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# PROBLEM 1

a. (7 points) Assume that the gas component of a galaxy, with a mass fraction $f_g$, is virialized and follow the overall density profile of the galaxy, a singular isothermal sphere mass profile,

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2}, \tag{1}$$

where $\sigma$ is the velocity dispersion of the galaxy. The galaxy has a central AGN, which is radiating at the Eddington luminosity,

$$L_{Edd} = 4\pi G c m_p M_{BH}/\sigma_T. \tag{2}$$

A fraction, $f_w$, of the energy radiated by the central AGN is deposited to the gas in the form of kinetic energy. This kinetic "feedback" energy from the AGN can drive the gas in the host galaxy to flow outward. Assume that the final gas outflow is in a spherical shell with a constant velocity, v, and half of the kinetic feedback energy is converted to the gravitational potential of the gas and the other half to the kinetic energy of the gas during the outflowing process. Use the conservation or transfer of energy to show that the final gas wind speed is

$$v^3 = \frac{G L_{Edd} f_w}{2\sigma^2}. \tag{3}$$

b. (3 points) If the wind speed is large enough to escape the potential well of the galaxy ($v = \sigma$), the central AGN will blow out the majority of gas in the galaxy and terminate the formation of stars. Show that this gives us the $M_{BH}$–$\sigma$ relation,

$$M_{BH} = \frac{1}{2\pi} \frac{\sigma_T}{G^2 c m_p} \frac{1}{f_w} \sigma^5, \tag{4}$$

where G is the gravitational constant, $\sigma_T$ is the Thomson cross section, c is the speed of light, and $m_p$ is the mass of a proton.

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# Astro #1

a) $\rho(r) = \frac{\sigma^2}{2\pi G r^2}$

$$M_G = \int \rho(r)\, dV$$

$$= \int_0^R \int_0^{2\pi} \int_0^{\pi} \frac{\sigma}{2\pi G r^2} r^2 \sin\theta \, dr\, d\theta\, d\varphi$$

$$= \int_0^R dr \int_0^{2\pi} d\varphi \int_0^{\pi} \frac{\sigma^2 \sin\theta}{2\pi G} d\theta$$

$$= \int_0^R dr \int_0^{2\pi} d\varphi \frac{\sigma^2}{2\pi G} \left( -\cos\theta \Big|_0^{\pi} \right)$$

$$-(-1) - -1$$

$$= \int_0^R dr \int_0^{2\pi} \frac{\sigma^2}{\pi G} d\varphi$$

$$= \int_0^R \frac{2\sigma^2}{G} dr$$

$$M_G = \frac{2\sigma^2 R}{G}$$

$$\Rightarrow \frac{1}{2} m_g v^2 = \frac{1}{2} f w L_{edd} t$$

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#1 (cont.)

b) $$v^3 = \frac{G L_{edd} f_w}{2\sigma^2}$$

$$L_{edd} = \frac{4\pi G c m_p M_{BH}}{\sigma_T}$$

* Assuming $v = \sigma$

$$\sigma^3 = \frac{G f_w}{2\sigma^2}\left(\frac{4\pi G c m_p M_{BH}}{\sigma_T}\right)$$

$$\frac{2\sigma^5 \sigma_T}{4\pi G^2 c m_p f_w} = M_{BH}$$

$$\hookrightarrow M_{BH} = \frac{\sigma_T \sigma^5}{2\pi G^2 c m_p f_w}$$