Dai? / Leighly?

PROBLEM 4

a. (3 points) The hot gaseous halo of galaxy clusters is pressure supported, and thus it follows the hydrostatic equilibrium equation. Write down the hydrostatic equilibrium equation and the ideal gas law.

b. (4 points) First derive the following equation

$$\frac{dP}{dr} = \frac{k}{\mu m_H}\left(T\frac{\partial\rho}{\partial r} + \rho\frac{\partial T}{\partial r}\right), \tag{1}$$

assuming that the mean molecular weight is a constant for the gas. Then derive the expression for the total gravitational mass

$$M = -\frac{kTr}{\mu m_H G}\left(\frac{\partial \ln\rho}{\partial \ln r} + \frac{\partial \ln T}{\partial \ln r}\right). \tag{2}$$

c. (3 points) Using a so-called β model for the gas density and assuming that the gas is isothermal, the expression for M can be written as

$$M = \frac{3\beta kTr}{\mu m_H G}\left(\frac{r^2}{r^2 + r_c^2}\right). \tag{3}$$

For a cluster with a temperature of T = 5 keV, a core radius r_c=400 kpc, mean molecular weight $\mu = 0.61$, and $\beta = 0.7$, find the total mass of the cluster within 2 Mpc.

Aug 2012

Astro #4

a) $PV = nRT$ (Ideal Gas Law)

$= NkT$

$$\frac{dP}{dr} = -\frac{GM}{r^2}\rho \quad \text{(HSE eqn)}$$

b) Rewrite Ideal gas law as

$$P = \frac{\rho}{m_H \mu} kT$$

$$\frac{dP}{dr} = \frac{k}{\mu m_H}\frac{d}{dr}(\rho T)$$

$$= \frac{k}{\mu m_H}\left(\rho\frac{\partial T}{\partial r} + T\frac{\partial \rho}{\partial r}\right)$$

* Rewriting HSE eqn

$$\frac{r^2 dP}{-G\rho dr} = m$$

$$\Rightarrow m = \frac{r^2}{-G\rho}\left(\frac{k}{\mu m_H}\left[\rho\frac{\partial T}{\partial r} + T\frac{\partial \rho}{\partial r}\right]\right)$$

$$= -\frac{kTr}{\mu m_H G}\left(\frac{r}{T}\frac{\partial T}{\partial r} + \frac{r\partial \rho}{\rho \partial r}\right)$$

$$= -\frac{kTr}{\mu m_H G}\left(\frac{\partial \ln \rho}{\partial \ln r} + \frac{\partial \ln(T)}{\partial \ln r}\right)$$

c) * β assumes $\beta = \frac{P_{gas}}{P_{tot}} \Rightarrow P_{gas} = \beta P_{tot}$

$$P = \frac{1}{\beta}\frac{\rho}{m_H \mu}kT, \quad T = \text{const.}$$

$$\Rightarrow m = \frac{r^2}{-G\rho}\frac{dP}{dr}$$

$$= \frac{r^2}{-G\rho}\frac{d}{dr}\left[\frac{1}{\beta}\frac{\rho}{m_H \mu}kT\right] ??$$

Aug 2012

#4 (cont.)

* Note: 1eV = 11,600 K

c) * Calculation only

$$M = \frac{3\beta k T r}{\mu m_H G}\left(\frac{r^2}{r^2 + r_c^2}\right)$$

$$= \frac{3(0.7)(1.38\cdot10^{-7}\,\frac{erg}{K})(5\text{ keV})(2\text{ Mpc})}{(0.61)(1.66\cdot10^{-24}\,g)(6.67\cdot10^{-8}\,\frac{cm^3}{g s^2})}\left[\frac{(2\text{ Mpc})^2}{(2\text{ Mpc})^2 + (400\text{ kpc})^2}\right]$$

$$= \frac{2.1(1.38\cdot10^{-7}\,\frac{g\,\cancel{cm^2}}{\cancel{s^2} K})(5000\cdot 11{,}600\text{ K})(2\cdot10^6\cdot 3.1\cdot10^{18}\,\cancel{cm})}{(0.61)(1.66\cdot10^{-24}\,g)(6.67\cdot10^{-8}\,\frac{\cancel{cm^3}}{\cancel{g}\,\cancel{s^2}})}\left[\frac{(2\cdot10^6)^2}{(2\cdot10^6)^2 + (4\cdot10^5)^2}\right]$$

$$= 1.48\cdot10^{57}\,g$$