Astronomy Qualifier - August 2011

Lots of necessary (and some unnecessary) "constants" and possibly useful integrals at end.

Problem 1: Wang

The inflationay theory of the very early Universe solves the horizon problem of standard cosmology.

a) [2 pts] What is the horizon problem?

b) [2 pts] Show that inflation solves the horizon problem if $a(t) \propto t^{\alpha}$ during inflation, with $\alpha > 1$.

c) [4 pts] Derive the requirement from inflation on the equation of state of the matter-energy in the universe.

d) [2 pts] Does any matter-energy component that has been studied in cosmology satisfy this requirement?

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#1

a) What is the horizon problem?

The horizon problem is an issue in cosmology where parts of the universe are not causally connected yet share the same properties. For example, if we look at two points of the CMB separated by 180°, we have no reason to suspect that they share any common properties, as it takes information roughly 27.199 billion years to travel b/w the two points, Yet we know that both of these CMB points share the same temperature even though they are not causally connected.

Dai

Problem 2:

a) [5 pts] Assume that a model for the dark matter halo of the Galaxy is:

$$\rho(r) = \frac{C_0}{(a^2 + r^2)},$$

where ρ is density, r is distance from the galactic center, and $a = 2.8$ kpc. Show that the amount of dark matter interior to a radius r is given by the expression:

$$M_r = 4\pi C_0 \left[r - a\ tan^{-1}\left(\frac{r}{a}\right)\right]$$

b) [2 pts] If 5.5×10^{11} $M_\odot$ of dark matter is located within 100 kpc of the Galactic center, determine C_0 in units of $M_\odot$/kpc. Repeat your calculation if 1.3×10^{12} $M_\odot$ is located within 230 kpc of the Galactic center.

c) [3 pts] Estimate the amount of dark matter (in solar masses) within a radius of 50 kpc of the Galactic center. Compare your answer to the mass of the stellar halo (choose a reasonable value for the latter).

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Astro #2

a) Show the amount of dark matter interior to a radius r is $m(r) = 4\pi C_0[r - a\tan^{-1}(\frac{r}{a})]$ when $\rho(r) = \frac{C_0}{a^2+r^2}$

$$m(r) = \int_0^r \rho(r')dr'$$
$$= \int_0^r \frac{C_0}{a^2+r^2} dr^3$$
$$= 4\pi C_0 \int_0^r \frac{r^2}{a^2+r^2} dr$$
$$= 4\pi C_0 \left(r - \frac{1}{a}\tan^{-1}\left(\frac{r}{a}\right)\Big|_0^r \right)$$
$$= 4\pi C_0 \left(r - \frac{1}{a}\tan^{-1}\left(\frac{r}{a}\right) - \left(0 - \frac{1}{a}\tan^{-1}\left(\frac{0}{a}\right)\right)\right)$$
$$= 4\pi C_0 \left(r - \frac{1}{a}\tan^{-1}\left(\frac{r}{a}\right)\right) \checkmark$$

b) If $5.5\cdot 10^{11} M_\odot$ of DM is located w/in 100 kpc of the Galactic center, find C_0 in terms of $M_\odot$/kpc. Repeat w/ $1.3\cdot 10^{12} M_\odot$ w/in 230 kpc

$$m(r) = 4\pi C_0 \left(r - \frac{1}{a}\tan^{-1}\left(\frac{r}{a}\right)\right)$$
$$\Rightarrow C_0 = \frac{m(r)}{4\pi\left(r - \frac{1}{a}\tan^{-1}\left(\frac{r}{a}\right)\right)}$$

(1) $$C_0 = \frac{5.5\cdot 10^{11} M_\odot}{4\pi\left(100 - \frac{1}{2.8}\tan^{-1}\left(\frac{100}{2.8}\right)\right)}$$
$$= 4.4\cdot 10^8 \frac{M_\odot}{kpc}$$

(2) $$C_0 = \frac{1.3\cdot 10^{12} M_\odot}{4\pi\left(230\,kpc - \frac{1}{2.8\,kpc}\tan^{-1}\left(\frac{230}{2.8}\right)\right)}$$
$$= 4.51\cdot 10^8 \frac{M_\odot}{kpc}$$

c) Estimate the amount of DM w/in 50 kpc of galactic center. Compare to mass of stellar halo.

$\rightarrow C_0 \approx 4.45\cdot 10^8$

$$m(r) = 4\pi\left(4.45\cdot 10^8 \frac{M_\odot}{kpc}\right)\left(50\,kpc - \frac{1}{2.8\,kpc}\tan^{-1}\left(\frac{50\,kpc}{2.8\,kpc}\right)\right)$$
$$= 2.77\cdot 10^{11} M_\odot$$

John

Problem 3: $\boxed{\text{B.O.B} - 7.4}$

Consider an eclipsing spectroscopic binary with the following properties:

- Orbital period is 6.31 yr.
- Maximum radial velocities of Star A and Star B are 5.4 $\mathrm{kms^{-1}}$ and 22.4 $\mathrm{km\ s^{-1}}$.
- Time period between first contact and minimum light is 0.58 d, and the length of the primary minimum is 0.64 d.
- The apparent bolometric magnitudes of the maximum, primary minimum, and secondary minimum are 5.40 magnitudes, 9.20 magnitudes, and 5.44 magnitudes, respectively.

Assuming circular orbits and that the plane of the system lies in our line of sight, find the following:

a) [2 pts] Ratio of the stellar masses.

b) [2 pts] Sum of the masses.

c) [2 pts] Individual masses.

d) [2 pts] Individual radii.

e) [2 pts] Ratio of the effective temperatures of the two stars.

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#3

$P = 6.31\ yr$

$V_{r,A} = 5.4\ \frac{km}{s}$

$V_{r,B} = 22.4\ \frac{km}{s}$

a) $\frac{m_A}{m_B} = \frac{V_B}{V_A}$

$= \frac{22.4}{5.4}$

$= 4.15$

b) $P^2 = \frac{4\pi^2}{G(m_1+m_2)} a^3$

$(m_1+m_2) = \frac{4\pi^2}{GP^2} a^3$

$*$ but $a = a_1 + a_2$

$= \frac{P}{2\pi} V_1 + V_2$

$= \frac{4\pi^2}{GP^2} \left(\frac{P}{2\pi} [V_1 + V_2] \right)^3$

$= \frac{4\pi^2 P}{G 8\pi^3} (V_1+V_2)^3$

$= \frac{P}{2\pi G} (V_1+V_2)^3$

$= \frac{P}{2\pi G} \frac{(V_{1,R}+V_{2,R})^3}{\sin^3 i}$

$= 1.02 \cdot 10^{31}\ kg$

c) $m_A = 4.15\ m_B$

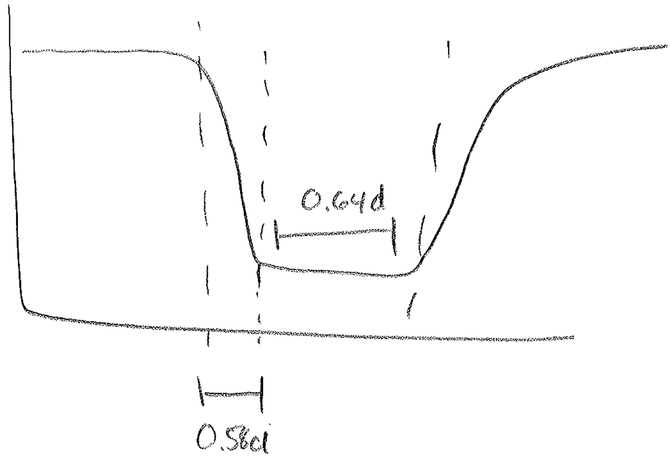
$\Rightarrow 5.15\ m_B = 1.02 \cdot 10^{31}\ kg$

$m_B = 1.98 \cdot 10^{30}\ kg$

$m_A = 8.23 \cdot 10^{30}\ kg$

#3 (cont.)

d)



$$r_B = \frac{v}{2}(0.58 \text{ days})$$

$$= \frac{\left(5.4\frac{km}{s} + 22.4\frac{km}{s}\right)}{2}\left(0.58 \cdot \cancel{365.25\frac{day}{yr}} \cdot 24\frac{hr}{day} \cdot 3600\frac{s}{hr}\right)$$

$$= 6.965 \cdot 10^5 \text{ km}$$

$$= 6.97 \cdot 10^8 \text{ m}$$

$$r_A = \frac{5.4\frac{km}{s} + 22.4\frac{km}{s}}{2}\left([0.58 + 0.64] \cdot 24 \cdot 3600\right)$$

$$= 1.47 \cdot 10^6 \text{ km}$$

$$= 1.47 \cdot 10^9 \text{ m}$$

e)

Kilre

Problem 4:

a) [4 pts] Compare the nucleosynthesis evolution of low-mass (stars like the sun) and high-mass (20 solar mass) stars. In particular, describe all of the hydrostatic and and/or explosive phases of element formation for each type of star. List the elements that are fused (or burned), the order that they happen during stellar evolution and the most likely products of those reactions.

b) [3 pts] What is the heaviest element that can be fused in low-mass and high-mass stars and why? What about iron fusion? When does it occur, or if not, why not? What about the heaviest elements such as precious metals? How are they formed? Describe the processes?

c) [3 pts] How do we know that nucleosynthesis occurs in stars? Give specific examples of observations that indicate element formation must occur in certain stars. What stage of evolution are these stars in, and how are the elements that we observe formed inside the star?

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Astro #4

a) In a low mass star, once it reaches the main sequence it will born H to He via the P-P chain for billions of years. As He ash builds in the core, eventually the H-burning will occur in a shell instead of in the core. As more ash boilds, the He core becomes degenerate and will spark in a He-flash up to several times to break the degenerecy and stably born He to C and O. But, unlike the He ash, the C/O ash that boilds up in the core is unable to break it's degenerecy + the star dies as a C/O white dwarf.

For the high mass star, it burn H to He via the CNO cycle. After that, it boilds up a core of concentric shells of various elements (listed in order below) Due to the large mass of the star, it is not necessary for the core to become degenerate to transition from burning one element to another.

$$H \rightarrow He \rightarrow C \rightarrow O \rightarrow Ne \rightarrow Mg \rightarrow Si \rightarrow Fe$$

Once the core becomes filled w/ enough Si ash to begin Si burning, the Ni^{56} ash which subsequently decays to Fe^{56} no more fusion occurs, as fusion of iron into another element requires the addition of energy from its surroundings, instead of releasing energy during the process.

b) In a low mass star, the heaviest elements fused are C/O; heavier elements are not fused b/c the central temperatures of low mass are not high enough for more advanced fusion processes to occur.

In high mass stars, the heaviest element that can be fused is Fe. Fusing elements heavier than iron requires adding energy to the system instead of releasing energy.

The reason we find elements heavier than iron is because during core-collapse SN, the large amounts of neutrons present allow s and r process elements to form. Since neutrons are electrically neutral, they don't need to overcome the Coloumb potential to collide w/ the atomic nuclei. However, these neutron rich isotopes are unstable, and

b) when they decay they leave a proton in the nucleus and create heavier elements.

c)

??

Problem 5:

A telescopic survey to find nearby "space rocks" can find moving objects to a magnitude of 18.5. The relationship between magnitude and flux for the "visible" passband used is:

$$mag = -2.5 log(f/f_0)$$

where f is the flux from the target and f_0 is the flux from a mag = 0 object (assume f_0 = 1.0E−8 W m^{-2}).

An approximately spherical space rock, 50 meters in diameter, with an albedo of 0.2, comes near the Earth. The rock shines in the visible only by reflected sunlight.

a) [1 pts] Calculate the flux of sunlight in the visible at a distance of 1 AU from the Sun. Assume the "visible" pass band encompasses 1/3 of the bolometric power output of the Sun.

b) [2 pts] From the parameters given calculate the visible power of the rock (power of reflected sunlight) when approximately 1 AU from the Sun.

c) [4 pts] What is the maximum distance from Earth that the survey could detect the rock? (The rock will not emit isotropically, of course, but only from its illuminated side. Just assume it reflects uniformly from half its surface (the "day" side)). Don't worry about the changing solar flux with distance- just assume the rock is near 1 AU from Sun.

d) [3 pt] Assume the rock has a density of a typical rocky asteroid. Assume it hits the Earth with a speed equal to the escape speed of the Earth. How many megatons of energy would be released by the impact? (1 MT = 4.2E15 Joules).

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Astro #5

a) $mag = -2.5 \log(f/f_0)$

$-\frac{2}{5} m = \log(f/f_0)$

$e^{-.4m} = \frac{f}{f_0}$

$f_0 e^{-.4m} = f$

$1 \cdot 10^{-8} \frac{W}{m^2} e^{-.4(4.7)} = f_0$

$1.526 \cdot 10^{-9} \frac{W}{m^2} = f_0$

$\Rightarrow f_{VIB} = 5.09 \cdot 10^{-10} \frac{W}{m^2}$

b) $f_{VIB} = 5.09 \cdot 10^{-10} \frac{W}{m^2}$

$(0.2) \cdot f_{VIB} \cdot \pi(25m)^2 = P_{rock}$ $\Longleftarrow$ $I = \frac{P}{SA}$

$P_{rock} = 1.99 \cdot 10^{-7}\ W \cdot 4$

c) $f_0 e^{-.4m} = f$

$\Rightarrow f_{18.5} = 6.11 \cdot 10^{-12} \frac{W}{m^2} = J/s$

$P_{18.5} = f_{18.5} \cdot \pi(25m)^2 \cdot 4$

$= 1.2 \cdot 10^{-8}\ W \cdot 4$

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#5 (cont.)

d) *Assume density of rocky asteroid is same as density of earth.

$$\rho = \frac{m}{V}$$

$$= \frac{5.97 \cdot 10^{27}\ g}{(6.37 \cdot 10^{8}\ cm)^3 \cdot \pi \cdot \frac{4}{3}}$$

$$= 5.51 \frac{g}{cm^3}$$

* Find escape velocity

$$\frac{1}{2} \cancel{m} v^2 = \frac{M\cancel{m}G}{R}$$

$$v = \sqrt{\frac{2MG}{R}}$$

$$V_{esc,\oplus} = \sqrt{\frac{2(5.97 \cdot 10^{27} g)(6.67 \cdot 10^{-8} \frac{cm^3}{g s^2})}{(6.37 \cdot 10^{8} cm)}}$$

$$= 1.12 \cdot 10^{6} \frac{cm}{s}$$

$$E_{asteroid} = \frac{1}{2} m V_{esc\oplus}^2$$

$$= \frac{1}{2}\left(\rho \cdot \frac{4}{3}\pi (2500\ cm)^3\right)\left(1.12 \cdot 10^{6} \frac{cm}{s}\right)^2$$

$$= 2.25 \cdot 10^{23} \frac{g\ cm^2}{s^2}$$

$$= 2.25 \cdot 10^{16}\ J$$

$$\boxed{= 5.36\ MT}$$

cm → m $(10^2)^2$
g → kg 10^3

Eddie

Problem 6:

The equation of radiative transfer in spherical coordinates is:

$$\mu \frac{\partial I_\nu}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial I_\nu}{\partial \mu} = -\chi_\nu I_\nu + \eta_\nu$$

a) [3pts] Show that the moment equations can be written:

$$\frac{1}{r^2}\frac{\partial}{\partial \tau_\nu}(r^2 H_\nu) = (J_\nu - S_\nu)$$

$$\frac{\partial K_\nu}{\partial \tau_\nu} + \frac{(J_\nu - 3K_\nu)}{(\chi_\nu r)} = H_\nu$$

b) [2pts] Introduce the Eddington factor $f_\nu = K_\nu / J_\nu$ and rewrite the moment equations in terms of it.

c) [2pts] Explain the problem with deriving a single second order equation for J_ν as is done in the plane-parallel case.

d) [3pts] Show that in fact with the sphericity factor:

$$ln(r^2 q_\nu) = \int_{r_c}^{r} [(3f_\nu - 1)/(r' f_\nu)] dr' + ln(r_c^2)$$

where r_c is the radius of the opaque core, the two moment equations can be combined to give:

$$\frac{\partial^2}{\partial X_\nu^2}(r^2 q_\nu f_\nu J_\nu) = q_\nu^{-1} r^2 (J_\nu - S_\nu)$$

where $dX_\nu = q_\nu d\tau_\nu$.

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Astro #6

RTE in spherical:

$$\mu \frac{\partial I_\nu}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial I_\nu}{\partial \mu} = -\chi_\nu I_\nu + \eta_\nu$$

a) $$\mu \frac{\partial I_\nu}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial I_\nu}{\partial \mu} = -\chi_\nu I_\nu + \eta_\nu$$

$$\frac{\mu}{-\chi_\nu}\frac{\partial I_\nu}{\partial r} + \frac{1-\mu^2}{-r\chi_\nu}\frac{\partial I_\nu}{\partial \mu} = I_\nu - S_\nu \qquad \text{where } S_\nu = \frac{\eta_\nu}{\chi_\nu}$$

$$\mu \frac{\partial I_\nu}{\partial \tau_\nu} + \frac{1-\mu^2}{\tau_\nu}\frac{\partial I_\nu}{\partial \mu} = I_\nu - S_\nu \qquad \text{where } d\tau_\nu = -\chi_\nu dr,\ \tau_\nu = -\chi_\nu r$$

Moment equations:

$$J_\nu = \frac{1}{2}\int_{-1}^{1} I_\nu \, d\mu$$

$$K_\nu = \frac{1}{2}\int_{-1}^{1} \mu^2 I_\nu \, d\mu$$

$$H_\nu = \frac{1}{2}\int_{-1}^{1} \mu I_\nu \, d\mu$$

$$\Rightarrow I_\nu = S_\nu + \mu \frac{\partial I_\nu}{\partial \tau_\nu} + \frac{1-\mu^2}{\tau_\nu}\frac{\partial I_\nu}{\partial \mu}$$

$$\hookrightarrow H_\nu = \frac{1}{2}\int_{-1}^{1} \cancel{\mu S_\nu}^{\,0} + \mu^2 \frac{\partial I_\nu}{\partial \tau_\nu} + \frac{\mu - \mu^3}{\tau_\nu}\frac{\partial I_\nu}{\partial \mu}\, d\mu$$

$$H_\nu = \quad + \frac{\partial K_\nu}{\partial \tau_\nu} + \frac{1}{\chi_\nu r}\left(J_\nu - 3K_\nu\right) \checkmark$$

* Integrating RTE over all μ yields

$$J_\nu - S_\nu = \frac{\partial K_\nu}{\partial \tau_\nu} + \frac{1}{\tau_\nu}\left(I_\nu - H_\nu\right)$$

$$= \frac{\partial K_\nu}{\partial \tau_\nu} + \frac{1}{\tau_\nu}\left(I_\nu - \left[\frac{\partial K_\nu}{\partial \tau_\nu} + \frac{1}{\chi_\nu}\left(J_\nu - 3K_\nu\right)\right]\right)$$

$$= \frac{\partial K_\nu}{\partial \tau}\left(1 - \frac{1}{\tau_\nu}\right) + \frac{1}{\tau_\nu}\left(I_\nu - \frac{1}{\chi_\nu}\left[J_\nu - 3K_\nu\right]\right) \quad ???$$

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#6 (cont.)

b) * Assuming

$$\frac{1}{r^2}\frac{\partial}{\partial \tau_\nu}(r^2 H_\nu) = J_\nu - S_\nu$$

$$\frac{\partial K_\nu}{\partial \tau_\nu} + \frac{(J_\nu - 3K_\nu)}{\chi_\nu r} = H_\nu$$

* if $f_\nu = \frac{K_\nu}{J_\nu}$

$$\frac{1}{J_\nu}\left[\frac{\partial K_\nu}{\partial \tau_\nu} + \frac{J_\nu - 3K_\nu}{\chi_\nu r} = H_\nu\right]$$

$$\frac{\partial f_\nu}{\partial \tau_\nu} + \frac{1 - 3f_\nu}{\chi_\nu r} = \frac{H_\nu}{J_\nu}$$

$$= \frac{H_\nu K_\nu}{f_\nu}$$

CONSTANTS

$$\sigma = 5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-4}; \quad c = 3.00 \times 10^{10} \text{ cm s}^{-1}; \quad T_\odot = 5,800\text{K}$$

$$G = 6.67 \times 10^{-8} \text{ g}^{-1} \text{ cm}^3 \text{ s}^{-2}; \quad k = 1.38 \times 10^{-16} \text{ erg K}^{-1}$$

$$m_H = 1.67 \times 10^{-24} \text{ g}; \quad m_e = 9.11 \times 10^{-28} \text{ g}; \quad M_\odot = 1.99 \times 10^{33} \text{ g}$$

$$M_{\text{earth}} = 5.97 \times 10^{27} \text{ g}; \quad M_G = 4.0 \times 10^{11} M_\odot$$

$$h = 6.63 \times 10^{-27} \text{ erg s}; \quad a = 7.56 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4}$$

$$R_\odot = 6.96 \times 10^{10} \text{ cm}; \quad R_{\text{earth}} = 6.37 \times 10^8 \text{ cm}$$

$$1 \text{ AU} = 1.496 \times 10^{13} \text{ cm}$$

$$1 \text{ parsec} = 3.09 \times 10^{18} \text{ cm}; \quad 1\mathring{A} = 10^{-8} \text{ cm}$$

$$M_V(\odot) = 4.8; \quad M_{bol}(\odot) = 4.7; \quad L_\odot = 3.9 \times 10^{33} \text{ ergs s}^{-1}$$

$$1 \text{ year} = 3.16 \times 10^7 \text{ s}$$