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Problem 5:

A telescopic survey to find nearby "space rocks" can find moving objects to a magnitude of 18.5. The relationship between magnitude and flux for the "visible" passband used is:

$$mag = -2.5log(f/f_0)$$

where f is the flux from the target and f_0 is the flux from a mag = 0 object (assume f_0 = 1.0E−8 W m^{-2}).

An approximately spherical space rock, 50 meters in diameter, with an albedo of 0.2, comes near the Earth. The rock shines in the visible only by reflected sunlight.

a) [1 pts] Calculate the flux of sunlight in the visible at a distance of 1 AU from the Sun. Assume the "visible" pass band encompasses 1/3 of the bolometric power output of the Sun.

b) [2 pts] From the parameters given calculate the visible power of the rock (power of reflected sunlight) when approximately 1 AU from the Sun.

c) [4 pts] What is the maximum distance from Earth that the survey could detect the rock? (The rock will not emit isotropically, of course, but only from its illuminated side. Just assume it reflects uniformly from half its surface (the "day" side)). Don't worry about the changing solar flux with distance- just assume the rock is near 1 AU from Sun.

d) [3 pt] Assume the rock has a density of a typical rocky asteroid. Assume it hits the Earth with a speed equal to the escape speed of the Earth. How many megatons of energy would be released by the impact? (1 MT = 4.2E15 Joules).

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Astro #5

a) $mag = -2.5 \log(f/f_0)$

$-\frac{2}{5} m = \log(f/f_0)$

$e^{-.4m} = \frac{f}{f_0}$

$f_0 e^{-.4m} = f$

$1 \cdot 10^{-8} \frac{W}{m^2} e^{-.4(4.7)} = f_0$

$1.526 \cdot 10^{-9} \frac{W}{m^2} = f_0$

$\Rightarrow f_{VIS} = 5.09 \cdot 10^{-10} \frac{W}{m^2}$

b) $f_{VIS} = 5.09 \cdot 10^{-10} \frac{W}{m^2}$

$(0.2) \cdot f_{VIS} \cdot \pi (25m)^2 = P_{rock}$ ⟵ $I = \frac{P}{SA}$

$P_{rock} = 1.99 \cdot 10^{-7} \, W \cdot 4$

c) $f_0 e^{-.4m} = f$

$\Rightarrow f_{18.5} = 6.11 \cdot 10^{-12} \frac{W}{m^2} = J/s$

$P_{18.5} = f_{18.5} \cdot \pi (25m)^2 \cdot 4$

$= 1.2 \cdot 10^{-8} \, W \cdot 4$

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#5 (cont.)

d) *Assume density of rocky asteroid is same as density of earth.

$$\rho = \frac{m}{V}$$

$$= \frac{5.97 \cdot 10^{27}\ g}{(6.37 \cdot 10^{8}\ cm)^3 \cdot \pi \cdot \frac{4}{3}}$$

$$= 5.51 \frac{g}{cm^3}$$

* Find escape velocity

$$\frac{1}{2} \cancel{m} v^2 = \frac{M \cancel{m} G}{R}$$

$$v = \sqrt{\frac{2MG}{R}}$$

$$V_{esc,\oplus} = \sqrt{\frac{2(5.97 \cdot 10^{27} g)(6.67 \cdot 10^{-8} \frac{cm^3}{g s^2})}{(6.37 \cdot 10^{8} cm)}}$$

$$= 1.12 \cdot 10^{6} \frac{cm}{s}$$

$$E_{asteroid} = \frac{1}{2} m V_{esc\oplus}^2$$

$$= \frac{1}{2}\left(\rho \cdot \frac{4}{3}\pi (2500 cm)^3\right)\left(1.12 \cdot 10^{6} \frac{cm}{s}\right)^2$$

$$= 2.25 \cdot 10^{23} \frac{g\, cm^2}{s^2}$$

$$= 2.25 \cdot 10^{16}\ J$$

$$\boxed{= 5.36\ MT}$$

cm → m $(10^2)^2$
g → kg 10^3