 1.66053886 $\times 10^{-24}$ g

John?

PROBLEM 1

A star of magnitude 0 delivers a flux density equal to $4.17 \times 10^{-11}\,\mathrm{erg\,s^{-1}\,cm^{-2}\,\AA^{-1}}$ in the K band ($\lambda = 2.2\mu$m).

a. Derive the flux density in units of $\mathrm{W\,m^{-2}\,Hz^{-1}}$ (2 points).

b. What is the count rate in terms of photons $\mathrm{s^{-1}\,cm^{-2}\,\AA^{-1}}$? (2 points)

c. What will be the diameter of a telescope whose diffraction limit at this wavelength is 0.05 arcsec? (2 points)

d. The telescope in part c has a focal ratio of 2. What would the size of a pixel in the detector have to be to critically sample the diffraction limit (NB: critically sampled means that the airy disk FWHM subtends two pixels)? (2 points)

e. The sky background at this wavelength is about $13.7\,\mathrm{mag\,arcsec^{-2}}$. Assuming that the detector and telescope present a quantum efficiency of 50%, what is the background rate per pixel for the detector imagined in part d? Assume that you are observing through a filter that has a width of $0.3\,\mu$m. (2 points)

Jan 2012

Astro #1

a) $f = 4.17 \cdot 10^{-11} \frac{erg}{s \cdot cm^2 \cdot \AA}$

$*$ Want f in $\frac{W}{m^2\, Hz}$

$W = 1\ J/s = 1 \frac{\frac{kg\, m^2}{s^2}}{s} = \frac{kg\, m^2}{s^3}$ $\quad \AA = 1 \cdot 10^{-8}\ cm = 1 \cdot 10^{10}\ m$

$Hz = \frac{1}{s}$

$erg = \frac{g \cdot cm^2}{s^2}$

$\Rightarrow f = 4.17 \cdot 10^{-11} \frac{\frac{g \cdot cm^2}{\cancel{cm}\ s^2}}{s \cdot cm^2 \cdot \AA}$

$= 4.17 \cdot 10^{-11} \frac{g}{s^3 \AA}$

$= 4.17 \cdot 10^{-14} \frac{kg}{s^3 \cdot \AA}$

$= 4.17 \cdot 10^{-14} \frac{J}{m^2 \AA}$

$= 4.17 \cdot 10^{-14} \frac{W s}{m^2 \AA}$

$= 4.17 \cdot 10^{-14} \frac{W}{m^2\, Hz\, \AA}$?? $\rightarrow$ multiply by λ in $\AA$? Typo?

b) $E_\gamma = h\nu$

$= (6.63 \cdot 10^{-27}\ erg \cdot s)\, \nu$

$*$ if all photons have $\lambda = 2.2\ \mu m$

$= 2.2 \cdot 10^{-6}\ m$

$= 2.2 \cdot 10^{-4}\ cm$

$c = \lambda \nu \rightarrow \nu = \frac{c}{\lambda}$

$= \frac{3 \cdot 10^{10} \frac{cm}{s}}{2.2 \cdot 10^{-4}\ cm}$

$= 1.36 \cdot 10^{14}$ [Hz] or $[\frac{1}{s}]$

$E_\gamma = 9.04 \cdot 10^{-13}\ erg$

$f = \left(4.17 \cdot 10^{-11} \frac{erg}{s \cdot cm^2 \cdot \AA}\right) \cdot \left(\frac{1}{9.04 \cdot 10^{-13}} \frac{photons}{erg}\right)$

$= 46.1 \frac{photons}{s \cdot cm^2 \cdot \AA}$

Kilic

PROBLEM 2

When a $5M_\odot$ star leaves the main sequence it enters the largely horizontal sub-giant branch. Models indicate that the star spends about 350,000 years on this section of the HR diagram before beginning its ascent on the red giant branch. Compute the expected Kelvin-Helmholtz time scale for this phase of stellar evolution and explain any differences by doing the following:

a. (4) Show that the gravitational energy ultimately radiated away is:

$$E_g = \frac{3GM^2}{10R},$$

where M and R are the the stellar mass and radius, respectively. Assume the virial theorem and that the density of the star at any distance from its center is equal to the star's average density, $M/\frac{4}{3}\pi R^3$.

b. (3) If $L = 10^3 L_\odot$ and $T_{eff} = 10^{3.9}$ K, estimate the time in years that this luminosity could be sustained if it is based solely on gravitational energy.

c. (3) Compare your answer in b. with the model-predicted time and explain why they are different. Make sure you explain what current theory tells us is going on inside the star.

Jan 2012

Astro #2

a) $U = \frac{-GMm}{r}$ (Gravitational Potential Energy) $\Rightarrow$ $dU = \frac{-Gm\,dm}{r}$

$E = \frac{1}{2}U$ (Virial Thm)

* but $dm = 4\pi r^2 \rho\, dr$, $\rho = \frac{M}{\frac{4}{3}\pi R^3}$

$\Rightarrow dm = 4\pi r^2 \left(\frac{m}{\frac{4}{3}\pi R^3}\right) dr$

$\Rightarrow dU = -Gm\frac{dm}{r}$

$= -G\,\frac{4}{3}\pi R^3 \rho\; 4\pi r^2 \rho\, dr$

$U = \int_0^R \frac{-16\pi^2 G r^3 \rho^2}{3r} r^2 dr$

$= \frac{-16\pi^2 G}{3} \int_0^R \rho^2 r^4 dr$

$= \frac{-16\pi^2 G}{3} \rho^2 \left(\frac{1}{5}R^5\right)$

$= -\frac{3GM^2}{5R}$

$\Rightarrow E = \frac{-3GM^2}{10R}$

b) $t_{KH} = \frac{\Delta E_g}{L_\odot}$

$= \frac{\frac{3GM^2}{10R}}{L}$

$= \frac{\frac{3G(5M_\odot)^2}{10R}}{10^3 L_\odot}$

* for stupidity's sake, assume $R \sim R_\odot$

$= \frac{25\,\frac{3GM_\odot^2}{10R_\odot}}{10^3 L_\odot}$

* $t_\odot$ can be expressly calculated using values on pg 1

$= \frac{25}{1000} t_\odot \approx 0.025 \cdot 10^7 \text{yr} = 2.5 \cdot 10^5 \text{yr}$

Jan 2012

c) Our KH timescale estimate is 250,000 yrs compared to our model estimate of 350,000 yrs.

This is b/c the KH timescale estimates how long the star would live if the only reason it shines is due to gravitational collapse. But we know that stars shine due to nuclear fusion, and that the release of energy from nuclear fusion of H → He is able to account for the timescales that objects around stars have existed; ie rocky planets/moons around sun dated via carbon dating.

Henry

PROBLEM 3

This problem concerns the important 21-cm line in astrophysics and its production mechanism.

a. (1) Briefly discuss the physics of the 21 cm line. What causes it? What's happening at the atomic level? Mention what kind of interstellar environment (density, temperature, state of hydrogen) is associated with this line.

b. (2) Estimate a minimum temperature that is necessary to excite this line and compare with a typical temperature of the interstellar environment which you identified in a. Discuss.

c. (3) Assuming a 2-level configuration, i.e., a ground state and one excited state, write down a rate equation which takes collisional excitation, collisional deexcitation, and spontaneous deexcitation into account. Use q_{up} and q_{down} to represent excitation and deexcitation collisional rate coefficients, respectively, A to represent the spontaneous downward rate, and N_g and N_{ex} to represent volume densities of ground and excited states.The sum of all the rates should equal zero.

d. (2) Based on your equation in c., show that the volume emissivity of 21 cm radiation, ϵ_{21}, is given by $\epsilon_{21cm} = (N_g)^2 q_{up} E_{21cm}$, where E_{21cm} is the energy associated with the transition. Assume that $q_{down} << A$.

e. (2) Suppose an interstellar cloud produces 21-cm radiation with an optical depth at its center of $\tau = 0.5$. The line's full width at half-maximum of the line $\Delta v = 10$ km/s. Find the thickness of the cloud in parsecs if $\tau = 5.2 \times 10^{-23} \frac{N_{col}}{T \Delta v}$, where N_{col} is column density in cm^2 and Δv is in km/s. Assume an order-of-magnitude temperature and density which is characteristic of the system that typically emits 21-cm radiation.

Dai? / Leighly?

PROBLEM 4

(1) Define the Eddington luminosity. (2pts)

(2) Derive the Eddington luminosity by balancing the radiation force and gravity for an electron. (3pts) gas?

(3) What is the Eddington luminosity for a $10^8 M_\odot$ AGN? (3pts)

(4) An AGN is observed to emit at a super-Eddington rate. What are the possible explanations? (2pts)

Jan 2012

Astro #4

a) The Eddington Luminosity is the maximal luminosity an object can have while remaining in hydrostatic equilibrium.

b) * For an electron gas in HSE:

$$\left|\frac{dP_{rad}}{dr}\right| < \left|\frac{dP}{dr}\right|$$

$$P_{rad} = \frac{1}{3}aT^4 \qquad \frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}, \quad \frac{dP}{dm} = \frac{-Gm}{4\pi r^4}, \quad \frac{dT}{dm} = \frac{-GmT}{4\pi r^4 P}\nabla$$

$$\nabla = \frac{-3\kappa l P}{16\pi a c G m T^4}$$

$$\frac{dP_{rad}}{dr} = \frac{4}{3}aT^3\frac{dT}{dr}$$

$$= \frac{4}{3}aT^3\frac{dT}{dm}\cdot\frac{dm}{dr}$$

$$= \frac{4}{3}aT^3\left(\frac{-GmT}{4\pi r^4 P}\cdot\frac{-3\kappa l P}{16\pi a c G m T^4}\right)\cdot 4\pi r^2\rho$$

$$= \frac{\kappa l \rho}{4\pi r^2 c}$$

$$\frac{dP}{dm}\cdot\frac{dm}{dr} = \frac{-Gm}{4\pi r^4}\cdot 4\pi r^2\rho$$

$$= \frac{-Gm\rho}{r^2}$$

$$\Rightarrow \frac{\kappa l \cancel{\rho}}{4\pi \cancel{r^2} c} < \frac{+Gm\cancel{\rho}}{\cancel{r^2}}$$

$$l < \frac{4\pi c G m}{\kappa}$$

c) $$l < \frac{4\cdot\pi\cdot 3\cdot 10^{10}\,\frac{cm}{s}\cdot 6.67\cdot 10^{-8}\,\frac{cm^3}{g s^2}\cdot 2\cdot 10^{41}g}{\kappa}$$

$$l < \frac{5.03\cdot 10^{45}\,\frac{cm^4}{s}}{\kappa} \ ?$$

d) ?

Wang

PROBLEM 5

The observed universe can be described by the cosmological parameters that include H_0, Ω_m, Ω_r, Ω_k, and Ω_X.

(1) Define what H_0, Ω_m, Ω_r, Ω_k, and Ω_X are. (1 pt)

(2) Write down the expansion rate of the universe as a function of redshift. (3 pts)

(3) Derive an expression for the age of the universe as a function of redshift z (2pts)

(4) Galaxies have been observed at $z \sim 8$. Estimate the age of the universe at $z = 8$. (4 pts)

PROBLEM 6

Consider a rigid satellite of mass m, radius r, and density ρ_m orbiting at a distance d from its massive primary planet of mass M, radius R, and density ρ_M (see the figure below).

a. (2 pts) Show that the angular speed of the satellite about the primary is $\omega = \sqrt{\frac{GM}{d^3}}$

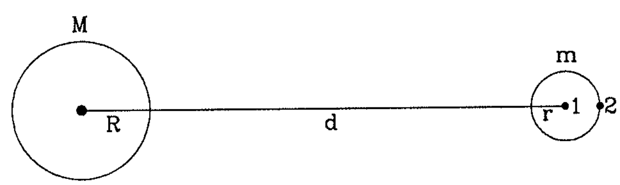
b. (3 pts) Find the differences in the gravitational acceleration between the center of the satellite (point 1) and the outer edge (point 2) due to the primary. Also find the differences in the centripetal acceleration between these two points. Show that the combination of the two effects is

$$\approx \frac{3GMr}{d^3}$$

c. (3 pts) The satellite will be tidally disrupted if the acceleration found in (b) is larger than the satellite's self gravitational acceleration. Show that the disruption occurs at

$$d = r\left(\frac{3M}{m}\right)^{(1/3)} = R\left(\frac{3\rho_M}{\rho_m}\right)^{(1/3)}$$

d. (2 pts) Assuming that the Earth and the Moon have the same density, at what distance would the Moon be disrupted? What about a moon around an Earth-size $1M_{\odot}$ ($3 \times 10^6 M_{earth}$) white dwarf star?



Jan 2012

Astro #6

a) $$p^2 = \frac{4\pi^2}{G(m_1+m_2)} a^3$$

$$= \frac{4\pi^2}{G(M+m)} d^3$$

* Assuming circular orbit at $M \gg m$

$$p^2 = \frac{4\pi^2}{GM} d^3$$

$$\left(\frac{2\pi}{\omega}\right)^2 = \frac{4\pi^2}{GM} d^3$$

$$\frac{4\pi^2}{\omega} = \frac{4\pi^2}{GM} d^3$$

$$\rightarrow \omega = \sqrt{\frac{GM}{d^3}}$$

b) $$ma = \frac{GMm}{r^2}$$

* at pt 1

$$a = \frac{GM}{(d+r)^2}$$

* Gravitational acceleration

* at pt 2

$$a = \frac{GM}{(d+2r)^2}$$

$$ma = m\frac{v^2}{r}$$

* Centripetal acceleration

$$a = \frac{v^2}{r}$$

* but $\omega = \frac{v}{r}$

$$\Rightarrow a = \omega^2 r$$

* at pt 1

$$a = \frac{GM}{d^3}(d+r)$$

* at pt 2

$$a = \frac{GM}{d^3}(d+2r)$$

$$\Rightarrow a_1 = -GM\left(\frac{1}{(d+r)^2} + \frac{d+r}{d^3}\right)$$

$$a_2 = -GM\left(\frac{1}{(d+2r)^2} + \frac{d+2r}{d^3}\right)$$

$$a_1 - a_2 = -GM\left[\left(\frac{1}{(d+r)^2} + \frac{d+r}{d^3}\right) - \left(\frac{1}{(d+2r)^2} + \frac{d+2r}{d^3}\right)\right]$$

Jan 2012

#6 (cont.)

c) $a = \frac{Gm}{r^2}$

$$\frac{Gm}{r^2} = \frac{3GMr}{d^3}$$

$$\frac{m}{r^2} = \frac{3Mr}{d^3}$$

$$d^3 m = 3M r^3$$

$$d^3 = \frac{3M}{m} r^3$$

$$d = \left(\frac{3M}{m}\right)^{1/3} r$$

$$= R\left(\frac{3\rho_M}{\rho_m}\right)^{1/3}$$

d) Math Stuffs

ASTRONOMY QUALIFYING EXAM
August 2012

Possibly Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \times 10^{10}$ cm
1 AU = 1.5 $\times 10^{13}$ cm
1 pc = 3.26 Ly. = 3.1 $\times 10^{18}$ cm
a = 7.56 $\times 10^{-15}$ erg cm^{-3} K^{-4}
c = 3 $\times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
k = 1.38 $\times 10^{-16}$ erg K^{-1}
e = 4.8 $\times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
G = 6.67 $\times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
h = 6.63 $\times 10^{-27}$ erg s
1 amu = 1.66053886 $\times 10^{-24}$ g

John

PROBLEM 1

Suppose a main sequence star at a distance of d = 8 pc from the Earth has been observed to have a maximum radial velocity of 0.1 m/s, and the radial velocity varies periodically with a period P =1.5 years. From this we conclude that the star must have an unseen companion, a planet.

Assume that the star has mass of 0.8 solar masses, and T=5000 K. For simplicity, assume circular orbits and an inclination angle of 90°.

a. Calculate the average separation between the star and the planet. (4 points)

b. Calculate the mass of the planet. (4 points)

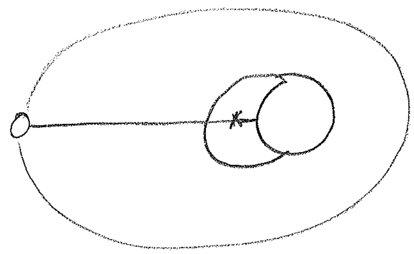
$$\frac{m_p^3}{(m_s + m_p)^2} \sin^3 i = \frac{P}{2\pi G} v_{1,r}^3$$

c. Can life exist on this planet? Explain. (2 points)

Aug 2012

Astro #1

a) * Assuming circular orbits



$M = .8 M_\odot$
$= 1.6 \cdot 10^{33}$ g

$T = 5000$ K

$$P^2 = \frac{4\pi^2}{G(M+m)} a^3$$

* rewrite in terms of reduced mass, $\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{m_s m_p}{m_s + m_p}$

* For star: $P^2 = \frac{4\pi^2}{G(m_s + \mu)} a^3$ $\qquad \frac{G m_s \mu}{a^2} = m_s \frac{V_s^2}{a} \rightarrow V_s = \sqrt{\frac{G\mu}{a}}$

$$\Rightarrow a = \left(\frac{G P^2 (m_s + \mu)}{4\pi^2}\right)^{1/3}$$

* For planet: $P^2 = \frac{4\pi^2}{G(\mu + m_p)} b^3$ $\qquad \frac{G m_p \mu}{b^2} = m_p \frac{V_p^2}{b}$

$$b = \left(\frac{G P^2 (\mu + m_p)}{4\pi^2}\right)^{1/3} \qquad V_p = \sqrt{\frac{G\mu}{b}}$$

* Separation always constant so center of mass is stationary

$$a + b = \left(\frac{GP^2}{4\pi^2}\right)^{1/3} \left((m_s + \mu)^{1/3} + (m_p + \mu)^{1/3} \right)$$

$$P^2 = \frac{4\pi^2}{G m_s} a^3 \qquad m_s = 0.8\, M_\odot$$

$$\Rightarrow \quad a = \left(\frac{G m_s P^2}{4\pi^2}\right)^{1/3}$$

$$= \left[\frac{6.67\cdot 10^{-8}\ \frac{cm^3}{g\cdot s^2}\cdot 0.8\cdot 2\cdot 10^{33} g\cdot (1.5\cdot\pi\cdot 10^7 s)^2}{4\pi^2}\right]^{1/3}$$

Correct

$$= 1.82\cdot 10^{13}\ cm$$
$$\approx 1.21\ AU$$

$$V_s = \sqrt{\frac{G\mu}{a}} \qquad V_s = \sqrt{\frac{G m_s}{a}}$$

$$a V_s^2 = G\left(\frac{m_s m_p}{m_s + m_p}\right)$$

$$\frac{a V_s^2}{G} = \alpha = \frac{m_s m_p}{m_s + m_p}$$

$$(m_s + m_p)\alpha = m_s m_p$$

$$m_s \alpha = m_p(m_s - \alpha)$$

$$\frac{m_s \alpha}{m_s - \alpha} = m_p = 2.72\cdot 10^{18} g$$

$$P^2 = \frac{4\pi^2}{G(m_s + m_p)} a^3$$

$$\frac{P^2}{a^3} = \frac{4\pi^2}{G m_s + m_p}$$

$$m_s + m_p = \frac{4\pi^2 a^3}{G P^2}$$

$$m_p = \frac{4\pi^2 a^3}{G P^2} - m_s$$

$$= \frac{4\cdot\pi^2\cdot(1.82\cdot 10^{13}\ cm)^3}{6.67\cdot 10^{-8}\ \frac{cm^3}{g s^2}\cdot(1.5\cdot\pi\cdot 10^7 s)^2} - 0.8\cdot 2\cdot 10^{33} g$$

$$= 0$$

$$P^2 = \frac{4\pi^2}{G(m_s + \mu)} a^3$$

$$P^2 G\left(m_s + \frac{m_s m_p}{m_s + m_p}\right) = 4\pi^2 a^3$$

$$\frac{m_s m_p}{m_s + m_p} = \frac{4\pi^2 a^3}{P^2 G} - m_s$$

$$m_s m_p = \left(\frac{4\pi^2 a^3}{P^2 G} - m_s\right)(m_s + m_p)$$

$$\left(m_s - \frac{4\pi^2 a^3}{P^2 G}\right) m_p = \left(\frac{4\pi^2 a^3}{P^2 G} - m_s\right) m_s$$

$$m_p = -m_s$$

$$\frac{a^3}{m_s + \mu} = \frac{b^3}{m_p + \mu}$$

$$a^3(m_p + \mu) = b^3(m_s + \mu)$$

$$a^3\left(m_p + \frac{m_s m_p}{m_s + m_p}\right) = b^3\left(m_s + \frac{m_s m_p}{m_s + m_p}\right)$$

$$\frac{a^3}{b^3} = \frac{m_p + \frac{m_s m_p}{m_s + m_p}}{m_p + \frac{m_s m_p}{m_s + m_p}}$$

$$= \frac{m_s(m_s + m_p) + m_s m_p}{m_p(m_s + m_p) + m_s m_p}$$

$$\frac{a}{b} \approx \frac{m_s}{m_p} \qquad = \frac{m_s^2 \cancel{+ 2 m_s m_p}}{\cancel{m_p^2} + 2 m_s m_p}$$

Dai? / Leighly?

PROBLEM 2

The Crab pulsar has a period of P = 0.0333 seconds, and a slow-down rate of $\dot{P} = 4.21\times10^{-13}$. The Crab nebula emits a total luminosity of 5×10^{31} W. A neutron star can be assumed to have a mass equal to $1.4\,M_\odot$ and a radius of 10km.

a. What is the size of the light cylinder for the crab pulsar? (2 points)

b. Show that the rate of rotational energy lost approximately equals the luminosity of the nebula. (3 points)

c. The energy per second emitted by a rotating magnetic dipole is

$$\frac{dE}{dt} = -\frac{32\pi^5 B^2 R^6 \sin^2\theta}{3\mu_0 c^3 P^4}.$$

Assuming that the rotational kinetic energy lost by the star is carried away by magnetic dipole radiation, derive an equation for the magnetic field at the pole of the neutron star. Use the parameters for the Crab pulsar to obtain a value of the magnetic field in Teslas. (2 points)

c. Discuss and explain the properties of various classes of pulsars. Sketch the $P\dot{P}$ diagram, show how it is populated by different classes of pulsars, and explain how we use this diagram to infer the magnetic field and age of these objects. (3 points)

Aug 2012

Astro #2

a) * See Fig 16.26 of B.O.B (pg 600) for description of light cylinder

$$R_c = \frac{cP}{2\pi}$$

$$= \frac{3\cdot 10^{10}\,\frac{cm}{s}\cdot(.0333\,s)}{2\pi}$$

$$= 1.59\cdot 10^{8}\,cm$$

b) $\dot{P} = 4.21\cdot 10^{-13}\,s$

$$RE = \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} I \left(\frac{P}{2\pi}\right)^{-2}$$

$$= \frac{2}{5} MR^2 \left(\frac{P}{2\pi}\right)^{-2}$$

$$= \frac{8\pi^2 MR^2}{5P^2}$$

$$[E] = \frac{g\,cm^2}{s^2} \quad [\dot{E}] = \frac{g\,cm^2}{s^3}$$

$$\dot{E} = \frac{8\pi^2 MR^2}{5}\left(-2\frac{\dot{P}}{P^3}\right)$$

$$= \frac{-16\pi^2 MR^2\dot{P}}{5P^3}$$

$$= \frac{-16\pi^2(1.4\cdot 2\cdot 10^{33}g)(1\cdot 10^6 cm)^2(4.21\cdot 10^{-13})}{5\ (0.0333\,s)^3}$$

$$= -1.01\cdot 10^{39}\,\frac{g\,cm^2}{s^3}$$

$$L = 5\cdot 10^{31}\,W$$

$$= 5\cdot 10^{31}\,\frac{kg\,m^2}{s^3}$$

$$= 5\cdot 10^{38}\,\frac{g\,cm^2}{s^3}$$

$\dot{E} \approx 2L \Rightarrow$ close enough, off by factor of 2 somewhere

Aug 2012

#2 (cont.)

c) $\frac{dE}{dt} = \frac{-32\pi^5 B^2 R^6 \sin^2\theta}{3\mu_0 c^3 P^4}$

$[T] = \frac{N}{A\,m}$ ← $\frac{kg\,m}{s^2}$

$[\mu_0] = 4\pi \cdot 10^{-7}\ \frac{N}{A^2}$

$\Rightarrow\ B = \left[\frac{3\dot{E}\mu_0 c^3 P^4}{-32\pi^5 R^6 \sin^2\theta}\right]^{1/2}$ ← B.O.B says assume $\theta = 90°$

↳ "light house assumption"

$B_{pole} = \left[\frac{3\left(-1.01\cdot 10^{32}\ \frac{Nm}{s}\right)\left(4\pi\cdot 10^{-7}\ \frac{N}{A^2}\right)\left(3\cdot 10^8\ \frac{m}{s}\right)^3 (0.0333\ s)^4}{-32(\pi)^5 (10\cdot 10^3\ m)^6 \sin^2(90°)}\right]$

$= \frac{\frac{N^2 m^4}{A^2}}{m^6} \rightarrow \frac{N^2}{m^2 A^2}$

$= 1.14\cdot 10^9\ T$ ($8\cdot 10^8\ T$ to account for extra factor of 2 error)

d) See pg 601 of B.O.B for diagram; info sparse

Kilic

PROBLEM 3

In this problem you are asked to discuss and compare two types of H fusion which occur in stars along with the chemical evolution of nitrogen.

a. (1 point) Write down the three reaction steps in the PPI reaction. Show all isotopes and bi-products involved.

b. (1 point) Write down the six reactions in the CN cycle. Show all isotopes and bi-products involved. Identify the two relatively fast reactions and the slowest reaction of the six. Why is carbon referred to as a catalyst?

c. (2 points) Make a qualitative comparison of PPI and the CN cycle in terms of the threshold temperature and temperature sensitivity of the energy generation coefficient ϵ, i.e., $d\epsilon/dT$. Discuss the relative amount that each cycle contributes to the total energy generation in the Sun's core.

d. (3 points) Explain the relevance of the CN cycle to the evolution of the total nitrogen abundance in a galaxy. Explain what stellar types (mass ranges) are thought to produce significant amounts of N.

e. (3 points) The nearby figure shows the universal behavior of the N/O abundance ratio as a function of metallicity, as measured by O/H. Note the flat behavior at low metallicities and the upward turn starting at around solar metallicity of about 8.7. Explain this change in slope.

Aug 2012

Astro #3

a) P-P I Chain

$$H^1_1 + H^1_1 \rightarrow H^2_1 + \nu + e^+ + \gamma$$
$$H^2_1 + H^1_1 \rightarrow He^3_2 + \gamma$$
$$\Big\} \times 2$$
$$He^3_2 + He^3_2 \rightarrow He^4_2 + 2H^1_1$$

b) CN cycle

$$C^{12}_6 + H^1_1 \rightarrow N^{13}_7 + \gamma$$

*Decay rxns are quickest

$$N^{13}_7 \rightarrow N^{13}_6 + e^+ + \nu$$
$$C^{13}_6 + H^1_1 \rightarrow N^{14}_7 + \gamma$$
$$N^{14}_7 + H^1_1 \rightarrow O^{15}_8 + \gamma$$

← limiting rxn

$$O^{15}_8 \rightarrow N^{15}_7 + e^+ + \nu$$
$$N^{15}_7 + H^1_1 \rightarrow C^{12}_6 + He^4_2$$

Carbon is referred to as a catalyst b/c while it is part of the rxn chain, the abundance of it is uneffected by the rxn itself (1 C^{12}_6 at both beginning and end)

c) P-P I chain: $\sim 1 \cdot 10^7$ K needed to start rxn, $E_{PP} \propto T^4$

CN cycle: $\sim 1.5 \cdot 10^7$ K needed to dominate over PP I chain, $E_{CN} \propto T^{18}$

W/in the sun, most energy produced via PP chain

Dai? / Leighly?

PROBLEM 4

a. (3 points) The hot gaseous halo of galaxy clusters is pressure supported, and thus it follows the hydrostatic equilibrium equation. Write down the hydrostatic equilibrium equation and the ideal gas law.

b. (4 points) First derive the following equation

$$\frac{dP}{dr} = \frac{k}{\mu m_H}\left(T\frac{\partial \rho}{\partial r} + \rho\frac{\partial T}{\partial r}\right), \tag{1}$$

assuming that the mean molecular weight is a constant for the gas. Then derive the expression for the total gravitational mass

$$M = -\frac{kTr}{\mu m_H G}\left(\frac{\partial \ln \rho}{\partial \ln r} + \frac{\partial \ln T}{\partial \ln r}\right). \tag{2}$$

c. (3 points) Using a so-called β model for the gas density and assuming that the gas is isothermal, the expression for M can be written as

$$M = \frac{3\beta kTr}{\mu m_H G}\left(\frac{r^2}{r^2 + r_c^2}\right). \tag{3}$$

For a cluster with a temperature of T = 5 keV, a core radius r_c=400 kpc, mean molecular weight $\mu = 0.61$, and $\beta = 0.7$, find the total mass of the cluster within 2 Mpc.

Aug 2012

Astro #4

a) $PV = nRT$ (Ideal Gas Law)

$= NkT$

$$\frac{dP}{dr} = -\frac{GM}{r^2}\rho \quad \text{(HSE eqn)}$$

b) Rewrite Ideal gas law as

$$P = \frac{\rho}{m_H \mu} kT$$

$$\frac{dP}{dr} = \frac{k}{\mu m_H}\frac{d}{dr}(\rho T)$$

$$= \frac{k}{\mu m_H}\left(\rho \frac{\partial T}{\partial r} + T\frac{\partial \rho}{\partial r}\right)$$

* Rewriting HSE eqn

$$\frac{r^2 dP}{-G\rho dr} = m$$

$$\Rightarrow m = \frac{r^2}{-G\rho}\left(\frac{k}{\mu m_H}\left[\rho\frac{\partial T}{\partial r} + T\frac{\partial \rho}{\partial r}\right]\right)$$

$$= -\frac{kTr}{\mu m_H G}\left(\frac{r}{T}\frac{\partial T}{\partial r} + \frac{r\partial \rho}{\rho \partial r}\right)$$

$$= -\frac{kTr}{\mu m_H G}\left(\frac{\partial \ln \rho}{\partial \ln r} + \frac{\partial \ln(T)}{\partial \ln r}\right)$$

c) * β assumes $\beta = \frac{P_{gas}}{P_{tot}} \Rightarrow P_{gas} = \beta P_{tot}$

$$P = \frac{1}{\beta}\frac{\rho}{m_H \mu} kT, \quad T = \text{const.}$$

$$\Rightarrow m = \frac{r^2}{-G\rho}\frac{dP}{dr}$$

$$= \frac{r^2}{-G\rho}\frac{d}{dr}\left[\frac{1}{\beta}\frac{\rho}{m_H \mu}kT\right] \; ??$$

Aug 2012

#4 (cont.)

$*$ Note: $1\,eV = 11{,}600\,K$

c) $*$ Calculation only

$$m = \frac{3\beta kTr}{\mu m_H G}\left(\frac{r^2}{r^2+r_c^2}\right)$$

$$= \frac{3(0.7)(1.38\cdot10^{-7}\,\frac{erg}{K})(5\,keV)(2\,Mpc)}{(0.61)(1.66\cdot16^{-24}\,g)(6.67\cdot10^{-8}\,\frac{cm^3}{g s^2})}\left[\frac{(2\,Mpc)^2}{(2\,Mpc)^2+(400\,kpc)^2}\right]$$

$$= \frac{2.1(1.38\cdot10^{-7}\,\frac{g\,cm^2}{s^2 K})(5000\cdot11{,}600\,K)(2\cdot10^6\cdot3.1\cdot10^{18}\,cm)}{(0.61)(1.66\cdot10^{-24}\,g)(6.67\cdot10^{-8}\,\frac{cm^3}{\cancel{g}\,s^2})}\left[\frac{(2\cdot10^6)^2}{(2\cdot10^6)^2+(4\cdot10^5)^2}\right]$$

$$= 1.48\cdot10^{57}\,g$$

Wang

PROBLEM 5

Inflationary paradigm is an integral part of modern cosmology.

a. Why do we need inflation in the extremely early universe? (2 pts)

b. What condition on the equation of state must be satisfied for inflation to occur? Explain. (2 pts)

c. Use the conservation of energy and momentum to derive the condition in (b). (6 pts)

Eddie

PROBLEM 6

a. (2 pts) Write down the equation of radiative transfer for a plane-parallel atmosphere and define all the terms.

b. (3 pts) Assuming that there is no external irradiation at the surface, show that

$$I_\lambda = S_\lambda(1 - e^{-\tau_\lambda})$$

c. (5 pts) What is I_λ in terms of S_λ for the optically thin and optically thick cases? Do you expect to see emission or absorption lines at the wavelengths of large opacity, κ_λ?

Aug 2012

#6

a) Write down equation of radiative transfer in plane-parallel + define all terms

$$\mu \frac{dI_\nu}{dz} = \eta_\nu - \chi_\nu I_\nu$$

$\mu = \cos \Theta$

η_ν = scattering coefficient

χ_ν = opacity

I_ν = intensity

b) Assuming there is no external irradiation at the surface, show: $I_\lambda = S_\lambda(1-e^{\tau_\lambda})$

$$\frac{dI_\lambda}{dz} = \eta_\lambda - \chi_\lambda I_\lambda$$

$$\frac{dI_\lambda}{\chi_\lambda dz} = \frac{\eta_\lambda}{\chi_\lambda} - I_\lambda$$

* let $d\tau = \chi_\lambda dz$, $\frac{\eta_\lambda}{\chi_\lambda} = S_\lambda$

$$\frac{dI_\lambda}{d\tau} = S_\lambda - I_\lambda$$

$$\left(\frac{dI_\lambda}{d\tau} + I_\lambda = S_\lambda\right) e^{\tau}$$

$$\frac{dI_\lambda}{d\tau} e^{+\tau} + I e^{+\tau} = S_\lambda e^{\tau}$$

$$\frac{d I e^{\tau}}{d\tau} = S_\lambda e^{\tau}$$

$$-I_0 + I e^{\tau} = S_\lambda [e^{\tau_\lambda} - 1]$$

$$I_\lambda = S_\lambda(1 - e^{-\tau})$$

Aug 2012

#6 (cont.)

c) Optically Thin $\rightarrow$ $\tau \ll 1$

$$I_\lambda = S_\lambda (1 - e^{-\tau_\lambda})$$

$$= S_\lambda (1 - [1 - \tau_\lambda + \tfrac{1}{2}\tau_\lambda^2 + \dots])$$

$$= [+\tau_\lambda - \tfrac{1}{2}\tau_\lambda^2 + \tfrac{1}{6}\tau_\lambda^3 - \dots] S_\lambda$$

$$= S_\lambda \tau_\lambda - \tfrac{1}{2}\tau_\lambda^2 S_\lambda$$

Optically Thick $\rightarrow$ $\tau \gg 1$

$$I_\lambda = S_\lambda (1 - \cancelto{0}{e^{-\tau_\lambda}})$$

$$I_\lambda = S_\lambda$$

In the optically thick case, you would expect to only see the source function. In the optically thin case, you would expect to see emission lines.

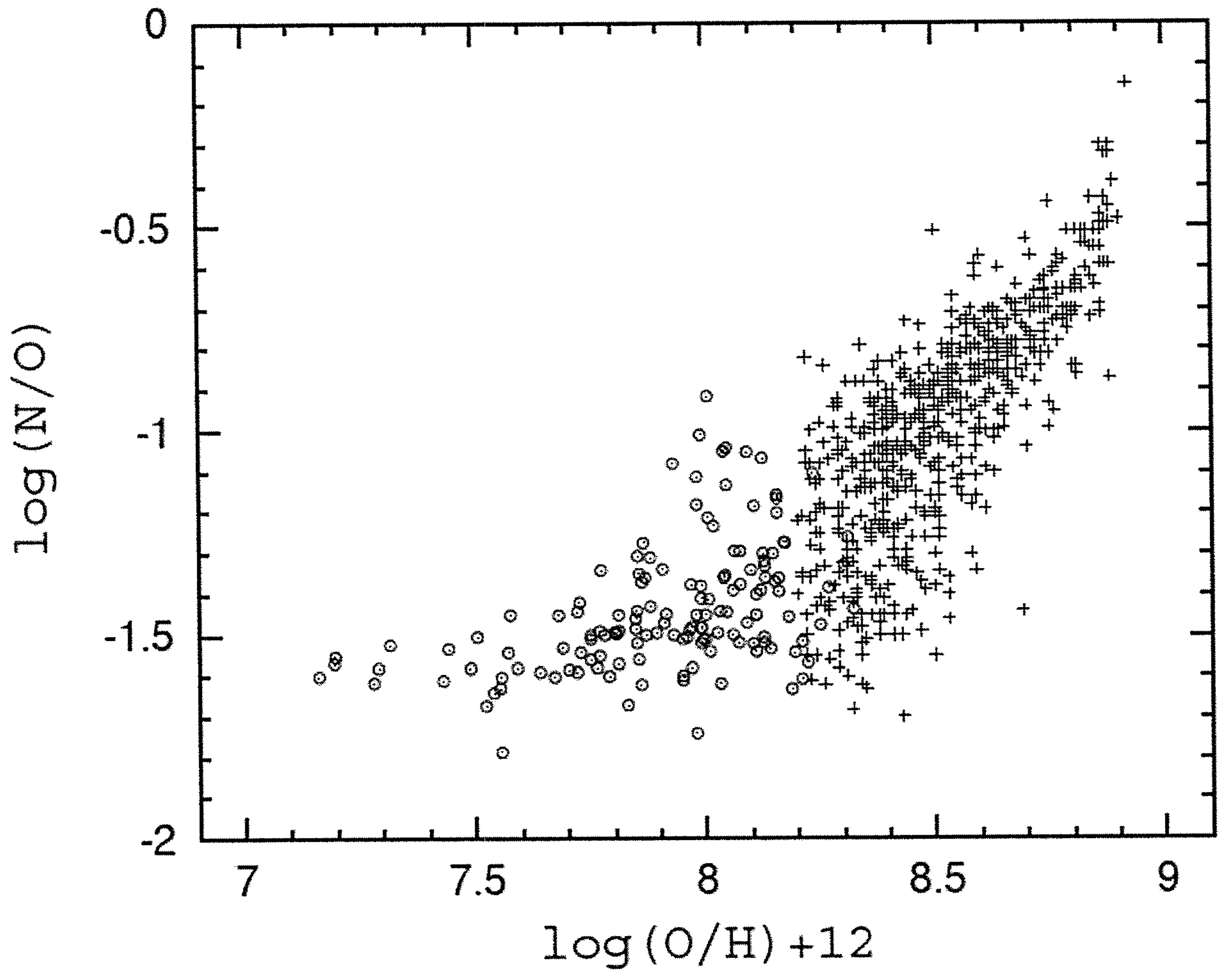


Fig. 4. The N/O–O/H for H II regions in spiral (pluses) and irregular (circles) galaxies.

ASTRONOMY QUALIFYING EXAM
January 2013

Possibly Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \times 10^{10}$ cm
1 AU = 1.5×10^{13} cm
1 pc = 3.26 Ly. = 3.1×10^{18} cm
$a = 7.56 \times 10^{-15}$ erg cm^{-3} K^{-4}
$c = 3 \times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
$k = 1.38 \times 10^{-16}$ erg K^{-1}
$e = 4.8 \times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
$G = 6.67 \times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
$h = 6.63 \times 10^{-27}$ erg s
1 amu = $1.66053886 \times 10^{-24}$ g

Dai? / Leighly?

PROBLEM 1

a. (7 points) Assume that the gas component of a galaxy, with a mass fraction f_g, is virialized and follow the overall density profile of the galaxy, a singular isothermal sphere mass profile,

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2}, \tag{1}$$

where σ is the velocity dispersion of the galaxy. The galaxy has a central AGN, which is radiating at the Eddington luminosity,

$$L_{Edd} = 4\pi G c m_p M_{BH}/\sigma_T. \tag{2}$$

A fraction, f_w, of the energy radiated by the central AGN is deposited to the gas in the form of kinetic energy. This kinetic "feedback" energy from the AGN can drive the gas in the host galaxy to flow outward. Assume that the final gas outflow is in a spherical shell with a constant velocity, v, and half of the kinetic feedback energy is converted to the gravitational potential of the gas and the other half to the kinetic energy of the gas during the outflowing process. Use the conservation or transfer of energy to show that the final gas wind speed is

$$v^3 = \frac{G L_{Edd} f_w}{2\sigma^2}. \tag{3}$$

b. (3 points) If the wind speed is large enough to escape the potential well of the galaxy ($v = \sigma$), the central AGN will blow out the majority of gas in the galaxy and terminate the formation of stars. Show that this gives us the M_{BH}–σ relation,

$$M_{BH} = \frac{1}{2\pi} \frac{\sigma_T}{G^2 c m_p} \frac{1}{f_w} \sigma^5, \tag{4}$$

where G is the gravitational constant, σ_T is the Thomson cross section, c is the speed of light, and m_p is the mass of a proton.

Jan 2013

Astro #1

a) $\rho(r) = \frac{\sigma^2}{2\pi G r^2}$

$$M_G = \int \rho(r)\, dV$$
$$= \int_0^R \int_0^{2\pi} \int_0^{\pi} \frac{\sigma}{2\pi G r^2} r^2 \sin\theta \, dr\, d\theta\, d\varphi$$
$$= \int_0^R dr \int_0^{2\pi} d\varphi \int_0^{\pi} \frac{\sigma^2 \sin\theta}{2\pi G} d\theta$$
$$= \int_0^R dr \int_0^{2\pi} d\varphi \frac{\sigma^2}{2\pi G} \left(-\cos\theta \Big|_0^{\pi} \right)$$
$$-(-1) - -1$$
$$= \int_0^R dr \int_0^{2\pi} \frac{\sigma^2}{\pi G} d\varphi$$
$$= \int_0^R \frac{2\sigma^2}{G} dr$$
$$M_G = \frac{2\sigma^2 R}{G}$$

$$\Rightarrow \frac{1}{2} m_g v^2 = \frac{1}{2} f w h e d a t$$

Jan 2013

#1 (cont.)

b) $$v^3 = \frac{G L_{edd} f_w}{2\sigma^2}$$

$$L_{edd} = \frac{4\pi G c m_p M_{BH}}{\sigma_T}$$

* Assuming $v = \sigma$

$$\sigma^3 = \frac{G f_w}{2\sigma^2}\left(\frac{4\pi G c m_p M_{BH}}{\sigma_T}\right)$$

$$\frac{2\sigma^5 \sigma_T}{4\pi G^2 c m_p f_w} = M_{BH}$$

$$\hookrightarrow M_{BH} = \frac{\sigma_T \sigma^5}{2\pi G^2 c m_p f_w}$$

Wang

PROBLEM 2

The Universe is dominated by dark energy today, but for a rough estimate of the age of the Universe at $2 < z < 100$, we can assume a matter dominated universe.

The Friedman Equation is

$$\dot{R}^2 + k = \frac{8\pi G}{3}\rho R^2, \tag{5}$$

where

$$\frac{8\pi G}{3}\rho = \Omega H^2. \tag{6}$$

(1) Derive the formula for the age of a matter-dominated universe at redshift z, assuming that we know t_0 (the age of the Universe today). (5 pts)

(2) What is the current measurement of t_0? Estimate the age of the Universe at $z = 10$ using this information. (2 pts)

(3) How does dark energy change the age of the Universe today, compared to a flat universe with matter only? (3 pts)

Leighly?

PROBLEM 3

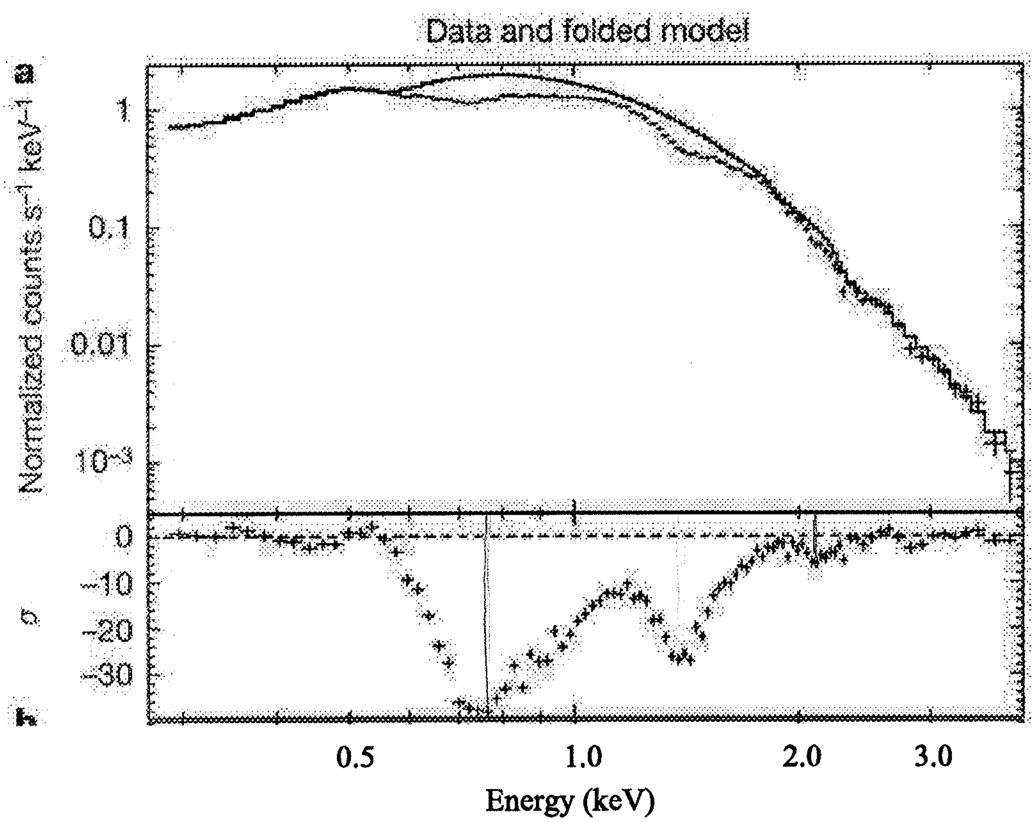
a. An electron in an electromagnetic field will experience a Lorentz force. Write down the equation for the Lorentz force. (2 points).

b. Consider an electron in a uniform magnetic field with a velcocity v. What is the frequency of light emitted by this electron if the velocity vector is oriented perpendicular to the magnetic field lines? (2 points)

c. The figure below shows the X-ray spectrum of an isolated neutron star. Direct your attention to the lower panel, which shows the difference between the spectrum and a blackbody continuum model. Three (possibly 4) absorption lines are seen. Please estimate the frequencies (in Hz) of these absorption lines. Which one is the fundamental frequency and which are harmonics? (2 points)

d. Estimate the magnetic field strength, in gauss, of the neutron star, ignoring general relativistic effects. (2 points)

e. Neutron stars are very compact, and general relativity should not be ignored. GR will affect the frequency of the absorption feature. Will the real feature have a higher frequency or lower frequency than estimated in part (d)? Explain. (2 points)



Jan 2013

Astro #3

a) $F = q\left(E + \frac{v}{c} \times B\right)$

b)

Henry

PROBLEM 4

Briefly define and discuss the relevance of the following terms to modern astronomy. 1 point per question

1. Cepheid variable star
2. Initial mass function
3. tunneling in the context of the PPI chain reaction
4. age-metallicity relation
5. damped Lyα system (DLA)
6. s-process
7. G dwarf problem
8. Tully-Fisher relation
9. Galactic thin disk
10. isophotal radius

Jan 2013

Astro #4

a) A Cepheid Variable star is a type of pulsating star that follows the period-luminosity relation, which states that the longer the star's period of pulsation, the more luminous the Cepheid is. By calibrating this relationship using parallax techniques to determine the distances to nearby Cepheids, the period-luminosity relation allows us to determine the distance to these sphecial class of stars simply by knowing how bright they are. Cepheids make up one rung of the astronomical distance ladder.

b) The initial mass function (IMF) of a galaxy is an attempt to estimate the amount of stars that will form in a galaxy of a certain mass, within a certain volume

c) In the context of the PP I chain, tunnelling is the ability of charged particles to overcome the coulomb barrier b/w them and begin the fusion process. Without the ability of the particles to tunnel, the hydrogen atoms would not be able to this barrier (at any velocity) and fusion would not occur. This quantum mechanical property of these H atoms is what allows stars to exist, as w/o fusion, the stars would only illuminate themselves by conversion of gravitational potential energy + would be unable to sustain themselves for long periods of time.

d) The age-metallicity relation is the correlation b/w the ages of stars in a galaxy and their chemical compositions. As early, massive stars die, they chemically enrich the surrounding medium. Therefore, as newer stars form from these gas clouds they contain a higher percentage of fusion byproducts like carbon and iron (metals) than stars that formed earlier. Therefore, when we look at stars of similar types, stars with higher metallicities must be younger than stars w/ lower metallicities. This provides us w/ a rough estimate of stellar ages w/in a galaxy.

Jan 2013

#4 (cont.)

e) Damped Lyman-α systems are concentrations of neutral hydrogen gas associated w/ quasars at high-z. It is believed that these systems contain most of the neutral hydrogen in the universe, and that they are correlated w/ the early stages of galaxy formation. Therefore study of the dynamics of these systems may allow us to better understand galaxy formation mechanisms.

f) The s-process is a nuclear fusion process by which elements heavier than Fe can be formed. In regions w/ a low neutron flux, like stars undergoing neutron cooling or SN remnants, the atom will fuse with the neutron and create an unstable particle, where the neutron will eventually β-decay, increasing the # of protons in the nucleus of the atom. Non-radioactive heavy elements can be formed by this process.

g) The G-dwarf problem arises from a discrepancy b/w theory + observation of stars in the solar neighborhood. Current models of chemical enrichment in the galaxy suggest that we should see many more G/F class stars w/ metallicities close to 0 than we do. This suggests that there was another method of chemical enrichment that occured earlier in the galaxy formation process

h) The Tulley-Fisher Relation is a relationship that exists b/w the luminosity and maximal rotation velocity of a spiral galaxy. Because of this relationship, it can be used as a rung on the distance ladder for nearby galaxy's.

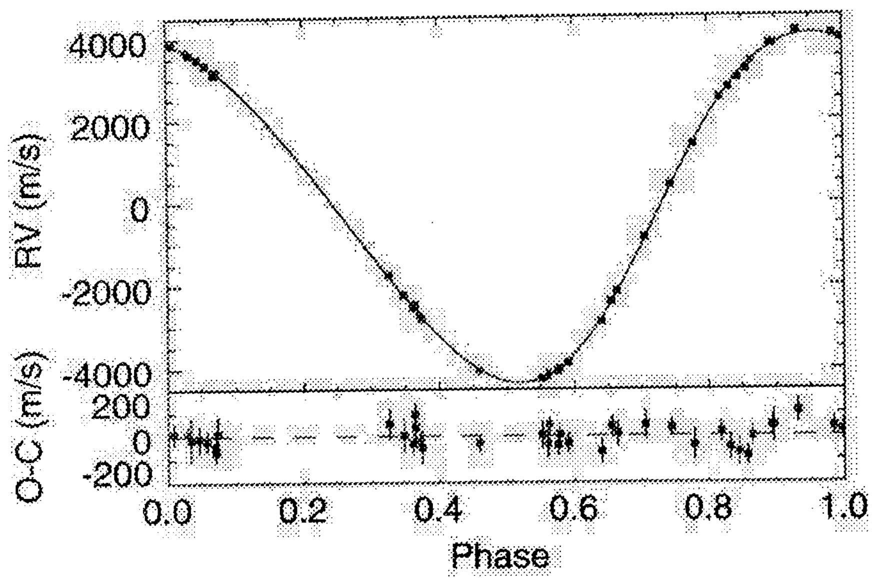
i) The galactic thin disk is the region w/in the disk of a spiral galaxy close to the mid-plane of the disk where most star formation occurs. Stars w/in the thin disk are younger and will over time drift away as their specific velocities carry them away from the mid-plane. Stars w/in the thin disk typically have higher metallicities according to the age metallicity relation.

j) The isophotal radius is a measurement of the approximate size of a galaxy. Near the edges of a galaxy, its luminosity becomes low, making an exact measurement of its size hard. Typically, the isophotal radius are measured by a % of the night-sky background brightness

John

PROBLEM 5

The following radial velocity phase curve is observed for a companion orbiting a star. Assume e=0 and P=79 days:



a. (7 pts) Derive a general expression for the companion mass.

b. (1 pt) What is the minimum mass of the companion, assuming the host star is a Solar analog?

c. (1 pt) What is the semi-major axis, a, of the companion in AU? Assume sini = 1 and the host star is a Solar analog.

d. (1 pt) The companion is observed to transit the primary star, producing a 2% drop in flux. Assuming the primary is a Solar analog, what is the radius of the companion in R_{Sun}?

Jan 2013

Astro #5

* See pg 184 - 188 B.O.B

a) $e = 0 \rightarrow$ circular orbit

$P = 79$ days

$$P^2 = \frac{4\pi^2}{G(m_1+m_2)} a^3$$

* m_1 is mass of star, m_2 mass of companion

$$\left(\frac{G P^2}{4\pi^2 a^3}\right)^{-1} - m_1 = m_2$$

$$P = \frac{2\pi a}{V} \rightarrow \frac{PV}{2\pi} = a$$

* but $V = \frac{V_r}{\sin i}$

$$\Rightarrow a = \frac{P V_r}{2\pi \sin i}$$

$$\Rightarrow \frac{4\pi^2 \left(\frac{P^3 V_r^3}{(2\pi)^3 \sin^3 i}\right)}{G P^2} - m_1 = m_2$$

$$\frac{V_r^3 P}{2\pi G \sin^3 i} - m_1 = m_2$$

b) * for minimum mass $\rightarrow \sin^3 i = 1$

$$\frac{V_r^3 P}{2\pi G} - m_1 = m_2$$

$$\frac{\left(4000 \frac{m}{s}\right)^3 \left(3.15 \cdot 10^7 \frac{s}{yr} \cdot 79\, yr\right)}{2\pi \left(6.67 \cdot 10^{-8}\right) \frac{cm^3}{g s^2}} - 2 \cdot 10^{33} g =$$

Eddie

PROBLEM 6

1. (4 pts) Show that the formal solution of the plane-parallel radiative transfer equation can be written:

$$I_\nu(\tau_1,\mu) = I_\nu(\tau_2,\mu)e^{-(\tau_2-\tau_1)/\mu} + \frac{1}{\mu}\int_{\tau_1}^{\tau_2} S_\nu(t)e^{-(t-\tau_1)/\mu}\, d\cancel{\mu}\tau \tag{7}$$

   where $S_\nu(t)$ is the source function, $\tau_{1,2}$ are optical depth points in the atmosphere, and μ is the cosine of the angle of the ray.

2. (2 pts) Apply Eqn. 7 to an arbitrary point in the atmosphere of a semi-infinite slab to find:

$$I_\nu(\tau,\mu) = \int_\tau^\infty S_\nu(t)e^{-(t-\tau)/\mu}\, dt/\mu \quad \text{for } 0 \le \mu \le 1 \tag{8}$$

$$I_\nu(\tau,\mu) = \int_0^\tau S_\nu(t)e^{-(\tau-t)/(+\mu)}\, dt/(+\mu) \quad \text{for} -1 \le \mu \le 0 \tag{9}$$

3. (2pts) Integrate Eqns 8 and 9 over angle to find

$$J_\nu(\tau) = 1/2\left[\int_\tau^\infty dt\, S_\nu(t)\int_1^\infty dw\, e^{-w(t-\tau)}/w + \int_0^\tau dt\, S_\nu(t)\int_1^\infty dw\, e^{-w(\tau-t)}/w\right] \tag{10}$$

   These integrals are of standard form (the first exponential integral):

$$E_1(x) = \int_1^\infty e^{-xt}/t\, dt$$

4. (1 pt) Show that in terms of E_1, J may be written:

$$J_\nu(\tau) = 1/2\int_\tau^\infty dt\, S_\nu(t)E_1(|t-\tau|)$$

5. (1 pt) Explain the nature of this final operator.

Jan 2013

#6

a) $$\frac{dI}{ds} = \mu \frac{dI}{dz} = -\kappa_\nu I_\nu + \eta_\nu$$

$$\mu \frac{dI}{-\kappa_\nu dz} = +I_\nu - \frac{\eta_\nu}{\kappa_\nu}$$

$$\mu \frac{dI}{d\tau} = I - S$$

$$\mu \frac{dI}{d\tau} - I = -S$$

$$\frac{dI}{d\tau} - \frac{1}{\mu} I = -\frac{1}{\mu} S$$

$$\frac{dI}{d\tau} e^{-\tau/\mu} - \frac{I}{\mu} e^{-\tau/\mu} = -\frac{1}{\mu} S e^{-\tau/\mu}$$

$$\frac{d(I e^{-\tau/\mu})}{d\tau} = \frac{-1}{\mu} S e^{-\tau/\mu}$$

$$\int_{\tau_1}^{\tau_2} \frac{d(I e^{-\tau/\mu})}{d\tau} = \int_{\tau_1}^{\tau_2} \frac{-1}{\mu} S e^{-\tau/\mu}$$

$$I e^{-\tau/\mu} \Big|_{\tau_1}^{\tau_2} = \int_{\tau_1}^{\tau_2} \frac{-S}{\mu e} {}^{\tau/\mu}$$

$$I(\tau,\mu) e^{-\tau_2/\mu} - I(\tau,\mu) e^{-\tau_1/\mu} = \int_{\tau_1}^{\tau_2} \frac{-S}{\mu} e^{-\tau/\mu}$$

$$I(\tau_1,\mu) e^{-\tau_1/\mu} = I(\tau_2,\mu) e^{-\tau_2/\mu} + \int_{\tau_1}^{\tau_2} \frac{S}{\mu} e^{-\tau/\mu}$$

$$I(\tau_1,\mu) = I(\tau_2,\mu) \exp(-(\tau_2-\tau_1)/\mu) + \frac{1}{\mu} \int_{\tau_1}^{\tau_2} S \exp[-(t-\tau_1)/\mu \, dt$$

b) * At an arbitrary point, incoming rays ($\mu < 0$); outgoing rays ($\mu > 0$)

⇒ Outgoing rays $\tau_1 = \tau$, $\tau_2 = \infty$

$$I(\tau,\mu) = I \exp[\cancel{-(\infty-\tau)/\mu}]^{0} + \frac{1}{\mu} \int_{\tau}^{\infty} S \exp[-(t-\tau)/\mu] dt$$

$$= \frac{1}{\mu} \int_{\tau}^{\infty} S \exp[-(t-\tau)/\mu] d\tau$$

#6 (cont.)

b) $\Rightarrow$ for incoming rays $\tau_1 = 0$ $\tau_2 = \tau$

$$I(\tau,\mu) = \int_0^{\tau} S(t)\exp[-(t-\tau)/-\mu]\frac{dt}{-\mu} + I_\nu(\infty,-\mu)\cancel{e^{-(\infty-0)/\mu}}^{\,0}$$

c) $$J_\nu(\tau) = \frac{1}{2}\int_{-1}^{1} I_\nu(\tau,\mu)\,d\mu$$

$$= \frac{1}{2}\left[\int_{-1}^{0} I_\nu(\tau,\mu)\,d\mu + \int_0^1 I_\nu\,d\mu\right]$$

$$= \frac{1}{2}\left[\int_{-1}^{0}\int_0^{\tau} S_\nu(t)e^{-(\tau-t)/(-\mu)}dt\cdot\frac{1}{-\mu}d\mu + \int_0^1\int_\tau^{\infty} S_\nu e^{-(t-\tau)/\mu}\frac{dt}{\mu}\,d\mu\right]$$

* assuming an isotropic source function

$$= \frac{1}{2}\left[\int_\tau^{\infty} dt\,S_\nu\int_0^1 \exp[-(t-\tau)/\mu]\frac{1}{\mu}d\mu + \int_0^{\tau} dt\,S_\nu\int_{-1}^{0}\exp[-(\tau-t)/+\mu]\frac{d\mu}{+\mu}\right]$$

* let $x = t-\tau$ $\qquad x' = \tau - t$

$y = \frac{1}{\mu} \rightarrow dy = \frac{-1}{\mu^2}d\mu \rightarrow y^{-2}dy = d\mu \qquad y = \frac{-1}{\mu}$

d) $$= \frac{1}{2}\left[\int_\tau^{\infty} S_\nu dt\int_{\infty}^{1} -\exp[-xy]\frac{dy}{y} + \int_0^{\tau} dt\,S_\nu\int_{+1}^{+\infty}\exp[-xy]\frac{dy}{y}\right]$$

$$= \frac{1}{2}\left[\int_\tau^{\infty} dt\,S_\nu\int_1^{\infty}\exp[-xy]\frac{dy}{y} + \int_0^{\tau} dt\,S_\nu\int_1^{\infty}\exp[-xy]\frac{dy}{y}\right]$$

$$= \frac{1}{2}\left[\int_\tau^{\infty} dt\,S_\nu\,E_1(t-\tau) + \int_0^{\tau} dt\,S_\nu\,E_1(\tau-t)\right]$$

$$= \frac{1}{2}\int_0^{\tau_{max}} dt\,S_\nu\,E_1(|t-\tau|)$$

e) Known as Λ-operator

ASTRONOMY QUALIFYING EXAM
August 2013

Possibly Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \times 10^{10}$ cm
1 AU = 1.5 $\times 10^{13}$ cm
1 pc = 3.26 Ly. = 3.1 $\times 10^{18}$ cm
a = 7.56 $\times 10^{-15}$ erg cm^{-3} K^{-4}
c = 3 $\times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
k = 1.38 $\times 10^{-16}$ erg K^{-1}
e = 4.8 $\times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
G = 6.67 $\times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
h = 6.63 $\times 10^{-27}$ erg s
1 amu = 1.66053886 $\times 10^{-24}$ g

Henry

PROBLEM 1

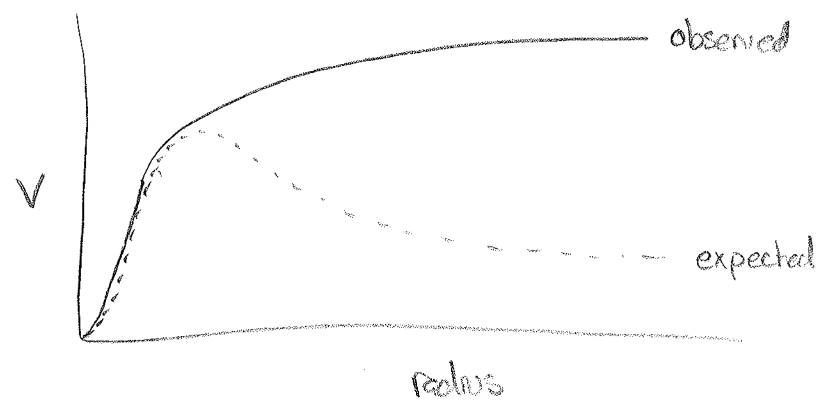
Briefly discuss the observational evidence for the following components of the Milky Way Galaxy. 1 point per question

1. dark matter
2. thin disk and thick disk
3. bulge
4. halo
5. cold ISM
6. hot ISM
7. magnetic field and cosmic rays
8. central black hole
9. population I and population II stars
10. accretion of dwarf galaxies

Aug 2013

Astro #1

a) Dark Matter - evidence from galactic rotation curves



b) Thin + Thick disk - Differences in stellar populations at various scale heights w/in the disks of spiral galaxies. Differences includes: specific velocity, metallicity, stellar density, stellar age

c) Bulge - Seen in COBE infrared maps of galaxy

d) Halo - Distribution of globular clusters and field stars w/ high specific velocities suggests there is a halo of stars spherically surrounding the galaxy. We also know that there must be a dark matter halo to explain the motions of stars outside of the solar radius R_0 w/in the galaxy

e) Cold ISM -

Wang

PROBLEM 2

Dark energy was discovered using the observations of Type Ia supernovae (SNe Ia).

(1) The measurement of X (using SN Ia observations) led to the discovery of the existence of dark energy. What is X? (2 pts)

(2) Express X in terms of the cosmological parameters that describe our Universe. Explain in as much detail as you can. (2 pts)

(3) If the peak brightness of a very large sample of SNe Ia has an observational uncertainty of 0.05 mag, and an intrinsic uncertainty of 0.12 mag, what is the resultant uncertainty in X? (3 pts)

(4) What are the systematic uncertainties of SNe Ia as a dark energy probe? How can these be mitigated? Explain in as much detail as you can. (3 pts)

Aug 2013

Astro #2

a) The Hubble parameter H_0

Dai?

PROBLEM 3

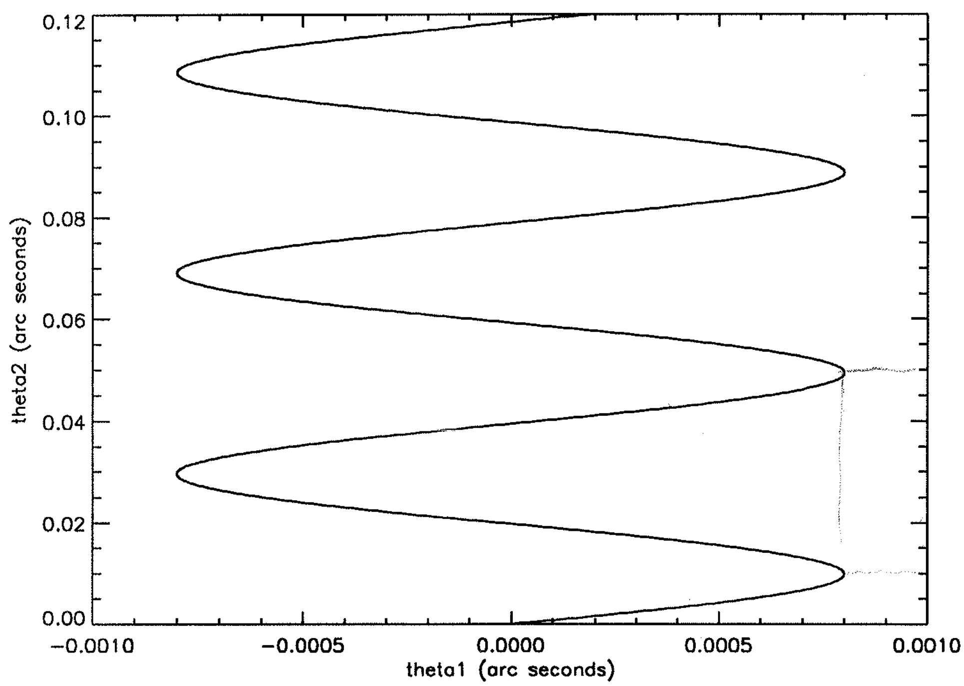
A new pulsar is discovered. It is observed to have a period of 1.3 seconds. It is observed for several years, and its motion on the sky is shown in the plot below, where the axes are orthogonal.

a. What is the distance to the pulsar in parsecs? (3 points).

b. What is the minimum velocity of the pulsar in km/s? (3 points).

c. What is maximum amplitude of the period variability observed during the monitoring time period in seconds? (3 points).

d. What is the size of light cylinder in km? (1 point)



Aug 2013

Astro #3

a) $d = \frac{1}{\alpha}$ pc

$= \frac{1}{.008}$

$= 125$ pc

Kilic

PROBLEM 4

a) [3 pts.] Describe the Pre Main Sequence (PMS) contraction of a 1 $M_\odot$ gas cloud up to the ZAMS stage. Draw the path in the HR diagram. What part of the HR diagram is this? Why is the path here and not in some other part of the HR diagram? Relate what is happening inside of the PMS object to its observable parameters in the HR diagram.

b) [3 pts.] Now describe the evolution of a 5 $M_\odot$ star from the time it arrives on the main sequence until it reaches the top of the second giant branch (or AGB). In particular, give the position in the HR diagram at various stages. Describe in detail the physics of the red giant phase (first ascent of the giant branch). What is the final fate of this star? How do we know?

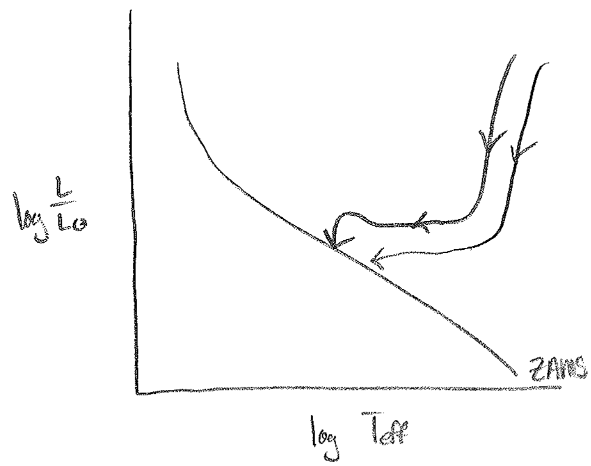
c) [2 pts.] How does the evolution of a 5 $M_\odot$ star differ from that of a 1 $M_\odot$ and 25 $M_\odot$ star? Compare the evolution of a 1 $M_\odot$ star with solar metallicity to that of a 1 $M_\odot$ star with low metallicity (*i.e.* a Pop II star). What are the final fates of stars of 1 and 25 $M_\odot$?

d) [2 pts.] Assuming that $(L/L_\odot) = (M/M_\odot)^\alpha$ where $\alpha = 3$, estimate the time spent on the main sequence for the 1, 5 and 25 $M_\odot$ stars. Describe the structure (*e.g.* the location of the convection and radiation zones) of these three stars while on the main sequence. Do not forget to indicate the energy sources in these stars.

Aug 2013

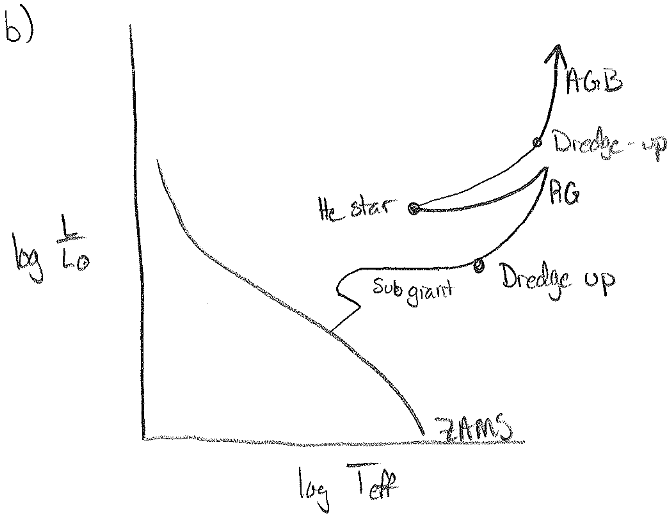
Astro #4

a)



These lines are known as Hyashi tracks, and they describe the pre-Main Sequence (PMS) evolution of a proto star. As the proto-star forms from a gas cloud, it changes from an object that is cool, has a high opacity, + is fully convective, into a warmer object w/ a radiative core.

b)



As the 5 $M_\odot$ star leaves the main sequence, due to its convective core, it must burn up almost all of the H in the core before becoming a shell burning star. Once this transition occurs, the He ash core begins to contract, and the envelope of the star becomes convective and grows. The star is now in the Red Giant (RG) phase of its life. During the phase, the convective enevelope will reach into the core and dredge up material and bring it to the surface, all the while the core continues to contract. Once the central temp reaches ~ 10^8 K, He fusion begins, where the core stops contraction and the envelope shrinks while the star stabilizes as a He-burning star. Once the He in the core is depleted, the C/O core shrinks until it becomes degenerate, while the envelope continues to grow and is eventually expelled from the star. The star ends its life as a C/O white dwarf.

Aug 2013

#4 (cont.)

c) There are only minor differences b/w the evolution of a $1 M_\odot$ star and that of a $5 M_\odot$ star, as they go through the same evolutionary phases, but the transition of the $1 M_\odot$ to a He burning star is different, as the core must first become degenerate before He fusion begins, leading to He flashes instead of a smoother transition. Both end as C/O WD.

However, due to the higher mass of the $25 M_\odot$ star, its evolution is quite different. The $25 M_\odot$ star is massive enough to begin fusing elements all the way up to Fe with very little change in radius or temperature. While there is some mass loss, as the star begins to approch the Eddington limit in the Red supergiant phase, there is still enough mass left to die as a Type II SN.

The effect of metallicity on a star is important, but it only plays a small role in a stars evolution. Due to the additional energy levels to absorb photons provided by the metals in a star, metal rich stars have a higher opacity and appear dimmer than their metal poor companions of a similar mass. Metal poor stars also tend to live shorter lives than metal rich stars.

d) $1 M_\odot$ - radiative core, convective envelope

$5 M_\odot$ + $25 M_\odot$ - convective core, radiative envelope

Energy source of all MS stars is H-fusion and gravitational collapse energy being radiated away

Time on MS: $1 M_\odot$ ~ 10 billion yrs

$5 M_\odot$ ~

$25 M_\odot$ ~

Dai? / Leighly?

PROBLEM 5

a. (2pts) Draw a typical velocity rotation curve for a spiral galaxy. What does the observed rotation curve tell us about the matter distribution in spiral galaxies?

b. (3pts) Describe the Tully-Fisher relationship for spiral galaxies and why it is important.

c. (5pts) Assume a spiral galaxy has a mass to light ratio γ. Use the virial theorem to derive an expression for the galaxy's dynamical mass in terms of γ, L, v_c, and R.

$$\frac{M_*}{L} = \gamma \qquad \frac{1}{\cancel{2}} M_* v_c^2 = +\frac{G M_* M}{\cancel{2} R}$$

$$\gamma L = M_* \qquad M_* + DM = M_T$$

$$a_c = \frac{v_c^2}{r} \qquad M_T = \gamma L + \qquad L + = \frac{-GMm}{2r}$$

$$a_c = \frac{G M_T}{R^2} = \frac{v_c^2}{R}$$

$$\frac{GM^2}{r} = M v_c^2$$

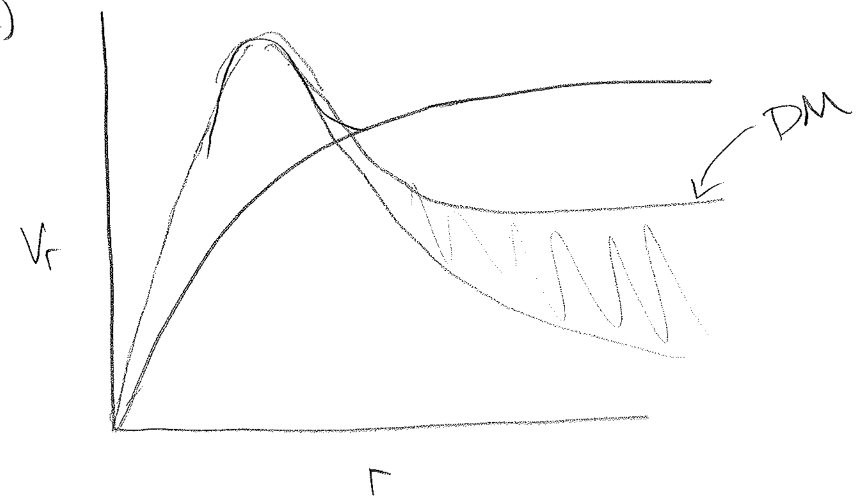
$$M_T = \frac{v_c^2 R}{G} = \gamma L + M_{DM} \qquad \gamma L = M_*$$

$$= \frac{v_c^2 R}{G} \frac{\gamma L}{\gamma L}$$

Aug 2013

Astro # 5

a)



The flat rotation curves of spiral galaxies at large radii tell us that $\rho \propto \frac{1}{r^2}$

The increasing interior part corresponds to rigid body rotation and $\rho \propto$

b) The Tulley - Fisher relationship describes a relation b/w the maximal rotation velocity and luminosity of a spiral galaxy based on the galaxy's Hubble type. By combining this relationship with other measurement it allows us to estimate the masses and distances to spiral galaxies.

c) $\Upsilon = \frac{M}{L}$

* Virial Thm states:

$T = -\frac{1}{2}U$

$E = \frac{1}{2}U$

Eddie? / Henry? / John?

PROBLEM 6

1. (4pts) Define and explain the difference between:
   (a) Effective Temperature
   (b) Excitation Temperature
   (c) Ionization Temperature

   To get full credit you need to use both words and equations.

2. (1pt) In what physical situation are all the temperatures defined above exactly the same?

3. (1pt) When in the history of the Universe are the almost exactly the same?

   The Boltzmann formula is:

$$\frac{n_i}{n_j} = \frac{g_i}{g_j} e^{-(\chi_i - \chi_j)/kT}$$

4. (4pts) Figure 1 shows an energy level diagram for sodium. At what temperature is the *total* population of the levels at 3.6eV equal to the population of the level at 3.2eV?

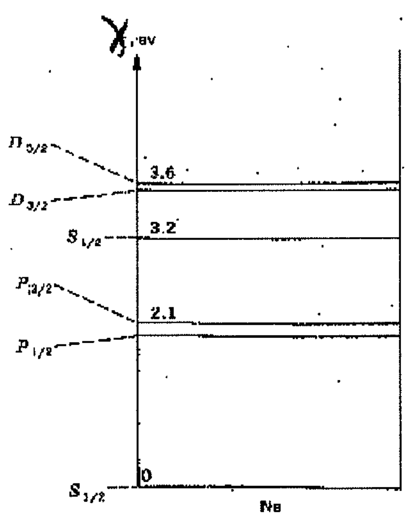


Fig. 1-7 An approximate term diagram for the electronic configuration of the element sodium. The excitation energy above the energy of the ground state is labeled by the quantum numbers of the configuration. The letter designates the orbital angular momentum of the electrons (in this case of a single-valence electron), and the subscript designates the total angular momentum of the states.

Figure 1:

ASTRONOMY QUALIFYING EXAM
August 2014

Possibly Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \times 10^{10}$ cm
1 AU = 1.5×10^{13} cm
1 pc = 3.26 Ly. = 3.1×10^{18} cm
$a = 7.56 \times 10^{-15}$ erg cm^{-3} K^{-4}
$c = 3 \times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
$k = 1.38 \times 10^{-16}$ erg K^{-1}
$e = 4.8 \times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
$G = 6.67 \times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
$h = 6.63 \times 10^{-27}$ erg s
1 amu = $1.66053886 \times 10^{-24}$ g

Kilic? / Henry? / Leighly?

PROBLEM 1

Use the Virial Theorem to:

a) (6 points) Derive the internal temperature of the Sun. How much hotter is this value compared to the Sun's effective surface temperature?

b) (4 points) Derive the Jeans Mass of a molecular cloud that is starting to collapse, thereby starting the star formation process.

Eddie

PROBLEM 2

a) (2 points) Calculate the zeroth and first moments of the plane-parallel radiation transport equation and explain why we can't simply solve the moment equations rather than the original equation.

b) (3 points) Explain the physical content of the grey atmosphere approximation and derive the radiation transport equation, its the zeroth and first moments in that approximation.

c) (5 points) The Rosseland mean opacity is defined as the grey opacity that reproduces the total flux with the following approximations:

1. The pressure is isotropic
2. The radiation field is in LTE

Use parts (a) and (b) to derive the Rosseland mean opacity. What is the meaning of these 2 approximations and when are they valid?

Dai?

PROBLEM 3

a) (2 points) What is the definition of the luminosity function of a class of astronomical objects?

b) (3 points) The Schechter luminosity function is commonly used to model the luminosity function of galaxies, which has the following functional form:

$$\phi(L) = \frac{\phi^*}{L^*}\left(\frac{L}{L^*}\right)^{-\alpha} \exp\frac{-L}{L^*},$$

with three parameters, ϕ^*, α, and L^*. What is the total luminosity of this class of objects? Simplify the formula using the Γ function,

$$\Gamma(x) = \int_0^\infty y^{(x-1)} e^{-y} dy.$$

c) (2 points) For a class of objects with $\phi^* = 0.016\ \mathrm{Mpc}^{-3}$, $\alpha = 0.9$, and $L^* = 10^{10} L_\odot$, what is the total number density of this class of objects ($\Gamma(0.1) = 9.5$)?

d) (3 points) A class of objects, located at a fixed distance, has a Schechter luminosity function with $\alpha = 0.9$. It is composed of two populations, one obscured and one normal. The normal population contributes to 70% of the total population intrinsically. The obscured population is dimmed by a factor of two by intervening obscuration.

A survey has a limit to detect the L^* objects of the normal population. Considering the two sub-classes of objects detected in this survey, what is the observed fraction of obscured objects to the total number of detections? Express the answers using the incomplete Γ function,

$$\Gamma(a, x) = \int_a^\infty y^{(x-1)} e^{-y} dy.$$

John?

PROBLEM 4

Sirius is a visual binary with a period of 49.94 yr. Its measured trigonometric parallax is 0.37921 ± 0.00158 arcsec and, assuming that the plane of the orbit is in the plane of the sky, the true angular extent of the semimajor axis of the reduced mass is 7.61 arcsec. The ratio of the distances of Sirius A and Sirius B from the center of mass is $a_A/a_B = 0.466$.

a) (3 points) Find the mass of each member of the system.

b) (3 points) The absolute bolometric magnitudes of Sirius A and Sirius B are 1.36 and 8.79, respectively. Determine their luminosities. Express your answers in terms of the luminosity of the Sun.

c) (2 points) The effective temperature of Sirius B is 24,790 K. Estimate its radius, and compare your answer to the radii of the Sun and Earth.

d) (2 points) Estimate the surface gravity of Sirius B in cgs units. Compare your answer to the surface gravity of the Sun and Earth.

Aug 2014

Astro #4

a) $\frac{a_A}{a_B} = 0.466 = \frac{m_B}{m_A}$ $a = \alpha d$ (in radians)

$$P^2 = \frac{4\pi^2}{G(m_1+m_2)} a^3 \rightarrow m_1 + m_2 = \frac{4\pi^2}{GP^2} a^3$$

$$d = \frac{1}{\alpha} \text{ pc}$$
$$= \frac{1}{.37921}$$
$$= 2.64 \text{ pc}$$
$$= 8.17 \cdot 10^{18} \text{ cm}$$

$$a = \alpha d$$
$$= 7.61 \cdot \frac{1^\circ}{3600 \text{ arcsec}} \cdot \frac{\pi \text{ rad}}{180^\circ} \cdot 8.17 \cdot 10^{18} \text{ cm}$$
$$= 3.02 \cdot 10^{14} \text{ cm}$$

$$\Rightarrow m_1 + m_2 = \frac{4\pi^2 (3.02 \cdot 10^{14})^3}{6.67 \cdot 10^{-8} \frac{cm^3}{g s^2} \cdot (49.94 \cdot \pi \cdot 10^7 \text{ s/yr})^2}$$

$$1.466 m_A = 6.59 \cdot 10^{33} g$$

$$m_A = 4.5 \cdot 10^{33} g$$

$$m_B = 2.09 \cdot 10^{33} g$$

b) $\frac{L_A}{L_\odot} = 100^{(M_\odot - M_A)/5}$

$$= 22.49 L_\odot$$

$$\frac{L_B}{L_\odot} = 100^{(M_\odot - M_B)/5}$$

$$= 0.024 L_\odot$$

#4 (cont.)

c) $L = 4\pi r^2 \sigma T^4$

$$r = \left(\frac{L}{4\pi\sigma T^4}\right)^{1/2}$$

$$= 5.88 \cdot 10^8 \text{ cm}$$

$R_\odot = 7 \cdot 10^{10}$ cm

$R_\oplus = 6.37 \cdot 10^8$ cm $\Rightarrow$ $r < R_\oplus$

d) $$mg = \frac{G m_1 m_2}{r^2}$$

$$g = \frac{G m_1}{r^2}$$

$$= \frac{6.67 \cdot 10^{-8} \cdot 2.09 \cdot 10^{33}}{(5.88 \cdot 10^8)^2}$$

$$= 4.03 \cdot 10^8 \ \frac{\text{cm}}{\text{s}^2}$$

$g_\oplus = 9.8 \cdot 10^2 \ \frac{\text{cm}}{\text{s}^2}$

$g_\odot = 2.7 \cdot 10^4 \ \frac{\text{cm}}{\text{s}^2}$

Kilrc

PROBLEM 5

a) (6 points) Compute the Kelvin-Helmholtz timescale for the Sun. Assume the virial theorem and that the density of the star at any distance from its center is equal to the star's average density.

b) (2 points) Assuming that 10 eV could be released by every atom in the Sun through chemical reactions, estimate how long the Sun could shine at its current rate through chemical processes alone. For simplicity, assume that the Sun is composed entirely of hydrogen.

c) (2 points) Assuming that the Sun is 100% hydrogen, and that only the inner 10% of the Sun's mass becomes hot enough to burn hydrogen, estimate how long the Sun could shine at its current rate through nuclear reactions alone. Assume that 0.7% of the mass of hydrogen is converted to energy in forming a helium nucleus.

Wang

PROBLEM 6

The Universe contains different components that have different equations of state. These include matter, radiation, and dark energy.

a) (1 point) What is the equation of state for matter? Explain.

b) (1 point) What is the equation of state for radiation? Explain.

c) (2 points) What is the best current estimate for the equation of state for dark energy? Explain.

d) (6 points) Derive the cosmic scale factor as a function of time for a cosmic component with constant equation of state w, assuming a flat Universe.

ASTRONOMY QUALIFYING EXAM
January 2014

Possibly Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \times 10^{10}$ cm
1 AU = 1.5 $\times 10^{13}$ cm
1 pc = 3.26 Ly. = 3.1 $\times 10^{18}$ cm
$a = 7.56 \times 10^{-15}$ erg cm^{-3} K^{-4}
$c = 3 \times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
$k = 1.38 \times 10^{-16}$ erg K^{-1}
$e = 4.8 \times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
$G = 6.67 \times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
$h = 6.63 \times 10^{-27}$ erg s
1 amu = 1.66053886 $\times 10^{-24}$ g

John?

PROBLEM 1

a) (3 points) Calculate the orbital semi-major axis (a_{sun}) of the Sun's orbit about the barycenter of the Solar System, in AU, in response to Jupiter's orbital motion. Since Jupiter constitutes ~70% of the non-solar mass of our Solar System, you can ignore Solar System bodies less massive than Jupiter in your computation. Assume $a_{Jupiter} = 5.2$ AU.

b) (2 points) To an external observer, what would be the transit depth of an Earth-size planet located at a=0.1AU (assume circular orbit) about a M dwarf star (Mass = 0.3 Msun; Radius = 0.8 Rsun)?

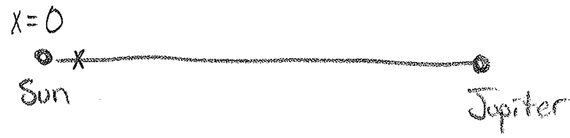
c) (3 points) To an external observer, what would be the transit duration (in hours) of an Earth-size planet located at a=0.1AU (assume circular orbit) about a M dwarf star (Mass = 0.3 Msun; Radius = 0.8 Rsun)?

d) (2 points) To an external observer located 20 pc away, what would be the angular separation in arcseconds between an Earth-size planet located at a=0.1 AU and its host star?

Jan 2014

Astro #1

a) * Bodies orbit mutual center of mass; Assuming point particles



* Assuming $M_J \sim \frac{M_\odot}{1000}$

$$X_{cm} = \frac{\sum_i x_i m_i}{\sum_i m_i}$$

$$= \frac{M_\odot \cdot 0 + 5.2\,AU \cdot \frac{M_\odot}{1000}}{M_\odot + \frac{M_\odot}{1000}}$$

$$= \frac{5.2\,AU/1000}{1 + \frac{1}{1000}}$$

$$= 7.79 \cdot 10^{10}\,cm$$

$$a_{sun} = X_{cm} - R_\odot$$

$$= 7.9 \cdot 10^{9}\,cm$$

b) * Assuming no emission from planet

$$\frac{\Delta L}{L} = \frac{4\pi\sigma T^4 (r_s^2 - r_p^2)}{4\pi\sigma T^4 r_s^2}$$

$$= \frac{r_s^2 - r_p^2}{r_s^2}$$

$$= 1 - \frac{r_p^2}{r_s^2}$$

$$= 1 - \boxed{8.4 \cdot 10^{-5}}$$

$$= .999916$$

$$\frac{R_\oplus}{R_\odot} = \frac{6.37 \cdot 10^6\,m}{6.95 \cdot 10^8\,m}$$

$$R_\oplus = 9.1 \cdot 10^{-3}\,R_\odot$$

$$= .009\,R_\odot$$

Jan 2014

#1 (cont.)

c) $F = \frac{GMm}{r^2} = m\frac{v^2}{r}$

$$\frac{GM}{r} = v^2$$

$$\Rightarrow v = \sqrt{\frac{GM}{r}}$$

$$= \sqrt{\frac{6.67 \cdot 10^{-8} \frac{g^{-1}}{cm^3 s^2} \cdot 2 \cdot 10^{33} g}{1.5 \cdot 10^{12} cm}}$$

$$= 9.43 \cdot 10^{6} \frac{cm}{s}$$

$R_\oplus = 6.37 \cdot 10^8$ cm

$R_\odot = 7 \cdot 10^{10}$ cm

* From first to last contact

$$t = \frac{2R_\odot + 2R_\oplus}{v}$$

$$= \frac{1.4 \cdot 10^{11} + 1.27 \cdot 10^{9}}{9.43 \cdot 10^{6}}$$

$$= 1.49 \cdot 10^{4} s$$

$$= 4.16 \text{ hrs}$$

d) $d = \frac{1}{\alpha}$ pc

$$\alpha = \left(\frac{1}{20}\right)''$$

$$= .05 \text{ arcsec}$$

Wang

PROBLEM 2

For a blackbody the number density of photons is

$$n_\gamma(\nu, T)\, d\nu = \frac{8\pi\nu^2}{c^3} \frac{1}{e^{h\nu/kT} - 1}\, d\nu.$$

The energy density is then

$$U_\gamma(\nu, T)\, d\nu = h\nu n_\gamma(\nu, T)\, d\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1}\, d\nu.$$

Assume that radiation is emitted at some prior epoch, t, in the history of the Universe when the scale factor was given by R. The radiation is observed today at time t_0 with scale factor $R_0 = 1$.

a) (6 points) Using your knowledge of how wavelengths of photons vary with the scale factor, show that

$$n_\gamma(\nu_0, T)\, d\nu_0 = \frac{8\pi\nu_0^2}{c^3} \frac{1}{e^{h\nu/kRT} - 1}\, d\nu_0.$$

b) (4 points) And therefore that

$$U_\gamma(\nu_0, T_0)\, d\nu_0 = \frac{8\pi h\nu_0^3}{c^3} \frac{1}{e^{h\nu/kT_0} - 1}\, d\nu_0,$$

so that

$$T/T_0 = 1/R$$

Kilic

PROBLEM 3

a) (3 points) Describe the burning process on the main sequence. Explain the difference of the sun on the main sequence and a $1.5M_{sun}$ star.

b) (3 points) Describe He burning in the lower mass stars and intermediate mass stars. What is the mass range for each approximately? Compare the timescale of helium burning (lifetime on the helium main sequence) to that of hydrogen burning (lifetime on the main sequence).

c) (4 points) Describe the following burning stages in stars: carbon burning, neon burning, oxygen burning, silicon burning.

Jan 2014

Astro #3

a) For a star on the main sequence, it is burning hydrogen to helium. For stars less than 1.3 $M_\odot$, they perform the nuclear fusion via the PP chain, while stars more massive than 1.3 $M_\odot$ use the CNO cycle. The reaction chains are shown below

P-P Chain:

$$\left.\begin{array}{l} 2H_1^1 \rightarrow H_1^2 + \nu_e + e^+ + \gamma \\ H_1^2 + H_1^1 \rightarrow He_2^3 + \gamma \end{array}\right\rangle \times 2$$

$$2He_2^3 + \rightarrow He_2^4 + 2H_1^1$$

CNO Cycle:

$$C_6^{12} + H_1^1 \rightarrow N_7^{13} + \gamma$$
$$N_7^{13} \rightarrow C_6^{13} + e^+ + \nu$$
$$C_6^{13} + H_1^1 \rightarrow N_7^{14} + \gamma$$
$$N_7^{14} + H_1^1 \rightarrow O_8^{15} + \gamma$$
$$O_8^{15} \rightarrow N_7^{15} + e^+ + \nu$$
$$N_7^{15} + H_1^1 \rightarrow C_6^{12} + He_2^4$$

b) He-burning is done via the triple-α rxn in stars, and produces carbon ash. In low mass stars, the He-core contracts to the point of degeneracy before He fusion begins, resulting in He flashes lifting the degeneracy before stable fusion starts. In higher mass stars, this is not necessary as the required central temperature can be reached simply via core contraction. He burning lasts for ~10% of time of H-burning (~120 Myr v 10 billion yr for 1 $M_\odot$ star).

$<2 M_\odot$

$<8 M_\odot$

Jan 2014

#3 (cont.)

c) Carbon burning: $C^{12}_{6} + He^{4}_{2} \rightarrow O^{16}_{8} + \gamma$

$T \approx 5 \cdot 10^{8}\,K$

$$2C^{12}_{6} \rightarrow Mg^{24*}_{12} \rightarrow Ne^{20}_{10} + \alpha$$
$$\rightarrow Na^{23}_{11} + p$$

Neon burning: $Ne^{20} + \gamma \leftrightarrow O^{16} + \alpha$

$Ne^{20} + \alpha \leftrightarrow Mg^{24} + \gamma$ $\quad T \approx 1.5 \cdot 10^{9}\,K$

$$\Rightarrow 2Ne^{20} \rightarrow Mg^{24} + O^{16}$$

Oxygen burning: $2O^{16}_{8} \rightarrow S^{32} \rightarrow Si^{28} + \alpha$

$T \approx 2 \cdot 10^{9}\,K$

Silicon burning: * combination of photodisintigration chain to generate α particles and successive fusions w/ α-particles from Si^{32} to Ni^{56} which decays to Fe^{56}

$T \approx 3 \cdot 10^{9}\,K$

Eddie? / Herry?

PROBLEM 4

a) **(3 points)** Imagine a large cloud of pure interstellar hydrogen having density n atoms/cm^3. Φ is the number of photons emitted by a star per second which are capable of photoionizing neutral hydrogen ($\lambda < 912$Å), while αn^2 is the number of recombinations per second per cm^3. If each photon results in a photoionization and the rate of photoionization equals the rate of recombination, find an expression for the Strömgren sphere R_s, i.e. the radius of the ionized gas cloud, in terms of n, Φ, and α.

b) **(2 points)** Find R_s in parsecs for an O star if $\Phi = 10^{49}$photons/s, n = 10 atoms/cm^3, and $\alpha = 2 \times 10^{-13}$.

c) **(2 points)** Find R_s in parsecs for the sun if $\Phi = 5 \times 10^{23}$ photons/s, while n and α remain the same.

d) **(3 points)** Could the cloud around the sun be seen by an astronomer on α-Centauri (distance=1.31pc) using a telescope which can just barely resolve objects which are 1" in angular size?

Constants:

1 parsec=3.086x10^{18}cm

1 radian = 206265 arcsec

Jan 2014

Astro #4

a) $[n] = \frac{\text{atoms}}{cm^3}$

$[\Phi] = \frac{\text{photons}}{s}$

* Assuming # of ionizations and recombinations are the same over a 1s interval

$$\Phi = \int \alpha n^2 dV$$

$$\Phi = \alpha n^2 \frac{4}{3}\pi r^3$$

$$\left(\frac{3\Phi}{4\alpha n^2 \pi}\right)^{1/3} = r$$

b) $$r = \left(\frac{3 \cdot 10^{49} \frac{1}{\cancel{s}}}{\pi \cdot 4 \cdot 2 \cdot 10^{-13} (10)^2 \frac{1}{cm^3}}\right)^{1/3}$$

$$= 4.92 \cdot 10^{19} \text{ cm}$$

$$\approx 15.96 \text{ pc}$$

c) $$r = \left(\frac{5 \cdot 10^{23}}{\pi \cdot 4 \cdot 2 \cdot 10^{-13} \cdot 10^2 \frac{1}{cm^3}}\right)^{1/3}$$

$$= 1.258 \cdot 10^{11} \text{ cm}$$

$$= 4.08 \cdot 10^{-8} \text{ pc}$$

Wang?/Dai?

PROBLEM 5

a) (5 points) There are four commonly used distances in extra-galactic astronomy, the co-moving line-of-sight distance, the co-moving transverse distance, the luminosity distance, and the angular diameter distance.

Given a cosmological object at a redshift z (z > 1), describe how to calculate these four distances. The cosmological parameters, H_0, Ω_m, and Ω_Λ are all given.

b) (1 point) What is the definition of the surface brightness of an astronomical object?

c) (3 points) For a cosmological extended object at z (z > 1) with a constant emissivity per unit area, show that the surface brightness, σ, of the object scales as $\sigma \propto (1+z)^{-4}$.

d) (1 point) If the object is nearby, show that the surface brightness is roughly a constant as a function of distance.

Lenghly?

PROBLEM 6

a) (3 points) A sequence of radio images from the quasar 3C 273 shows a blob of radio emission moving away from the nucleus with an angular velocity of 0.0008 arcsec yr^{-1}. Assuming that the radio knot is moving in the plane of the sky, and using the distance of $d = 440h^{-1}$ Mpc for 3C 273, derive the apparent transverse velocity v_{app} away from the nucleus. What is the value, in units of c, for normalized Hubble constant $h = 0.71$? Is this physically reasonable?

b) (4 points) Next, assuming that instead of moving in the plane of the sky, the blob is moving at an angle ϕ to our line of sight with an actual speed v (as distinguished from the apparent velocity v_{app}). Derive an equation for v/c in terms of the apparent transverse velocity and ϕ.

c) (3 points) Show that $v/c < 1$ for angles satisfying

$$\frac{v_{app}^2/c^2 - 1}{v_{app}^2/c^2 + 1} < \cos\phi < 1$$

ASTRONOMY QUALIFYING EXAM

January, 2015

Possibly Useful Quantities

$L_{\odot} = 3.9 \times 10^{33}$ erg s^{-1}
$M_{\odot} = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_{\odot} = 7 \times 10^{10}$ cm
1 AU=1.5×10^{13} cm
1 pc= 3.26 $Ly =$ 3.1×10^{18} cm
a=7.56×10^{-15} erg cm^{-3} K^{-4}
c=3×10^{10} cm s^{-1}
σ =ac/4= 5.7×10^{-5} erg cm^{-2} K^{-4} s^{-1}
k=1.38×10^{-16} erg K^{-1}
e=4.8×10^{-10} esu
1 fermi=10^{-13} cm
N$_A = 6.02 \times 10^{23}$ moles g^{-1}
G=6.67×10^{-8}g^{-1} cm^3 s^{-2}
m$_e = 9.1 \times 10^{-28}$ g
h= 6.63×10^{-27} erg s
1 amu= $1.66053886 \times 10^{-24}$ g
$r_e = 2.8179 \times 10^{-13}$ cm; (electron radius)

Eddie?/ Kilic?

PROBLEM 1

1. For this problem recall the thermodynamic identity $P = -\left.\frac{\partial E}{\partial V}\right|_S$
   (a) (1 point) Write down the polytropic equation of state.
   (b) (1 point) For a polytrope, the constant is a function of what thermodynamic quantity?
   (c) (2 points) For a polytropic equation of state derive the relationship between pressure and energy density. *Hint: Define the energy per unit mass $u = E/M$ and the specific volume per unit mass $v = V/M$ and then relate u to the energy density $\varepsilon = E/V$*
   (d) (3 points) Multiply the equation of hydrostatic equilibrium by $4\pi r^3$ and derive the Virial Theorem.
   (e) (3 points) Use the Virial Theorem to find the total energy of a star with a polytropic equation of state. Show that $\gamma = 4/3$ gives zero total energy and that $\gamma = 5/3$ corresponds to the classic case that the internal energy is half the gravitational energy in magnitude.

Kilic

PROBLEM 2

2. Consider a $3M_\odot$ main sequence star with $L = 80L_\odot$, $X = 0.7, Y = 0.28, Z = 0.02$ and

$$\epsilon_{\rm nuc} = \epsilon_c\left(1 - \frac{m}{0.1M}\right) \tag{1}$$

for $m < 0.1M$; m is the mass variable and M is the stellar mass.

(a) (2 points) Calculate and draw the luminosity profile, l, as a function of mass, m.

(b) (2 points) What is the numerical value of ϵ_c in erg/g/s?

(c) (3 points) Assuming radiative energy transport, calculate the H mass fraction as a function of mass and time, $X = X(m,t)$. What is the central value of X after 100 Myr? Draw X as a function of m at 100 Myr. (Assume that the energy generation per unit mass from hydrogen is 6×10^{18} erg g^{-1}).

(d) (2 points) Assuming that the inner 20% of the mass is convective, draw the new X profile as a function of m.

(e) (1 point) What is the H-burning lifetime for the star in (c) and (d)? How much is the lifetime extended due to convection?

John?

PROBLEM 3

3. A star has a temperature T = 6700 K, mass M = 1.4 $M_\odot$, and radius R = 1.25 $R_\odot$. There is a super-Earth exoplanet orbiting the star with a semi-major axis a = 5 AU in a circular orbit. The planet has a radius of 2 R_j (R_j = Jupiters radius). Assume the only source of energy for the planet is the star, all light falling on the planet is absorbed, and the star+planet are perfect blackbodies.

   (a) (8 points) Derive the temperature of the planet, in units of Kelvin. Assume that the temperature is uniform over the entire planet.

   (b) (2 points) Derive the period of the planet.

Henry

PROBLEM 4

4. The following two questions refer to the Milky Way Galaxy.

   (a) (3 points) List at least seven components of the Milky Way, which must include the most massive component.

   (b) (7 points) What are the observational evidences for these components?

John?

PROBLEM 5

5. The star beta Pic was observed to have a parallax of 51.44 milli-arcsec from the Hipparcos satellite.

   (a) (3 points) Given the apparent K-band magnitude of beta Pic, $m_K = 3.53$, what is the absolute K-band magnitude of beta pic, M_K ?

   (b) (2 points) A planet (beta Pic b) has recently been directly imaged around the star beta Pic. The planet has an apparent K-band magnitude of $m_K = 12.73$. How much less flux is the planet emitting in the K-band compared to its host star?

   (c) (3 points) The beta Pic b planet is oberved to be separated from its host star by 0.5 arcseconds. How far away is the planet located from its host star, in units of AU?

   (d) (2 points) Assume aliens live on the planet beta Pic b, and an alien observes the Earth/Sun system from his/her planet. Whats the angular separation (in units of milli-arcseconds) he/she would measure for the Earth and Sun?

Dai? / Henry?

PROBLEM 6

6. The following questions refer to the Milky Way Galaxy and its chemical evolution.

   (a) (1 point) Contrast thin disk and halo stars in terms of their kinematics, and metallicity.

   (b) (1 point) Give a plausible model for the formation of the Milky Way which explains the differences discussed in part a.

   (c) (2 points) According to chemical evolution theory, why does the value of [Fe/O] in any one location in the Galaxy tend to increase with time? If the initial mass function were flatter (higher fraction of massive stars), how would you expect that to affect the evolution of the local value of [Fe/O]? Explain.

   (d) (1 point) Sketch a plot of the rotation curve of the Milky Way and describe its behavior for both the bulge and disk. Make sure to include axis titles.

   (e) (3 points) Derive a functional relation between surface density σ (mass/pc^2), tangential (circular) velocity v, and galactocentric distance r for the disk. What is implied about the surface density in the disk? Explain.

   (f) (2 points) Given the rotation properties of the bulge, what is implied about the behavior of its volume density as a function of galactocentric distance? Use simple algebra to prove your point.

ASTRONOMY QUALIFYING EXAM

August, 2015

Possibly Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \times 10^{10}$ cm
1 AU=1.5×10^{13} cm
1 pc= 3.26 Ly = 3.1×10^{18} cm
a=7.56×10^{-15} erg cm^{-3} K^{-4}
c=3×10^{10} cm s^{-1}
σ =ac/4= 5.7×10^{-5} erg cm^{-2} K^{-4} s^{-1}
k=1.38×10^{-16} erg K^{-1}
e=4.8×10^{-10} esu
1 fermi=10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
G=6.67×10^{-8}g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
h= 6.63×10^{-27} erg s
1 amu= $1.66053886 \times 10^{-24}$ g
$r_e = 2.8179 \times 10^{-13}$ cm; (electron radius)

PROBLEM 1

1. (a) (2 points) Write or derive an equation for hydrostatic equilibrium in a form that is suitable for the interior of the sun, i.e., express dP/dr in terms of G, m, ρ, and r, where m is the mass interior to radius r and ρ is the mass density.

   (b) (1 point) Rewrite the equation with m as the independent variable, i.e, dP/dm = ...

   (c) (1 point) Use the dP/dm equation to obtain an approximate expression for the pressure at the center of the sun, in terms of G, M, and R, where M is the total mass of the sun and R is the solar radius.

   (d) (1 point) To the nearest powers of ten, what are the temperature and the density at the center of the sun?

   (e) (1 point) Write the "bottleneck" reaction (the least probable of the major reactions) for fusing hydrogen to helium in the core of the sun.

   (f) (2 points) At the middle of the solar photosphere, where the optical depth at 5000 Å is about 1, what (to the nearest 1000 K) is the temperature? Is the mass density at this depth much greater than, much less than, or about equal to the density of air at sea level? Is hydrogen mostly ionized, mostly neutral, mostly locked up in diatomic molecules, or in some other form? What is the dominant source of opacity at 5000 Å? Identify the atomic process as specifically as you can.

   (g) (2 points) In the approximation of local thermodynamic equilibrium (LTE), estimate the fraction of *all* hydrogen (ionized, neutral, molecular) that is in the Balmer (n = 2) level of neutral hydrogen.

PROBLEM 2

2. (a) (6 points) A star located 19.6 pc from the Sun has a temperature T = 8000 K and radius R = 0.5 Rsun. There is a planet orbiting the star with a semi-major axis a = 5 AU in a circular orbit. The planet has a radius of 2 Rj (Rj = Jupiters radius). Assume the only source of energy for the planet is the star, all light falling on the planet is absorbed, and the star+planet are perfect blackbodies.

   Estimate the temperature of the planet.

   (b) (2 points) Assume the planet described in part (1) transits its host star. What would be the observed transit depth?

   (c) (2 points) Assume aliens live on the planet, and an alien observes the Earth/Sun system from his/her planet. Whats the angular separation (in units of milli-arcseconds) he/she would measure for the Earth and Sun?

PROBLEM 3

3. (a) (1 point) Write down the general radiative transfer equation (RTE) in plane-parallel geometry and define all the terms including units.

   (b) (1 point) Define the 3 Eddington moments.

   (c) (1 point) Explain what the grey approximation is.

   (d) (1 point) Make the grey approximation and derive the 2 ordinary differential equations for the moments from the RTE.

   (e) (1 point) What does radiative equilibrium tell you about H in this case?

   (f) (1 point) Make the "two-stream" approximation

$$I = \begin{cases} I^+ & \mu \geq 0 \\ I^- & \mu < 0 \end{cases}$$

   and obtain the Eddington moments in this case.

   (g) (1 point) From the two-stream case above, find the Eddington factors $f = K/J$, and $h(0) = H(0)/J(0)$, where $H(0)$ is the value of H at the surface $\tau = 0$. Assume no external illumination.

   (h) (2 points) Using the Eddington factors found above, solve the moment equations for J.

   (i) (1 point) Assume $S = B$, where B is the grey planck function and find the temperature as a function of τ.

Aug 2015

#3

a) $\mu \frac{dI}{dz} = \eta_\nu - \chi_\nu I_\nu$

$\mu = \cos\theta$

η_ν = emissivity

χ_ν = opacity

I_ν = radiation intensity

b) $J_\nu = \frac{1}{2}\int_{-1}^{1} I_\nu \, d\mu$

$H_\nu = \frac{1}{2}\int_{-1}^{1} \mu I_\nu \, d\mu$

$K_\nu = \frac{1}{2}\int_{-1}^{1} \mu^2 I_\nu \, d\mu$

c) The grey approximation says that χ is independent of ν

d) * In the grey approximation

$\mu \frac{dI}{dz} = \eta - \chi I$

$\cancel{\mu \frac{dI}{dt} = S - I}$

$\frac{1}{2}\int_{-1}^{1} \mu \frac{dI}{dz} = \frac{dH}{dz} = -\frac{1}{2}\int_{-1}^{1} \chi_\nu (I_\nu - S_\nu) d\mu$

$= -\chi (J - S)$

$\frac{1}{2}\int_{-1}^{1} \mu^2 \frac{dI}{dz} = \frac{dK}{dz} = \frac{1}{2}\int_{-1}^{1} \chi (I - S) \mu d\mu$

$= \chi H$

e) * In the case of radiative equilibrium (energy only carried by radiation

$\int_0^\infty \frac{dH_\nu}{dz} d\nu = \frac{dH}{dz} = -\int \chi (J - S)$

$= 0$ b/c absorption = emission

PROBLEM 4

4. (a) (7 points) Assume that the gas component of a galaxy, with a mass fractionf_g, is virialized and follows the overall density profile of the galaxy, a singular isothermal sphere mass profile,

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2}$$

where σ is the velocity dispersion of the galaxy. The galaxy has a central AGN, which is radiating at the Eddington luminosity,

$$L_{Edd} = \frac{4\pi G c m_p M_{BH}}{\sigma_T}$$

A fraction, f_w, of the energy radiated by the central AGN is deposited into the gas in the form of kinetic energy. This kinetic "feedback" energy from the AGN can drive the gas in the host galaxy to flow outward. Assume that the final gas outflow is in a spherical shell with a constant velocity, v, and half of the kinetic feedback energy is converted to the gravitational potential of the gas and the other half to the kinetic energy of the gas during the outflowing process. Use the conservation or transfer of energy to show that the final gas wind speed is

$$v^3 = \frac{G L_{Edd} f_w}{2\sigma^2 f_g}$$

(b) (3 points) If the wind speed is large enough to escape the potential well of the galaxy, ($v = \sigma$), the central AGN will blow out the majority of gas in the galaxy and terminate the formation of stars. Show that this gives us the $M_{BH} - \sigma$ relation,

$$M_{BH} = \frac{1}{2\pi} \frac{\sigma_T}{G^2 c m_p} \frac{f_g}{f_w} \sigma^5$$

where G is the gravitational cnstant, σ_T is the Thomson cross section, c is the speed of light, and m_p is the mass of a proton.

PROBLEM 5

5. (a) (3 points) Describe the pre-MS to AGB evolution of a solar-metallicity 1 $M_\odot$ star. Plot the evolution in $\log T_{\rm central}$ vs. $\log \rho_{\rm central}$ and H-R diagrams. Also plot a Kippenhahn diagram showing the interior structure of the star (including burning and convective regions) as a function of time. Describe each phase of evolution, including the radiative properties and nuclear burning reactions and regions.

   (b) (3 points) Describe the pre-MS to AGB evolution of a solar-metallicity 5 $M_\odot$ star. Plot the evolution in $\log T_centਰal$ vs. $\log \rho_central$, H-R, and Kippenhahn diagrams. Describe each phase of evolution, including the radiative properties and nuclear burning reactions and regions, emphasizing the differences between this star and a 1 $M_\odot$ star.

   (c) (2 points) Describe the evolution of a 40 $M_\odot$ star. Will this star create a Red Supergiant and why?

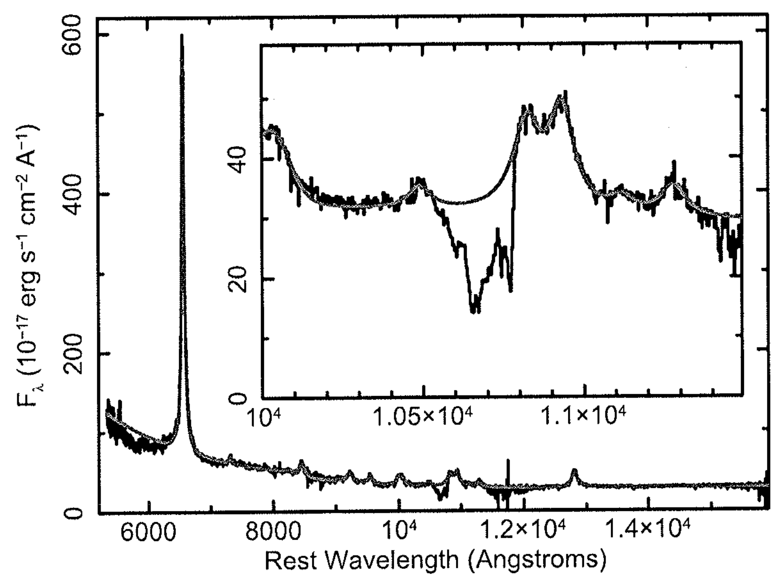
   (d) (2 points) Describe WNL, WNE, WC, and WO stars. These stars form an evolutionary sequence. Explain the connection between them.

PROBLEM 6

6. A fraction of quasars have broad, blueshifted absorption lines that indicate high-velocity outflows emerging from the central engine. Generally, an absorption profile can be described as:

$$\frac{I}{I_0} = exp(-\tau(\lambda))$$

where I is the observed flux density in the spectrum, I_0 is the intrinsic continuum (without absorption) and $\tau(\lambda)$ is the optical depth of the absorption trough originating from absorption by a single ion. An example of a broad absorption spectrum is shown below. The absorption is occurring in metastable helium in the 10830Å transition. A range of gas outflow velocities causes the absorption line to be broad.



Analysis of broad absorption lines is complicated by *partial covering*: the absorbing outflow does not cover all of the continuum emitting source, but rather covers only a fraction of it, C_f. Then, the absorption line looks shallower than it would be if the absorber covered the whole thing, and the inferred *apparent* optical depth is lower. However, this situation can be resolved, and the true optical depth and covering fraction can be determined if there are two lines in the spectrum that arise from the same lower level, because their true optical depth ratio is fixed by atomic physics. Specifically, the true optical depth ratio will be proportional to the ratio of $f_{ik}\lambda$, where f_{ik} and λ are the oscillator strength and wavelength of the transition. In that case, the intensity ratio can be expressed in these two equations:

$$I_s = (1 - C_f) + C_f e^{-\tau_s}$$

$$I_w = (1 - C_f) + C_f e^{-\tau_w}$$

where the subscripts w and s stand for weaker and stronger lines, respectively, and τ_s/τ_w is related by the ratio of their respective $f_{ik}\lambda$ values.

(a) (5 points) Consider a doublet, e.g., C IV. The first excited state has fine structure, so there are two possible transitions from the ground state to the first excited state, at 1548.2 and 1550.8Å (a doublet). The oscillator strengths for these two transtions are 0.190 and 0.0952 respectively. This means that the ratio of the optical depths τ_s/τ_w for these two transitions is effectively 2.

For the case of this doublet, solve the equations above for the covering fraction C_f and τ_s.

(b) (1 point) When scientists analyze an absorption line, they are often interested in measuring the column density N (in particles per cm^2) of the ion responsible for it. The optical depth and column density are related by the following equation:

$$\tau(\lambda) = \frac{\pi e^2}{m_e c^2} f_{ik} \lambda^2 N(\lambda),$$

where e is the charge on an electron, m_e is the mass of an electron, and c is the speed of light. Both τ and N are functions of lambda because as the absorption line is spread of a range of wavelengths due to the range of velocities over which the outflow is distributed.

Show that

$$\tau(v) = \frac{\pi e^2}{m_e c} f_{ik} \lambda N(v)$$

where v is velocity, λ is in Angstroms, and $N(v)$ is in atoms $\text{cm}^{-2}(\text{km}\,\text{s}^{-1})^{-1}$.

(c) (4 points) Further, show that

$$\tau(v) = 2.654 \times 10^{-15} f_{ik} \lambda N(v).$$

Astro Qualifier

Jan. 2016

- There are 6 problems. Attempt them all as partial credits will be given.
- Write on only one side of the provided paper for your solutions.
- Write your alias (NOT YOUR REAL NAME) on the top of every page of your solutions.
- Number each page of your solution with the problem number and page number (e.g. Problem 3, p. 2 is the second page for the solution to problem 3.)
- Do not staple your exam when done.
- You must show your work to receive full credit.

Constants:

$G = 6.67259 \times 10^{-8}$ dyne cm^2 g^{-2}
$c = 2.99792458 \times 10^{10}$ cm s^{-1}
$h = 6.6260755 \times 10^{-27}$ erg s
$k = 1.380658 \times 10^{-16}$ erg K^{-1}
$\sigma = 5.67051 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}
$m_p = 1.6726231 \times 10^{-24}$ g
$m_n = 1.674929 \times 10^{-24}$ g
$m_e = 9.1093897 \times 10^{-28}$ g
$m_H = 1.673534 \times 10^{-24}$ g
$e = 4.803206 \times 10^{-10}$ esu
1 eV $= 1.60217733 \times 10^{-12}$ erg
1 $M_\odot = 1.989 \times 10^{33}$ g
1 $L_\odot = 3.826 \times 10^{38}$ erg s^{-1}
1 pc $= 3.0857 \times 10^{18}$ cm
1 AU $= 1.4960 \times 10^{13}$ cm

1. Briefly define and discuss the relevance of the following terms to modern astronomy.
   (a) (1 point) Cepheid variable star
   (b) (1 point) Initial mass function
   (c) (1 point) tunneling in the context of the PPI chain reaction
   (d) (1 point) age-metallicity relation
   (e) (1 point) damped Lyα system (DLA)
   (f) (1 point) s-process
   (g) (1 point) G dwarf problem
   (h) (1 point) Tully-Fisher relation
   (i) (1 point) Thin disk
   (j) (1 point) isophotal radius

2. The specific intensity at the surface of stars is given by

$$I_\nu(u) = \int_0^\infty S_\nu(t)\frac{dt}{u}e^{-t/u}, \tag{1}$$

where S_ν is the source function, t is the optical depth, and $u = \cos\theta$. In addition, the moments of order n of the radiative field $M_\nu(n)$ are

$$M_\nu(n) = \frac{1}{2}\int_{-1}^{1} I_\nu(u)u^n du, \tag{2}$$

where $M_\nu(0) = J_\nu$ and $M_\nu(1) = H_\nu$.

   (a) If the source function inside the star is $S(\tau) = a + b\tau$, where a and b are functions of ν but not τ, calculate the specific intensity I_ν at the surface, for outgoing directions ($u \geq 0$).
   (b) the average intensity J_ν.
   (c) the Eddington flux H_ν.

3. Assume that as a pulsar slows down, the quantity

$$\frac{d\ln P}{dt} = b,$$

where b is positive constant and P is the rotation period.

   (a) (5 points) If at time $t = 0$, $P = P_0$, find an expression for $P(t)$, the period as a function of time.
   (b) (3 points) If the initial rotation energy is E_0, find an expression for $E(t)$, the energy as a function of time
   (c) (2 points) If $P_0 = 10^{-3}$ s, at what time is the period 3 s?

4. (a) (3 points) Use the Virial Theorem to derive expressions for the quantized radius AND energy of a Bohr hydrogen atom.

   (b) (1 point) Calculate the energy AND wavelength of light emitted by a Brackett gamma ($n = 7$ to $n = 4$) emission photon.

   (c) (3 points) Assume you observe a massive, hot star that exhibits a Brackett gamma emission line that has a P-Cygni line profile. What is the physical interpretation of this line profile? Discuss how such a profile arises; include a picture that describes where each region of the line profile arises from.

   (d) (3 points) Describe the process by which winds are driven in massive stars. Also describe how the Doppler effect aids wind driving in these stars.

5. (a) (2 points) Calculate the force due to radiation pressure experienced by an object of radius, r, and density, ρ, in a circular orbit with semimajor axis, a, around the Sun. Assume that the object absorbs all radiation and re-emits it isotropically in its rest-frame.

   (b) (2 points) If the object were stationary, this force would act only in the radial direction away from the Sun. However, because of our object's orbital velocity, the direction of the incoming photons has a small non-radial component in the object's rest-frame, and the radiation pressure exerted by the Sun in part a) has a small non-radial component. Expressing the object's orbital velocity in terms of the Sun's mass and a, solve for the non-radial component of the radiation pressure. (This non-radial component can be thought of as a photon headwind known as Poynting-Robertson drag.)

   (c) (2 points) This headwind causes the orbital semimajor axis to decay over time. Write the time derivative of the semimajor axis due to Poynting-Robertson drag for the object in part a. Assume that the radial component of the radiation pressure force is very small compared to the Sun's gravitational pull, so we only need to consider the Poynting-Robertson component.

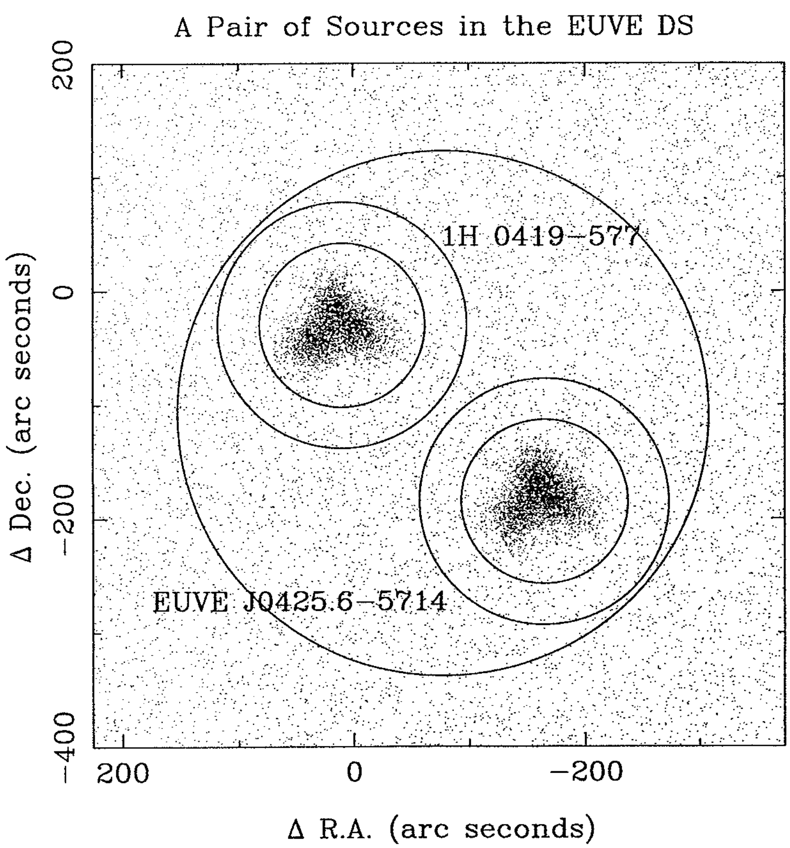
   (d) (2 points) If the object is orbiting at 1 AU, its radius is 100 μm and its density is 1 g/cm^3, then calculate the amount of time it takes to spiral into the Sun.

   (e) (2 points) The object in part (d) is typical of zodiacal dust (dust particles in the Solar system, primarily located between the Sun and Jupiter). What does the above calculation say about the theory that zodiacal dust was formed at the beginning of the Solar system?

6. The image below is derived from data collected by the *Extreme Ultraviolet Explorer* satellite (EUVE). It shows photons near 0.1 keV collected from a luminous Seyfert (active) galaxy 1H 0419−577 (upper left) that was the target of the observation, and a serendipitously discovered Am Herculis star, an accreting magnetic white dwarf star (lower right). The three-lobed structure of the image is an artifact of the telescope.

   (a) (5 points) This observation had a total exposure time of 171,841 seconds. During the observation, 5529 photons total were collected in the region 60 arcseconds in radius around the Seyfert, and 4733 photons were collected from the region 60 arcseconds in radius around the AM Her. In the background region, which is 230 arcseconds in radius, and excludes the regions around the Seyfert and around the AM Her that are each 105 arc seconds in radius, 6291 photons were collected. What are the average net count rates and uncertainties from the Seyfert and from the AM Her? Are these values significantly different? (Hint: you may assume that Poisson statistics apply.)

(b) (5 points) During the first 10,000 seconds of the observation, only 308, 288, and 394 photons were collected from the Seyfert, the AM Her and the background region, respectively. What were the net count rates and uncertainties from the Seyfert and from the AM Her during this time period. Are these values significantly different?



ASTRONOMY QUALIFYING EXAM
January 2017

Possibly Useful Quantities

$L_{\odot} = 3.9 \times 10^{33}$ erg s^{-1}
$M_{\odot} = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_{\odot} = 7 \times 10^{10}$ cm
1 AU = 1.5×10^{13} cm
1 pc = 3.26 Ly. = 3.1×10^{18} cm
$a = 7.56 \times 10^{-15}$ erg cm^{-3} K^{-4}
$c = 3 \times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
$k = 1.38 \times 10^{-16}$ erg K^{-1}
$e = 4.8 \times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
$G = 6.67 \times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
$h = 6.63 \times 10^{-27}$ erg s
1 amu = $1.66053886 \times 10^{-24}$ g

PROBLEM 1

37

Use the Virial Theorem to:

a) (6 points) Derive the internal temperature of the Sun. How much hotter is this value compared to the Sun's effective surface temperature?

b) (4 points) Derive the Jeans Mass of a molecular cloud that is starting to collapse, thereby starting the star formation process.

Int energy < Gravitational

PROBLEM 2

8?

a) (4 points) The observed wavelength λ_0 is related to the emitted wavelength by

$$\lambda_0/\lambda = 1/R = 1 + z,$$

where R is the scale factor, and z is the redshift. The energy density in radiation from a black body is given by:

$$U(\nu, T)d\nu = 8\pi h\nu^3/c^2 \left(e^{h\nu/(kT)} - 1\right)^{-1} d\nu$$

Remembering that volumes increase like $V_0/V = 1/R^3$. Show that:

$$U(\nu_0, T)d\nu_0 = 8\pi h\nu_0^3/c^2 \left(e^{h\nu_0/(RkT)} - 1\right)^{-1} d\nu_0$$

b) (4 points) Given that

$$\int_0^\infty U(\nu, T)d\nu = aT^4$$

or equivalently

$$\int_0^\infty x^3/(e^x - 1)dx = \pi^4/15$$

show that the temperature of the Cosmic Background Radiation (CBR) must scale as 1/R.

c) (2 points) Compare the energy density in the CBR with that in diffuse starlight. Assume that the diffuse starlight has a brightness temperature of 10,000 K and a volume filling factor of 10^{-14}.

10

PROBLEM 3

The most easily observed white dwarf in the sky is in the constellation of Eridanus. Three stars make up the 40 Eridani system: 40 Eri A is a 4th magnitude star similar to the sun; 40 Eri B is a 10th magnitude white dwarf; and 40 Eri C is an 11th magnitude red M5 star. This problem deals only with the latter two stars, which are separated from 40 Eri A by 400 AU.

✓ a) **(4 points)** The period of the 40 Eri B and C system is 247.9 years. The system's measured trigonometric parallax is 0.201 arcseconds, and the true angular extent of the semimajor axis of the reduced mass is 6.89 arc seconds. The ratio of the distances of 40 Eri B and C from the center of mass is $a_B/a_C = 0.37$. Find the masses of 40 Eri B and C in terms of the mass of the sun.

✓ b) **(2 points)** The absolute bolometric magnitude of 40 Eri B is 9.6. Determine its luminosity in terms of the luminosity of the sun. Note that the absolute bolometric luminosity of the sun is $M_{bol} = 4.74$, while its luminosity is 3.839×10^{26} W.

$\frac{L}{L_\odot} = 100^{(M_\odot - M_*)/5}$

✓ c) **(2 points)** The effective temperature of 40 Eri B is 16,900 K. Calculate its radius and compare your answer with the radius of the Earth (6.378×10^6 m).

✓ d) **(2 points)** Sirius B is another famous white dwarf star. It has a mass of $1.053\,M_\odot$. Do you expect it to be larger or smaller than 40 Eri B? Explain.

2

PROBLEM 4

You are planning to conduct high resolution optical spectroscopy toward Barnard's Star, whose current coordinates are α = 17:57:48.5 and δ=+04:41:36.2. It has a V-band magnitude of 9.51 (Vega system).

✓ **a) (1 point)** What is constantly changing about Barnard's Star that needs to be considered when planning observations? What time of year is be to observed this star from the ground and why?

b) (1 point) What is the flux density of the star in V-band if the flux zero-point is 3.636×10^{-20} erg cm^{-2} s^{-1} Hz^{-1}?

✓ **c) (1 point)** This observation will be source noise limited, what distribution describes the uncertainty of these measurements and what is the simplest equation for the uncertainty σ in this case (1 point).

d) (4 points) Derive an expression for the number of photons observed in a given Δt and calculate the number in a single resolution element for the ARCES spectrograph on the APO 3.5m at a wavelength of 5175 Angstroms. The ARCES spectrograph has a resolution of R$\sim$31,500 in the optical band. Assume that the V-band flux density is the flux density at 5175 Angstroms.

e) (1 point) What is the maximum exposure time to avoid detector non-linearity? The detector goes non-linear at 35,000 ADU and the gain of the detector is 3.8 e$^-$ ADU^{-1}.

f) (2 points) Demonstrate mathematically that multiple short exposures are equivalent to a single long exposure. Why is a single long exposure a bad idea in the first place and why do we typically take multiple exposures during observations?

PROBLEM 5

Consider a satellite of mass m and radius s that is in a circular orbit about a planet with mass M and radius R. Assume the planet and satellite are separated by a distance r.

a) (3 points) Tidal forces arise because the gravitational force exerted by one body on another is not constant across it. For instance, something on the near edge of the satellite will feel a stronger gravitational pull toward the planet than the center of the satellite will. Thus, the tidal force is differential. Derive the tidal force (relative to the satellite's center) that a small object of mass u will feel if it is sitting on the edge of the satellite nearest to the planet. In this derivation, assume that $r \gg s$.

b) (3 points) Find the distance, d, from the planet where the tidal force that the small object experiences is equal to the gravitational pull exerted by the satellite's gravity.

c) (2 points) Express this distance in terms of the densities of the planet (ρ_{M}) and the satellite (ρ_{m}).

d) (2 points) Mars' moon Phobos has a density of 2 g/cm^3. It currently orbits Mars at a distance of 9400 km but this distance decreases by 2 cm every year. Using your work in parts a–c, calculate the amount of time before Phobos will be destroyed by the tidal forces of Mars. (The density and radius of Mars are 4 g/cm^3 and 3400 km, respectively.)

8+

PROBLEM 6

Briefly define and discuss the relevance of the following terms to modern astronomy. 1 point per question

a 1. Cepheid variable star

b 2. Initial mass function

c 3. tunneling in the context of the PPI chain reaction

d 4. age-metallicity relation

e 5. damped Lyα system (DLA) neutral H^2 associated w/ quasars

f 6. s-process AGB + SN

g 7. G dwarf problem not enough low metal stars

h 8. Tully-Fisher relation luminosity, maximal rotation velocity

i 9. Galactic thin disk

j 10. isophotal radius

Midterm Exam #1
Monday Oct 6

Answer the questions on a separate sheet of paper. For each question begin a new sheet. Put your name and the question and page number on each sheet.
That is: Eddie Baron, Question 1, Page 2/7

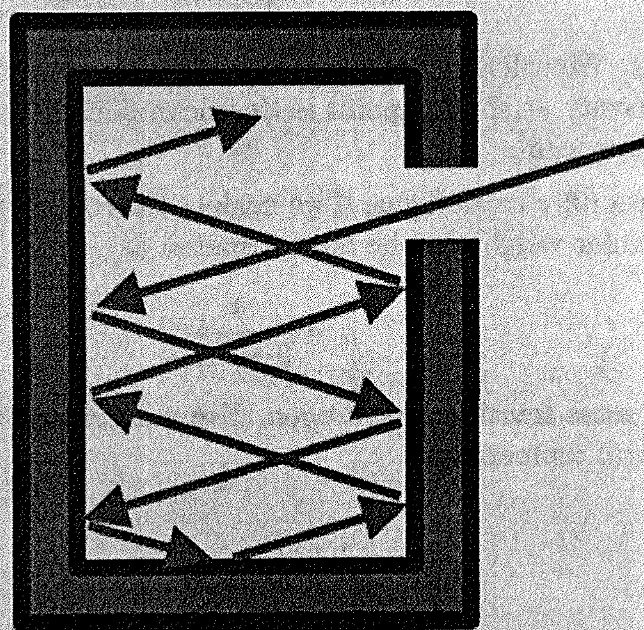


Figure 1: An illustration of a theoretical perfect blackbody. Ignore the hole that is in this illustration and assume the box is in a heat bath and the interior walls are perfect absorbers.

50 1. We calculated in our homework that the energy density in radiation inside the blackbody enclosure is :

$$U_\nu(T) = \frac{4\pi}{c} B_\nu(T),$$

where T is the temperature of the heat bath and $B_\nu(T)$ is the Planck function.

(a) (10 points) Write down the equation of radiative transfer for any arbitrary ray (whether or not it is inside the blackbody). Use the path length s along the ray as the independent variable.

(b) (5 points) What is the value of I_ν everywhere in the blackbody?

(c) (10 points) Using the result above, find the value of the moments, J_ν, H_ν, K_ν inside the blackbody.

(d) (5 points) Write down and evaluate the equation of radiative transfer inside the blackbody using the result found above.

(e) (10 points) Use the result above to derive the source function S_ν in thermal equilibrium.

(f) (10 points) Explain why we can generalize the result found above to the case of LTE.

30 2. (a) (10 points) Write down the equation of hydrostatic equilibruim.

(b) (10 points) Assume that the pressure at the surface, $r = R$ is $P(R) = 0$. Use this to approximately evaluate the derivative

$$\frac{dP}{dr}$$

at the mid point. We will assume that at the mid point $r = R/2$ and $m(R/2) = M/2$ and that the density at the midpoint is the mean density at that point. From this find the central pressure.

(c) (10 points) For a fully ionized gas, if we neglect the contribution of the metals that the mean molecular weight can be approximated as

$$\mu = \frac{4}{3+5X},$$

where X is the mass fraction of hydrogen. Use your answer from the previous part to find the central temperature.

) If we are measuring along the ray, we have $d\tau_\nu = -\alpha_\nu ds$. This means the equation of radiative transfer is

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

where I_ν and S_ν are both dependent upon τ_ν.

) Since this is a perfect blackbody, we have $I_\nu = B_\nu$, where B_ν is the Planck function.

) We have

$$J_\nu = \frac{1}{2}\int_{-1}^{1} B_\nu \, d\mu = \frac{1}{2}\left[B_\nu \mu\right]\Big|_{-1}^{1} = \frac{1}{2}(2B_\nu) = \boxed{B_\nu}$$

$$H_\nu = \frac{1}{2}\int_{-1}^{1} \mu B_\nu \, d\mu = \frac{1}{2}\left[\frac{1}{2}\mu^2 B_\nu\right]\Big|_{-1}^{1} = \boxed{0}$$

$$K_\nu = \frac{1}{2}\int_{-1}^{1} \mu^2 B_\nu \, d\mu = \frac{1}{2}\left[\frac{1}{3}\mu^3 B_\nu\right]\Big|_{-1}^{1} = \frac{1}{6}\left[2B_\nu\right] = \boxed{\frac{1}{3}}$$

$$= \frac{1}{3}$$

) We have $\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$, and since $I_\nu = B_\nu$, this becomes

$$\frac{dB_\nu}{d\tau_\nu} = -B_\nu + S_\nu$$

Midterm Exam #2
Monday Nov 7

Answer the questions on a separate sheet of paper. For each question begin a new sheet. Put your name and the question and page number on each sheet. That is: Eddie Baron, Question 1, Page 2/7

1. (a) (10 points) For a polytropic equation of state, derive the relationship between pressure and energy density. *Hint:* $P = n^2 \left.\frac{\partial u/n}{\partial n}\right|_S$ and ignore the proportionality constant between n and ρ, $n = \frac{N_A}{\mu}\rho$, that is set $\frac{N_A}{\mu} = 1$.

   (b) (10 points) Multiply the equation of hydrostatic equilibrium by $4\pi r^3$ and derive the Virial Theorem.

   (c) (10 points) Use the Virial Theorem to find the total energy of a star with a polytropic equation of state.

2. (a) (25 points) The entropy of a gas that consists of matter and radiation is given by

$$S = \text{constant} + \frac{N_A k}{\mu} \ln \frac{T^{3/2}}{\rho} + \frac{4a}{c}\frac{T^3}{\rho}$$

   where the last term is the entropy in radiation. Ignore the entropy in radiation and show that:

$$S = \frac{3}{2}\frac{N_A k}{\mu} \ln \frac{P}{\rho^{5/3}} + \text{constant}',$$

   that is, that $S = S(K)$.

   (b) (25 points) For a gas that consists of matter and radiation with gas fraction β, that is

$$P = P_g + P_{rad}$$
$$P_g = \beta P$$
$$P_{rad} = (1-\beta)P$$

   Show that:

$$T = \left(\frac{N_A k}{\mu}\frac{3}{a}\frac{1-\beta}{\beta}\right)^{1/3} \rho^{1/3}$$

   and that

$$P = \left[\left(\frac{N_A k}{\mu}\right)^4 \frac{3}{a}\frac{1-\beta}{\beta^4}\right]^{1/3} \rho^{4/3}$$

   and thus that $n = 3$ corresponds to the Eddington standard model.