Eddie

PROBLEM 6

a. (2 pts) Write down the equation of radiative transfer for a plane-parallel atmosphere and define all the terms.

b. (3 pts) Assuming that there is no external irradiation at the surface, show that

$$I_\lambda = S_\lambda(1 - e^{-\tau_\lambda})$$

c. (5 pts) What is I_λ in terms of S_λ for the optically thin and optically thick cases? Do you expect to see emission or absorption lines at the wavelengths of large opacity, κ_λ?

Aug 2012

#6

a) Write down equation of radiative transfer in plane-parallel + define all terms

$$\mu \frac{dI_\nu}{dz} = \eta_\nu - \chi_\nu I_\nu$$

$\mu = \cos\theta$

η_ν = scattering coefficient

χ_ν = opacity

I_ν = intensity

b) Assuming there is no external irradiation at the surface, show: $I_\lambda = S_\lambda(1 - e^{\tau_\lambda})$

$$\frac{dI_\lambda}{dz} = \eta_\lambda - \chi_\lambda I_\lambda$$

$$\frac{dI_\lambda}{\chi_\lambda dz} = \frac{\eta_\lambda}{\chi_\lambda} - I_\lambda$$

* let $d\tau = \chi_\lambda dz$, $\frac{\eta_\lambda}{\chi_\lambda} = S_\lambda$

$$\frac{dI_\lambda}{d\tau} = S_\lambda - I_\lambda$$

$$\left(\frac{dI_\lambda}{d\tau} + I_\lambda = S_\lambda\right) e^{\tau}$$

$$\frac{dI_\lambda}{d\tau} e^{+\tau} + I e^{+\tau} = S_\lambda e^{\tau}$$

$$\frac{d\, I e^{\tau}}{d\tau} = S_\lambda e^{\tau}$$

$$-I_0 + I e^{\tau} = S_\lambda\left[e^{\tau_\lambda} - 1\right]$$

$$I_\lambda = S_\lambda\left(1 - e^{-\tau}\right)$$

Aug 2012

#6 (cont.)

c) Optically Thin $\rightarrow$ $\tau \ll 1$

$$I_\lambda = S_\lambda (1 - e^{-\tau_\lambda})$$

$$= S_\lambda \left(1 - \left[1 + \tau_\lambda + \tfrac{1}{2}\tau_\lambda^2 + \dots\right]\right)$$

$$= \left[+\tau_\lambda - \tfrac{1}{2}\tau_\lambda^2 + \tfrac{1}{6}\tau_\lambda^3 - \dots\right] S_\lambda$$

$$= S_\lambda \tau_\lambda - \tfrac{1}{2}\tau_\lambda^2 S_\lambda$$

Optically Thick $\rightarrow$ $\tau \gg 1$

$$I_\lambda = S_\lambda (1 - e^{-\tau_\lambda \to \infty})$$

$$I_\lambda = S_\lambda$$

In the optically thick case, you would expect to only see the source function. In the optically thin case, you would expect to see emission lines.