John?

PROBLEM 1

a) (3 points) Calculate the orbital semi-major axis (a_{sun}) of the Sun's orbit about the barycenter of the Solar System, in AU, in response to Jupiter's orbital motion. Since Jupiter constitutes ~70% of the non-solar mass of our Solar System, you can ignore Solar System bodies less massive than Jupiter in your computation. Assume $a_{Jupiter} = 5.2$ AU.

b) (2 points) To an external observer, what would be the transit depth of an Earth-size planet located at a=0.1AU (assume circular orbit) about a M dwarf star (Mass = 0.3 Msun; Radius = 0.8 Rsun)?

c) (3 points) To an external observer, what would be the transit duration (in hours) of an Earth-size planet located at a=0.1AU (assume circular orbit) about a M dwarf star (Mass = 0.3 Msun; Radius = 0.8 Rsun)?

d) (2 points) To an external observer located 20 pc away, what would be the angular separation in arcseconds between an Earth-size planet located at a=0.1 AU and its host star?

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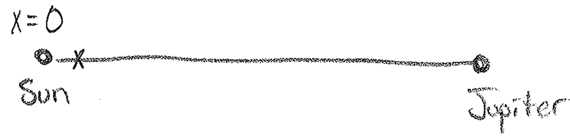
Astro #1

a) * Bodies orbit mutual center of mass; Assuming point particles



* Assuming $M_J \sim \frac{M_\odot}{1000}$

$$X_{cm} = \frac{\sum_i x_i m_i}{\sum_i m_i}$$

$$= \frac{M_\odot \cdot 0 + 5.2\,AU \cdot \frac{M_\odot}{1000}}{M_\odot + \frac{M_\odot}{1000}}$$

$$= \frac{5.2\,AU/1000}{1 + \frac{1}{1000}}$$

$$= 7.79 \cdot 10^{10}\,cm$$

$$a_{sun} = X_{cm} - R_\odot$$

$$= 7.9 \cdot 10^{9}\,cm$$

b) * Assuming no emission from planet

$$\frac{\Delta L}{L} = \frac{4\pi\sigma T^4 (r_s^2 - r_p^2)}{4\pi\sigma T^4 r_s^2}$$

$$= \frac{r_s^2 - r_p^2}{r_s^2}$$

$$= 1 - \frac{r_p^2}{r_s^2}$$

$$= 1 - \boxed{8.4 \cdot 10^{-5}}$$

$$= .999916$$

$$\frac{R_\oplus}{R_\odot} = \frac{6.37 \cdot 10^6\,m}{6.95 \cdot 10^8\,m}$$

$$R_\oplus = 9.1 \cdot 10^{-3}\,R_\odot$$

$$= .009\,R_\odot$$

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#1 (cont.)

c) $F = \frac{GMm}{r^2} = m\frac{v^2}{r}$

$$\frac{GM}{r} = v^2$$

$$\Rightarrow v = \sqrt{\frac{GM}{r}}$$

$$= \sqrt{\frac{6.67 \cdot 10^{-8} \frac{g^{-1}}{cm^3 s^2} \cdot 2 \cdot 10^{33} g}{1.5 \cdot 10^{12} cm}}$$

$$= 9.43 \cdot 10^6 \frac{cm}{s}$$

$R_\oplus = 6.37 \cdot 10^8$ cm

$R_\odot = 7 \cdot 10^{10}$ cm

* From first to last contact

$$t = \frac{2R_\odot + 2R_\oplus}{v}$$

$$= \frac{1.4 \cdot 10^{11} + 1.27 \cdot 10^9}{9.43 \cdot 10^6}$$

$$= 1.49 \cdot 10^4 \text{ s}$$

$$= 4.16 \text{ hrs}$$

d) $d = \frac{1}{\alpha}$ pc

$$\alpha = \left(\frac{1}{20}\right)''$$

$$= .05 \text{ arcsec}$$