Dai? / Lenghly?

# PROBLEM 2

The Crab pulsar has a period of $P = 0.0333$ seconds, and a slow-down rate of $\dot{P} = 4.21 \times 10^{-13}$. The Crab nebula emits a total luminosity of $5 \times 10^{31}$ W. A neutron star can be assumed to have a mass equal to $1.4\,M_\odot$ and a radius of 10km.

a. What is the size of the light cylinder for the crab pulsar? (2 points)

b. Show that the rate of rotational energy lost approximately equals the luminosity of the nebula. (3 points)

c. The energy per second emitted by a rotating magnetic dipole is

$$\frac{dE}{dt} = -\frac{32\pi^5 B^2 R^6 \sin^2\theta}{3\mu_0 c^3 P^4}.$$

Assuming that the rotational kinetic energy lost by the star is carried away by magnetic dipole radiation, derive an equation for the magnetic field at the pole of the neutron star. Use the parameters for the Crab pulsar to obtain a value of the magnetic field in Teslas. (2 points)

c. Discuss and explain the properties of various classes of pulsars. Sketch the $P\dot{P}$ diagram, show how it is populated by different classes of pulsars, and explain how we use this diagram to infer the magnetic field and age of these objects. (3 points)

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# Astro #2

a) * See Fig 16.26 of B.O.B (pg 600) for description of light cylinder

$$R_c = \frac{cP}{2\pi}$$

$$= \frac{3\cdot 10^{10}\ \frac{cm}{s} \cdot (.0333\ s)}{2\pi}$$

$$= 1.59 \cdot 10^{8}\ cm$$

b) $\dot{P} = 4.21 \cdot 10^{-13}\ s$

$$RE = \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} I \left(\frac{P}{2\pi}\right)^{-2}$$

$$= \frac{2}{5} MR^2 \left(\frac{P}{2\pi}\right)^{-2}$$

$$= \frac{8\pi^2 MR^2}{5P^2}$$

$[E] = \frac{g\,cm^2}{s^2}$ $\quad [\dot{E}] = \frac{g\,cm^2}{s^3}$

$$\dot{E} = \frac{8\pi^2 MR^2}{5}\left(-2\frac{\dot{P}}{P^3}\right)$$

$$= \frac{-16\pi^2 MR^2\dot{P}}{5P^3}$$

$$= \frac{-16\pi^2 (1.4 \cdot 2 \cdot 10^{33} g)(1 \cdot 10^{6} cm)^2 (4.21 \cdot 10^{-13})}{5\ (0.0333\ s)^3}$$

$$= -1.01 \cdot 10^{39}\ \frac{g\,cm^2}{s^3}$$

$$L = 5 \cdot 10^{31}\ W$$

$$= 5 \cdot 10^{31}\ \frac{kg\,m^2}{s^3}$$

$$= 5 \cdot 10^{38}\ \frac{g\,cm^2}{s^3}$$

$\dot{E} \approx 2L \Rightarrow$ close enough, off by factor of 2 somewhere

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#2 (cont.)

$$[T] = \frac{N}{A\,m} \leftarrow \frac{kg\,m}{s^2}$$

$$[\mu_0] = 4\pi \cdot 10^{-7}\ \frac{N}{A^2}$$

c) $$\frac{dE}{dt} = \frac{-32\pi^5 B^2 R^6 \sin^2\theta}{3\mu_0 c^3 P^4}$$

$$\Rightarrow B = \left[\frac{3\dot{E}\mu_0 c^3 P^4}{-32\pi^5 R^6 \sin^2\theta}\right]^{1/2}$$ ← B.O.B says assume $\theta = 90°$

↳ "light house assumption"

$$B_{pole} = \left[\frac{3\left(-1.01\cdot 10^{32}\ \frac{Nm}{s}\right)\left(4\pi\cdot 10^{-7}\ \frac{N}{A^2}\right)\left(3\cdot 10^8\ \frac{m}{s}\right)^3 (0.0333\ s)^4}{-32(\pi)^5 (10\cdot 10^3\ m)^6 \sin^2(90°)}\right]$$

$$= \frac{\frac{N^2 m^4}{A^2}}{m^6} \rightarrow \frac{N^2}{m^2 A^2}$$

$= 1.14 \cdot 10^9\ T$ ( $8 \cdot 10^8\ T$ to account for extra factor of 2 error )

d) See pg 601 of B.O.B for diagram; info sparse