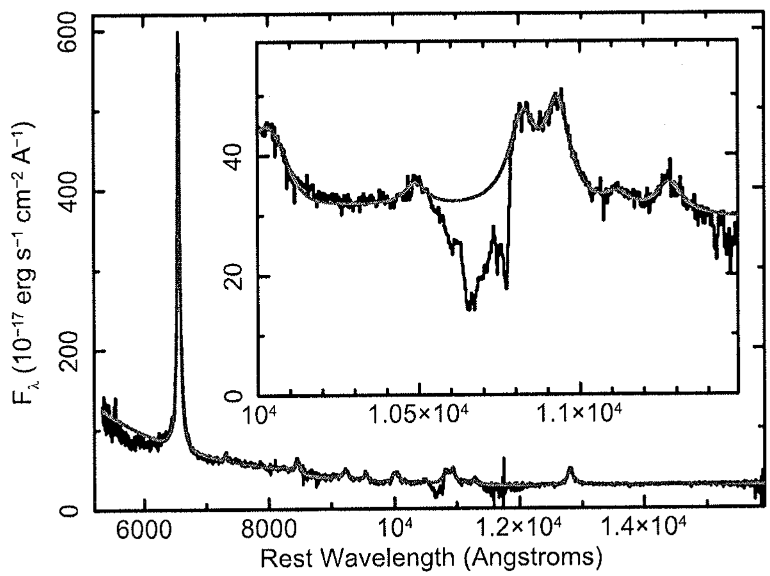
PROBLEM 6

6. A fraction of quasars have broad, blueshifted absorption lines that indicate high-velocity outflows emerging from the central engine. Generally, an absorption profile can be described as:

$$\frac{I}{I_0} = exp(-\tau(\lambda))$$

where I is the observed flux density in the spectrum, I_0 is the intrinsic continuum (without absorption) and $\tau(\lambda)$ is the optical depth of the absorption trough originating from absorption by a single ion. An example of a broad absorption spectrum is shown below. The absorption is occurring in metastable helium in the 10830Å transition. A range of gas outflow velocities causes the absorption line to be broad.



Analysis of broad absorption lines is complicated by *partial covering*: the absorbing outflow does not cover all of the continuum emitting source, but rather covers only a fraction of it, C_f. Then, the absorption line looks shallower than it would be if the absorber covered the whole thing, and the inferred *apparent* optical depth is lower. However, this situation can be resolved, and the true optical depth and covering fraction can be determined if there are two lines in the spectrum that arise from the same lower level, because their true optical depth ratio is fixed by atomic physics. Specifically, the true optical depth ratio will be proportional to the ratio of $f_{ik}\lambda$, where f_{ik} and λ are the oscillator strength and wavelength of the transition. In that case, the intensity ratio can be expressed in these two equations:

$$I_s = (1 - C_f) + C_f e^{-\tau_s}$$

$$I_w = (1 - C_f) + C_f e^{-\tau_w}$$

where the subscripts w and s stand for weaker and stronger lines, respectively, and τ_s/τ_w is related by the ratio of their respective $f_{ik}\lambda$ values.

(a) (5 points) Consider a doublet, e.g., C IV. The first excited state has fine structure, so there are two possible transitions from the ground state to the first excited state, at 1548.2 and 1550.8Å (a doublet). The oscillator strengths for these two transtions are 0.190 and 0.0952 respectively. This means that the ratio of the optical depths τ_s/τ_w for these two transitions is effectively 2.

For the case of this doublet, solve the equations above for the covering fraction C_f and τ_s.

(b) (1 point) When scientists analyze an absorption line, they are often interested in measuring the column density N (in particles per cm^2) of the ion responsible for it. The optical depth and column density are related by the following equation:

$$\tau(\lambda) = \frac{\pi e^2}{m_e c^2} f_{ik} \lambda^2 N(\lambda),$$

where e is the charge on an electron, m_e is the mass of an electron, and c is the speed of light. Both τ and N are functions of lambda because as the absorption line is spread of a range of wavelengths due to the range of velocities over which the outflow is distributed.

Show that

$$\tau(v) = \frac{\pi e^2}{m_e c} f_{ik} \lambda N(v)$$

where v is velocity, λ is in Angstroms, and $N(v)$ is in atoms $\mathrm{cm}^{-2}(\mathrm{km\,s}^{-1})^{-1}$.

(c) (4 points) Further, show that

$$\tau(v) = 2.654 \times 10^{-15} f_{ik} \lambda N(v).$$