John ?

PROBLEM 1

A star of magnitude 0 delivers a flux density equal to $4.17 \times 10^{-11}\,\mathrm{erg\,s^{-1}\,cm^{-2}\,\AA^{-1}}$ in the K band ($\lambda = 2.2\mu$m).

a. Derive the flux density in units of $\mathrm{W\,m^{-2}\,Hz^{-1}}$ (2 points).

b. What is the count rate in terms of $\mathrm{photons\,s^{-1}\,cm^{-2}\AA^{-1}}$? (2 points)

c. What will be the diameter of a telescope whose diffraction limit at this wavelength is 0.05 arcsec? (2 points)

d. The telescope in part c has a focal ratio of 2. What would the size of a pixel in the detector have to be to critically sample the diffraction limit (NB: critically sampled means that the airy disk FWHM subtends two pixels)? (2 points)

e. The sky background at this wavelength is about $13.7\,\mathrm{mag\,arcsec^{-2}}$. Assuming that the detector and telescope present a quantum efficiency of 50%, what is the background rate per pixel for the detector imagined in part d? Assume that you are observing through a filter that has a width of $0.3\,\mu$m. (2 points)

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Astro #1

a) $f = 4.17 \cdot 10^{-11} \frac{erg}{s \cdot cm^2 \cdot \mathring{A}}$

* Want f in $\frac{W}{m^2 Hz}$

$$W = 1\ J/s = 1 \frac{\frac{kg\ m^2}{s^2}}{s} = \frac{kg\ m^2}{s^3} \qquad \mathring{A} = 1 \cdot 10^{-8} cm = 1 \cdot 10^{10} m$$

$$Hz = \frac{1}{s}$$

$$erg = \frac{g \cdot cm^2}{s^2}$$

$$\Rightarrow f = 4.17 \cdot 10^{-11} \frac{\frac{g \cdot cm^2}{\cancel{cm}\ s^2}}{s \cdot cm^2 \cdot \mathring{A}}$$

$$= 4.17 \cdot 10^{-11} \frac{g}{s^3 \mathring{A}}$$

$$= 4.17 \cdot 10^{-14} \frac{kg}{s^3 \cdot \mathring{A}}$$

$$= 4.17 \cdot 10^{-14} \frac{J}{m^2 \mathring{A}}$$

$$= 4.17 \cdot 10^{-14} \frac{W s}{m^2 \mathring{A}}$$

$$= 4.17 \cdot 10^{-14} \frac{W}{m^2\ Hz\ \mathring{A}}$$?? → multiply by λ in $\mathring{A}$? Typo?

b) $E_\gamma = h\nu$

$$= (6.63 \cdot 10^{-27} erg \cdot s)\, \nu$$

* if all photons have $\lambda = 2.2\ \mu m$

$$= 2.2 \cdot 10^{-6} m$$

$$= 2.2 \cdot 10^{-4} cm$$

$$c = \lambda \nu \rightarrow \nu = \frac{c}{\lambda}$$

$$= \frac{3 \cdot 10^{10} \frac{cm}{s}}{2.2 \cdot 10^{-4} cm}$$

$$= 1.36 \cdot 10^{14} [Hz] \text{ or } [\tfrac{1}{s}]$$

$$E_\gamma = 9.04 \cdot 10^{-13} erg$$

$$f = \left(4.17 \cdot 10^{-11} \frac{erg}{s \cdot cm^2 \cdot \mathring{A}}\right) \cdot \left(\frac{1}{9.04 \cdot 10^{-13}} \frac{photons}{erg}\right)$$

$$= 46.1 \frac{photons}{s \cdot cm^2 \cdot \mathring{A}}$$