John

PROBLEM 1

Suppose a main sequence star at a distance of d = 8 pc from the Earth has been observed to have a maximum radial velocity of 0.1 m/s, and the radial velocity varies periodically with a period P =1.5 years. From this we conclude that the star must have an unseen companion, a planet.

Assume that the star has mass of 0.8 solar masses, and T=5000 K. For simplicity, assume circular orbits and an inclination angle of 90°.

a. Calculate the average separation between the star and the planet. (4 points)

b. Calculate the mass of the planet. (4 points)

$$\frac{m_p^3}{(m_s + m_p)^2} \sin^3 i = \frac{P}{2\pi G} v_{1,r}^3$$

c. Can life exist on this planet? Explain. (2 points)

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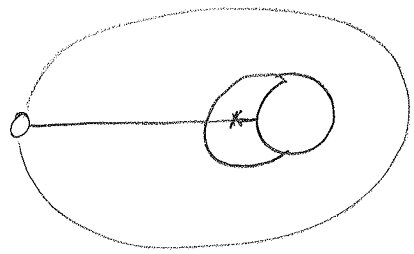
Astro #1

a) * Assuming circular orbits



$M = .8\, M_\odot$
$= 1.6 \cdot 10^{33}\, g$

$T = 5000\, K$

$$P^2 = \frac{4\pi^2}{G(M+m)} a^3$$

* rewrite in terms of reduced mass, $\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{m_s m_p}{m_s + m_p}$

* For star: $P^2 = \frac{4\pi^2}{G(m_s + \mu)} a^3$ $\qquad \frac{G m_s \mu}{a^2} = m_s \frac{V_s^2}{a} \rightarrow V_s = \sqrt{\frac{G\mu}{a}}$

$$\Rightarrow a = \left(\frac{G P^2 (m_s + \mu)}{4\pi^2}\right)^{1/3}$$

* For planet: $P^2 = \frac{4\pi^2}{G(\mu + m_p)} b^3$ $\qquad \frac{G m_p \mu}{b^2} = m_p \frac{V_p^2}{b}$

$$b = \left(\frac{G P^2 (\mu + m_p)}{4\pi^2}\right)^{1/3} \qquad V_p = \sqrt{\frac{G\mu}{b}}$$

* Separation always constant so center of mass is stationary

$$a + b = \left(\frac{GP^2}{4\pi^2}\right)^{1/3} \left((m_s + \mu)^{1/3} + (m_p + \mu)^{1/3} \right)$$

$$P^2 = \frac{4\pi^2}{G m_s} a^3 \qquad m_s = 0.8\, M_\odot$$

$$\Rightarrow \quad a = \left(\frac{G m_s P^2}{4\pi^2}\right)^{1/3}$$

$$= \left[\frac{6.67\cdot 10^{-8}\ \frac{cm^3}{g\cdot s^2}\cdot 0.8\cdot 2\cdot 10^{33} g\cdot (1.5\cdot\pi\cdot 10^7 s)^2}{4\pi^2}\right]^{1/3}$$

Correct

$$= 1.82\cdot 10^{13}\, cm$$
$$\approx 1.21\ AU$$

$$V_s = \sqrt{\frac{G\mu}{a}} \qquad V_s = \sqrt{\frac{G m_s}{a}}$$

$$a V_s^2 = G\left(\frac{m_s m_p}{m_s+m_p}\right)$$

$$\frac{a V_s^2}{\cancel{G}}\,\alpha = \frac{m_s m_p}{m_s+m_p}$$

$$(m_s+m_p)\,\alpha = m_s m_p$$

$$m_s\alpha = m_p(m_s-\alpha)$$

$$\frac{m_s\alpha}{m_s-\alpha} = m_p = 2.72\cdot 10^{18} g$$

$$P^2 = \frac{4\pi^2}{G(m_s+m_p)} a^3$$

$$\frac{P^2}{a^3} = \frac{4\pi^2}{G\, m_s+m_p}$$

$$m_s + m_p = \frac{4\pi^2 a^3}{G P^2}$$

$$m_p = \frac{4\pi^2 a^3}{G P^2} - m_s$$

$$= \frac{4\cdot\pi^2\cdot(1.82\cdot 10^{13}\, cm)^3}{6.67\cdot 10^{-8}\ \frac{cm^3}{g\, s^2}\cdot(1.5\cdot\pi\cdot 10^7 s)^2} - 0.8\cdot 2\cdot 10^{33} g$$

$$= 0$$

$$P^2 = \frac{4\pi^2}{G(m_s+\mu)} a^3$$

$$P^2 G\left(m_s + \frac{m_s m_p}{m_s+m_p}\right) = 4\pi^2 a^3$$

$$\frac{m_s m_p}{m_s+m_p} = \frac{4\pi^2 a^3}{P^2 G} - m_s$$

$$m_s m_p = \left(\frac{4\pi^2 a^3}{P^2 G} - m_s\right)(m_s+m_p)$$

$$\left(m_s - \frac{4\pi^2 a^3}{P^2 G}\right) m_p = \left(\frac{4\pi^2 a^3}{P^2 G} - m_s\right) m_s$$

$$m_p = -m_s$$

$$\frac{a^3}{m_s+\mu} = \frac{b^3}{m_p+\mu}$$

$$a^3(m_p+\mu) = b^3(m_s+\mu)$$

$$a^3\left(m_p + \frac{m_s m_p}{m_s+m_p}\right) = b^3\left(m_s + \frac{m_s m_p}{m_s+m_p}\right)$$

$$\frac{a^3}{b^3} = \frac{m_p + \frac{m_s m_p}{m_s+m_p}}{m_p + \frac{m_s m_p}{m_s+m_p}}$$

$$= \frac{m_s(m_s+m_p) + m_s m_p}{m_p(m_s+m_p) + m_s m_p}$$

$$\frac{a}{b} \approx \frac{m_s}{m_p} \qquad = \frac{m_s^2 \cancel{+2 m_s m_p}}{\cancel{m_p^2} + 2 m_s m_p}$$