John?

PROBLEM 4

Sirius is a visual binary with a period of 49.94 yr. Its measured trigonometric parallax is 0.37921 ± 0.00158 arcsec and, assuming that the plane of the orbit is in the plane of the sky, the true angular extent of the semimajor axis of the reduced mass is 7.61 arcsec. The ratio of the distances of Sirius A and Sirius B from the center of mass is $a_A/a_B = 0.466$.

a) (3 points) Find the mass of each member of the system.

b) (3 points) The absolute bolometric magnitudes of Sirius A and Sirius B are 1.36 and 8.79, respectively. Determine their luminosities. Express your answers in terms of the luminosity of the Sun.

c) (2 points) The effective temperature of Sirius B is 24,790 K. Estimate its radius, and compare your answer to the radii of the Sun and Earth.

d) (2 points) Estimate the surface gravity of Sirius B in cgs units. Compare your answer to the surface gravity of the Sun and Earth.

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Astro #4

a) $\frac{a_A}{a_B} = 0.466 = \frac{m_B}{m_A}$ $\qquad a = \alpha d$ (in radians)

$$P^2 = \frac{4\pi^2}{G(m_1+m_2)} a^3 \rightarrow m_1 + m_2 = \frac{4\pi^2}{GP^2} a^3$$

$$d = \frac{1}{\alpha}\,pc$$
$$= \frac{1}{.37921}$$
$$= 2.64\,pc$$
$$= 8.17 \cdot 10^{18}\,cm$$

$$a = \alpha d$$
$$= 7.61 \cdot \frac{1^\circ}{3600\,arcsec} \cdot \frac{\pi\,rad}{180^\circ} \cdot 8.17 \cdot 10^{18}\,cm$$
$$= 3.02 \cdot 10^{14}\,cm$$

$$\Rightarrow m_1 + m_2 = \frac{4\pi^2 (3.02 \cdot 10^{14})^3}{6.67 \cdot 10^{-8} \frac{cm^3}{g s^2} \cdot (49.94 \cdot \pi \cdot 10^7\, s/yr)^2}$$

$$1.466\, m_A = 6.59 \cdot 10^{33}\,g$$

$$m_A = 4.5 \cdot 10^{33}\,g$$

$$m_B = 2.09 \cdot 10^{33}\,g$$

b) $\frac{L_A}{L_\odot} = 100^{(M_\odot - M_A)/5}$

$$= 22.49\, L_\odot$$

$$\frac{L_B}{L_\odot} = 100^{(M_\odot - M_B)/5}$$

$$= 0.024\, L_\odot$$

#4 (cont.)

c) $L = 4\pi r^2 \sigma T^4$

$$r = \left(\frac{L}{4\pi\sigma T^4}\right)^{1/2}$$

$$= 5.88 \cdot 10^8 \text{ cm}$$

$R_\odot = 7 \cdot 10^{10}$ cm

$R_\oplus = 6.37 \cdot 16^8$ cm

$\Rightarrow \quad r < R_\oplus$

d) $$mg = \frac{GM_1 M_2}{r^2}$$

$$g = \frac{GM_1}{r^2}$$

$$= \frac{6.67 \cdot 10^{-8} \cdot 2.09 \cdot 10^{33}}{(5.88 \cdot 10^8)^2}$$

$$= 4.03 \cdot 10^8 \ \frac{cm}{s^2}$$

$g_\oplus = 9.8 \cdot 10^2 \ \frac{cm}{s^2}$

$g_\odot = 2.7 \cdot 10^4 \ \frac{cm}{s^2}$