Kilroy

PROBLEM 2

When a $5M_\odot$ star leaves the main sequence it enters the largely horizontal sub-giant branch. Models indicate that the star spends about 350,000 years on this section of the HR diagram before beginning its ascent on the red giant branch. Compute the expected Kelvin-Helmholtz time scale for this phase of stellar evolution and explain any differences by doing the following:

a. (4) Show that the gravitational energy ultimately radiated away is:

$$E_g = \frac{3GM^2}{10R},$$

where M and R are the the stellar mass and radius, respectively. Assume the virial theorem and that the density of the star at any distance from its center is equal to the star's average density, $M/\frac{4}{3}\pi R^3$.

b. (3) If $L = 10^3 L_\odot$ and $T_{\text{eff}} = 10^{3.9}$ K, estimate the time in years that this luminosity could be sustained if it is based solely on gravitational energy.

c. (3) Compare your answer in b. with the model-predicted time and explain why they are different. Make sure you explain what current theory tells us is going on inside the star.

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a) $U = \frac{-GMm}{r}$ (Gravitational Potential Energy) $\Rightarrow dU = \frac{-GM\,dm}{r}$

$E = \frac{1}{2}U$ (Virial Thm)

* but $dm = 4\pi r^2 \rho\, dr$, $\rho = \frac{M}{\frac{4}{3}\pi R^3}$

$\Rightarrow dm = 4\pi r^2 \left(\frac{M}{\frac{4}{3}\pi R^3}\right) dr$

$\Rightarrow dU = -GM \frac{dm}{r}$

$= -G \frac{4}{3}\pi R^3 \rho \; 4\pi r^2 \rho\, dr$

$U = \int_0^R \frac{-16\pi^2 G r^3 \rho^2}{3r} r^2 dr$

$= \frac{-16\pi^2 G}{3} \int_0^R \rho^2 r^4 dr$

$= \frac{-16\pi^2 G}{3} \rho^2 \left(\frac{1}{5}R^5\right)$

$= -\frac{3GM^2}{5R}$

$\Rightarrow E = \frac{-3GM^2}{10R}$

b) $t_{KH} = \frac{\Delta E_g}{L_\odot}$

$= \frac{\frac{3GM^2}{10R}}{L}$

$= \frac{\frac{3G(5M_\odot)^2}{10R}}{10^3 L_\odot}$

* for stupidity's sake, assume $R \sim R_\odot$

$= \frac{25\,\frac{3GM_\odot^2}{10R_\odot}}{10^3 L_\odot}$

* $t_\odot$ can be expressly calculated using values on pg 1

$= \frac{25}{1000} t_\odot \approx 0.025 \cdot 10^7 \text{yr} = 2.5 \cdot 10^5 \text{yr}$

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c) Our KH timescale estimate is 250,000 yrs compared to our model estimate of 350,000 yrs.
This is b/c the KH timescale estimates how long the star would live if the only reason it shines is due to gravitational collapse. But we know that stars shine due to nuclear fusion, and that the release of energy from nuclear fusion of H → He is able to account for the timescales that objects around stars have existed; ie rocky planets/moons around sun dated via carbon dating.