Eddie

Problem 6:

The equation of radiative transfer in spherical coordinates is:

$$\mu\frac{\partial I_\nu}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial I_\nu}{\partial \mu} = -\chi_\nu I_\nu + \eta_\nu$$

a) [3pts] Show that the moment equations can be written:

$$\frac{1}{r^2}\frac{\partial}{\partial \tau_\nu}(r^2 H_\nu) = (J_\nu - S_\nu)$$

$$\frac{\partial K_\nu}{\partial \tau_\nu} + \frac{(J_\nu - 3K_\nu)}{(\chi_\nu r)} = H_\nu$$

b) [2pts] Introduce the Eddington factor $f_\nu = K_\nu / J_\nu$ and rewrite the moment equations in terms of it.

c) [2pts] Explain the problem with deriving a single second order equation for J_ν as is done in the plane-parallel case.

d) [3pts] Show that in fact with the sphericity factor:

$$ln(r^2 q_\nu) = \int_{r_c}^{r} [(3f_\nu - 1)/(r' f_\nu)] dr' + ln(r_c^2)$$

where r_c is the radius of the opaque core, the two moment equations can be combined to give:

$$\frac{\partial^2}{\partial X_\nu^2}(r^2 q_\nu f_\nu J_\nu) = q_\nu^{-1} r^2 (J_\nu - S_\nu)$$

where $dX_\nu = q_\nu d\tau_\nu$.

Aug 2011

Astro #6

RTE in spherical:

$$\mu \frac{\partial I_\nu}{\partial r} + \frac{1-\mu^2}{r} \frac{\partial I_\nu}{\partial \mu} = -\chi_\nu I_\nu + \eta_\nu$$

a) $$\mu \frac{\partial I_\nu}{\partial r} + \frac{1-\mu^2}{r} \frac{\partial I_\nu}{\partial \mu} = -\chi_\nu I_\nu + \eta_\nu$$

$$\frac{\mu}{-\chi_\nu} \frac{\partial I_\nu}{\partial r} + \frac{1-\mu^2}{-r\chi_\nu} \frac{\partial I_\nu}{\partial \mu} = I_\nu - S_\nu \qquad \text{where } S_\nu = \frac{\eta_\nu}{\chi_\nu}$$

$$? \; \mu \frac{\partial I_\nu}{\partial \tau_\nu} + \frac{1-\mu^2}{\tau_\nu} \frac{\partial I_\nu}{\partial \mu} = I_\nu - S_\nu \qquad \text{where } d\tau_\nu = -\chi_\nu dr$$
$$\tau_\nu = -\chi_\nu r$$

Moment equations:

$$J_\nu = \frac{1}{2}\int_{-1}^{1} I_\nu d\mu$$

$$K_\nu = \frac{1}{2}\int_{-1}^{1} \mu^2 I_\nu d\mu$$

$$H_\nu = \frac{1}{2}\int_{-1}^{1} \mu I_\nu d\mu$$

$$\Rightarrow I_\nu = S_\nu + \mu \frac{\partial I_\nu}{\partial \tau_\nu} + \frac{1-\mu^2}{\tau_\nu} \frac{\partial I_\nu}{\partial \mu}$$

$$\hookrightarrow H_\nu = \frac{1}{2}\int_{-1}^{1} \cancelto{0}{\mu S_\nu} + \mu^2 \frac{\partial I_\nu}{\partial \tau_\nu} + \frac{\mu - \mu^3}{\tau_\nu} \frac{\partial I_\nu}{\partial \mu} d\mu$$

$$H_\nu = \quad + \frac{\partial K_\nu}{\partial \tau_\nu} + \frac{1}{\chi_\nu r}\left(J_\nu - 3K_\nu\right) \checkmark$$

* Integrating RTE over all μ yields

$$J_\nu - S_\nu = \frac{\partial K_\nu}{\partial \tau_\nu} + \frac{1}{\tau_\nu}(I_\nu - H_\nu)$$

$$= \frac{\partial K_\nu}{\partial \tau_\nu} + \frac{1}{\tau_\nu}\left(I_\nu - \left[\frac{\partial K_\nu}{\partial \tau_\nu} + \frac{1}{\chi_\nu}(J_\nu - 3K_\nu)\right]\right)$$

$$= \frac{\partial K_\nu}{\partial \tau}\left(1 - \frac{1}{\tau_\nu}\right) + \frac{1}{\tau_\nu}\left(I_\nu - \frac{1}{\chi_\nu}[J_\nu - 3K_\nu]\right) \quad ???$$

Aug 2011

#6 (cont.)

b) * Assuming

$$\frac{1}{r^2}\frac{\partial}{\partial \tau_\nu}(r^2 H_\nu) = J_\nu - S_\nu$$

$$\frac{\partial K_\nu}{\partial \tau_\nu} + \frac{(J_\nu - 3K_\nu)}{\chi_\nu r} = H_\nu$$

* if $f_\nu = \frac{K_\nu}{J_\nu}$

$$\frac{1}{J_\nu}\left[\frac{\partial K_\nu}{\partial \tau_\nu} + \frac{J_\nu - 3K_\nu}{\chi_\nu r} = H_\nu\right]$$

$$\frac{\partial f_\nu}{\partial \tau_\nu} + \frac{1 - 3f_\nu}{\chi_\nu r} = \frac{H_\nu}{J_\nu}$$

$$= \frac{H_\nu K_\nu}{f_\nu}$$