PROBLEM 6

Consider a rigid satellite of mass m, radius r, and density ρ_m orbiting at a distance d from its massive primary planet of mass M, radius R, and density ρ_M (see the figure below).

a. (2 pts) Show that the angular speed of the satellite about the primary is $\omega = \sqrt{\frac{GM}{d^3}}$

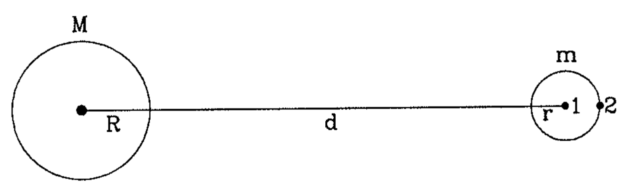
b. (3 pts) Find the differences in the gravitational acceleration between the center of the satellite (point 1) and the outer edge (point 2) due to the primary. Also find the differences in the centripetal acceleration between these two points. Show that the combination of the two effects is

$$\approx \frac{3GMr}{d^3}$$

c. (3 pts) The satellite will be tidally disrupted if the acceleration found in (b) is larger than the satellite's self gravitational acceleration. Show that the disruption occurs at

$$d = r(\frac{3M}{m})^{(1/3)} = R(\frac{3\rho_M}{\rho_m})^{(1/3)}$$

d. (2 pts) Assuming that the Earth and the Moon have the same density, at what distance would the Moon be disrupted? What about a moon around an Earth-size $1M_\odot$ ($3 \times 10^6 M_{earth}$) white dwarf star?



Jan 2012

Astro #6

a) $$P^2 = \frac{4\pi^2}{G(m_1+m_2)} a^3$$

$$= \frac{4\pi^2}{G(M+m)} d^3$$

* Assuming circular orbit at $M \gg m$

$$P^2 = \frac{4\pi^2}{GM} d^3$$

$$\left(\frac{2\pi}{w}\right)^2 = \frac{4\pi^2}{GM} d^3$$

$$\frac{4\pi^2}{w} = \frac{4\pi^2}{GM} d^3$$

$$\rightarrow w = \sqrt{\frac{GM}{d^3}}$$

b) $$ma = \frac{GMm}{r^2}$$

* at pt 1

$$a = \frac{GM}{(d+r)^2}$$

* Gravitational acceleration

* at pt 2

$$a = \frac{GM}{(d+2r)^2}$$

$$ma = m\frac{v^2}{r}$$

$$a = \frac{v^2}{r}$$

* but $w = \frac{v}{r}$

* Centripetal acceleration

$$\Rightarrow a = w^2 r$$

* at pt 1

$$a = \frac{GM}{d^3}(d+r)$$

* at pt 2

$$a = \frac{GM}{d^3}(d+2r)$$

$$\Rightarrow a_1 = -GM\left(\frac{1}{(d+r)^2} + \frac{d+r}{d^3}\right)$$

$$a_2 = -GM\left(\frac{1}{(d+2r)^2} + \frac{d+2r}{d^3}\right)$$

$$a_1 - a_2 = -GM\left[\left(\frac{1}{(d+r)^2} + \frac{d+r}{d^3}\right) - \left(\frac{1}{(d+2r)^2} + \frac{d+2r}{d^3}\right)\right]$$

Jan 2012

#6 (cont.)

c) $a = \frac{Gm}{r^2}$

$$\frac{Gm}{r^2} = \frac{3GMr}{d^3}$$

$$\frac{m}{r^2} = \frac{3Mr}{d^3}$$

$$d^3 m = 3Mr^3$$

$$d^3 = \frac{3M}{m} r^3$$

$$d = \left(\frac{3M}{m}\right)^{1/3} r$$

$$= R\left(\frac{3\rho_M}{\rho_m}\right)^{1/3}$$

d) Math Stuffs