Dai

Problem 2:

a) [5 pts] Assume that a model for the dark matter halo of the Galaxy is:

$$\rho(r) = \frac{C_0}{(a^2 + r^2)},$$

where ρ is density, r is distance from the galactic center, and $a = 2.8$ kpc. Show that the amount of dark matter interior to a radius r is given by the expression:

$$M_r = 4\pi C_0 \left[r - a\ tan^{-1}\left(\frac{r}{a}\right)\right]$$

b) [2 pts] If 5.5×10^{11} $M_\odot$ of dark matter is located within 100 kpc of the Galactic center, determine C_0 in units of $M_\odot$/kpc. Repeat your calculation if 1.3×10^{12} $M_\odot$ is located within 230 kpc of the Galactic center.

c) [3 pts] Estimate the amount of dark matter (in solar masses) within a radius of 50 kpc of the Galactic center. Compare your answer to the mass of the stellar halo (choose a reasonable value for the latter).

Aug 2011

Astro #2

a) Show the amount of dark matter interior to a radius r is $M(r) = 4\pi C_0[r - a\tan^{-1}(\frac{r}{a})]$ when $\rho(r) = \frac{C_0}{a^2+r^2}$

$$M(r) = \int_0^r \rho(r')dr'$$
$$= \int_0^r \frac{C_0}{a^2+r^2} dr^3$$
$$= 4\pi C_0 \int_0^r \frac{r^2}{a^2+r^2} dr$$
$$= 4\pi C_0 \left(r - \frac{1}{a}\tan^{-1}\left(\frac{r}{a}\right)\Big|_0^r \right)$$
$$= 4\pi C_0 \left(r - \frac{1}{a}\tan^{-1}\left(\frac{r}{a}\right) - \left(0 - \frac{1}{a}\tan^{-1}\left(\frac{0}{a}\right)\right)\right)$$
$$= 4\pi C_0 \left(r - \frac{1}{a}\tan^{-1}\left(\frac{r}{a}\right)\right) \checkmark$$

b) If $5.5 \cdot 10^{11}\ M_\odot$ of DM is located w/in 100 kpc of the Galactic center, find C_0 in terms of $M_\odot$/kpc. Repeat w/ $1.3 \cdot 10^{12}\ M_\odot$ w/in 230 kpc

$$M(r) = 4\pi C_0 \left(r - \frac{1}{a}\tan^{-1}\left(\frac{r}{a}\right)\right)$$
$$\Rightarrow C_0 = \frac{M(r)}{4\pi\left(r - \frac{1}{a}\tan^{-1}\left(\frac{r}{a}\right)\right)}$$

(1) $$C_0 = \frac{5.5 \cdot 10^{11}\ M_\odot}{4\pi\left(100 - \frac{1}{2.8}\tan^{-1}\left(\frac{100}{2.8}\right)\right)}$$
$$= 4.4 \cdot 10^8 \frac{M_\odot}{1\text{kpc}}$$

(2) $$C_0 = \frac{1.3 \cdot 10^{12}\ M_\odot}{4\pi\left(230\text{kpc} - \frac{1}{2.8\text{kpc}}\tan^{-1}\left(\frac{230}{2.8}\right)\right)}$$
$$= 4.51 \cdot 10^8 \frac{M_\odot}{\text{kpc}}$$

c) Estimate the amount of DM w/in 50 kpc of galactic center. Compare to mass of stellar halo.

$\rightarrow C_0 \approx 4.45 \cdot 10^8$

$$M(r) = 4\pi\left(4.45 \cdot 10^8 \frac{M_\odot}{1\text{kpc}}\right)\left(50\text{kpc} - \frac{1}{2.8\text{kpc}}\tan^{-1}\left(\frac{50\text{kpc}}{2.8\text{kpc}}\right)\right)$$
$$= 2.77 \cdot 10^{11}\ M_\odot$$