PROBLEM 3

3. (a) (1 point) Write down the general radiative transfer equation (RTE) in plane-parallel geometry and define all the terms including units.

   (b) (1 point) Define the 3 Eddington moments.

   (c) (1 point) Explain what the grey approximation is.

   (d) (1 point) Make the grey approximation and derive the 2 ordinary differential equations for the moments from the RTE.

   (e) (1 point) What does radiative equilibrium tell you about H in this case?

   (f) (1 point) Make the "two-stream" approximation

$$I = \begin{cases} I^+ & \mu \geq 0 \\ I^- & \mu < 0 \end{cases}$$

   and obtain the Eddington moments in this case.

   (g) (1 point) From the two-stream case above, find the Eddington factors $f = K/J$, and $h(0) = H(0)/J(0)$, where $H(0)$ is the value of H at the surface $\tau = 0$. Assume no external illumination.

   (h) (2 points) Using the Eddington factors found above, solve the moment equations for J.

   (i) (1 point) Assume $S = B$, where B is the grey planck function and find the temperature as a function of τ.

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#3

a) $$\mu \frac{dI}{dz} = \eta_\nu - \kappa_\nu I_\nu$$

$\mu = \cos\theta$
η_ν = emissivity
κ_ν = opacity
I_ν = radiation intensity

b) $$J_\nu = \frac{1}{2}\int_{-1}^{1} I_\nu \, d\mu$$

$$H_\nu = \frac{1}{2}\int_{-1}^{1} \mu I_\nu \, d\mu$$

$$K_\nu = \frac{1}{2}\int_{-1}^{1} \mu^2 I_\nu \, d\mu$$

c) The grey approximation says that χ is independent of ν

d) * In the grey approximation

$$\mu \frac{dI}{dz} = \eta - \kappa I$$

$$\cancel{\mu \frac{dI}{d\tau} = S - I}$$

$$\frac{1}{2}\int_{-1}^{1} \mu \frac{dI}{dz} = \frac{dH}{dz} = \frac{1}{2}\int_{-1}^{1} \chi_\nu (I_\nu - S_\nu)\, d\mu$$

$$= -\chi (J - S)$$

$$\frac{1}{2}\int_{-1}^{1} \mu^2 \frac{dI}{dz} = \frac{dK}{dz} = \frac{1}{2}\int_{-1}^{1} \chi (I - S)\, \mu \, d\mu$$

$$= \chi H$$

e) * In the case of radiative equilibrium (energy only carried by radiation

$$\int_0^\infty \frac{dH_\nu}{dz} d\nu = \frac{dH}{dz} = -\int \chi (J - S)$$

$= 0$ b/c absorption = emission