# ASTRONOMY QUALIFYING EXAM
## January 2014

### Possibly Useful Quantities

$L_{\odot} = 3.9 \times 10^{33}$ erg s$^{-1}$
$M_{\odot} = 2 \times 10^{33}$ g
$M_{\mathrm{bol}\odot} = 4.74$
$R_{\odot} = 7 \times 10^{10}$ cm
1 AU = 1.5 $\times 10^{13}$ cm
1 pc = 3.26 Ly. = 3.1 $\times 10^{18}$ cm
a = 7.56 $\times 10^{-15}$ erg cm$^{-3}$ K$^{-4}$
c = 3 $\times 10^{10}$ cm s$^{-1}$
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm$^{-2}$ K$^{-4}$ s$^{-1}$
k = 1.38 $\times 10^{-16}$ erg K$^{-1}$
e = 4.8 $\times 10^{-10}$ esu
1 fermi = $10^{-13}$ cm
$N_A = 6.02 \times 10^{23}$ moles g$^{-1}$
G = 6.67 $\times 10^{-8}$ g$^{-1}$ cm$^3$ s$^{-2}$
$m_e = 9.1 \times 10^{-28}$ g
h = 6.63 $\times 10^{-27}$ erg s
1 amu = 1.66053886 $\times 10^{-24}$ g

John?

# PROBLEM 1

**a) (3 points)** Calculate the orbital semi-major axis ($a_{sun}$) of the Sun's orbit about the barycenter of the Solar System, in AU, in response to Jupiter's orbital motion. Since Jupiter constitutes ~70% of the non-solar mass of our Solar System, you can ignore Solar System bodies less massive than Jupiter in your computation. Assume $a_{Jupiter} = 5.2$ AU.

**b) (2 points)** To an external observer, what would be the transit depth of an Earth-size planet located at a=0.1AU (assume circular orbit) about a M dwarf star (Mass = 0.3 Msun; Radius = 0.8 Rsun)?

**c) (3 points)** To an external observer, what would be the transit duration (in hours) of an Earth-size planet located at a=0.1AU (assume circular orbit) about a M dwarf star (Mass = 0.3 Msun; Radius = 0.8 Rsun)?

**d) (2 points)** To an external observer located 20 pc away, what would be the angular separation in arcseconds between an Earth-size planet located at a=0.1 AU and its host star?

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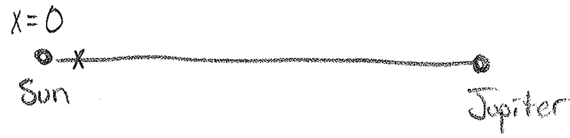
# Astro #1

a) * Bodies orbit mutual center of mass; Assuming point particles




* Assuming $M_J \sim \frac{M_\odot}{1000}$

$$X_{cm} = \frac{\sum_i x_i m_i}{\sum_i m_i}$$

$$= \frac{M_\odot \cdot 0 + 5.2\,AU \cdot \frac{M_\odot}{1000}}{M_\odot + \frac{M_\odot}{1000}}$$

$$= \frac{5.2\,AU/1000}{1 + \frac{1}{1000}}$$

$$= 7.79 \cdot 10^{10}\,cm$$

$$a_{sun} = X_{cm} - R_\odot$$

$$= 7.9 \cdot 10^{9}\,cm$$

b) * Assuming no emission from planet

$$\frac{\Delta L}{L} = \frac{4\pi\sigma T^4 (r_s^2 - r_p^2)}{4\pi\sigma T^4 r_s^2}$$

$$= \frac{r_s^2 - r_p^2}{r_s^2}$$

$$= 1 - \frac{r_p^2}{r_s^2}$$

$$= 1 - \boxed{8.4 \cdot 10^{-5}}$$

$$= .999916$$

$$\frac{R_\oplus}{R_\odot} = \frac{6.37 \cdot 10^6\,m}{6.95 \cdot 10^8\,m}$$

$$R_\oplus = 9.1 \cdot 10^{-3}\,R_\odot$$

$$= .009\,R_\odot$$

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#1 (cont.)

c) $$F = \frac{GMm}{r^2} = m\frac{v^2}{r}$$

$$\frac{GM}{r} = v^2$$

$$\Rightarrow v = \sqrt{\frac{GM}{r}}$$

$$= \sqrt{\frac{6.67 \cdot 10^{-8} \frac{g^{-1}}{cm^3 s^2} \cdot 2 \cdot 10^{33} g}{1.5 \cdot 10^{12} cm}}$$

$$= 9.43 \cdot 10^6 \frac{cm}{s}$$

$R_\oplus = 6.37 \cdot 10^8$ cm

$R_\odot = 7 \cdot 10^{10}$ cm

* From first to last contact

$$t = \frac{2R_\odot + 2R_\oplus}{v}$$

$$= \frac{1.4 \cdot 10^{11} + 1.27 \cdot 10^9}{9.43 \cdot 10^6}$$

$$= 1.49 \cdot 10^4 \text{ s}$$

$$= 4.16 \text{ hrs}$$

d) $$d = \frac{1}{\alpha} \text{ pc}$$

$$\alpha = \left(\frac{1}{20}\right)''$$

$$= .05 \text{ arcsec}$$

Wang

# PROBLEM 2

For a blackbody the number density of photons is

$$n_\gamma(\nu, T)\,d\nu = \frac{8\pi\nu^2}{c^3}\frac{1}{e^{h\nu/kT}-1}\,d\nu.$$

The energy density is then

$$U_\gamma(\nu, T)\,d\nu = h\nu n_\gamma(\nu, T)\,d\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1}\,d\nu.$$

Assume that radiation is emitted at some prior epoch, t, in the history of the Universe when the scale factor was given by R. The radiation is observed today at time $t_0$ with scale factor $R_0 = 1$.

**a) (6 points)** Using your knowledge of how wavelengths of photons vary with the scale factor, show that

$$n_\gamma(\nu_0, T)\,d\nu_0 = \frac{8\pi\nu_0^2}{c^3}\frac{1}{e^{h\nu/kRT}-1}\,d\nu_0.$$

**b) (4 points)** And therefore that

$$U_\gamma(\nu_0, T_0)\,d\nu_0 = \frac{8\pi h\nu_0^3}{c^3}\frac{1}{e^{h\nu/kT_0}-1}\,d\nu_0,$$

so that

$$T/T_0 = 1/R$$

Kilic

# PROBLEM 3

**a) (3 points)** Describe the burning process on the main sequence. Explain the difference of the sun on the main sequence and a $1.5M_{sun}$ star.

**b) (3 points)** Describe He burning in the lower mass stars and intermediate mass stars. What is the mass range for each approximately? Compare the timescale of helium burning (lifetime on the helium main sequence) to that of hydrogen burning (lifetime on the main sequence).

**c) (4 points)** Describe the following burning stages in stars: carbon burning, neon burning, oxygen burning, silicon burning.

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# Astro #3

a) For a star on the main sequence, it is burning hydrogen to helium. For stars less than 1.3 $M_\odot$, they perform the nuclear fusion via the PP chain, while stars more massive than 1.3 $M_\odot$ use the CNO cycle. The reaction chains are shown below

P-P Chain:

$$\left.\begin{aligned} 2H_1^1 &\rightarrow H_1^2 + \nu_e + e^+ + \gamma \\ H_1^2 + H_1^1 &\rightarrow He_2^3 + \gamma \end{aligned}\right\} \times 2$$

$$2He_2^3 \rightarrow He_2^4 + 2H_1^1$$

CNO Cycle:

$$C_6^{12} + H_1^1 \rightarrow N_7^{13} + \gamma$$

$$N_7^{13} \rightarrow C_6^{13} + e^+ + \nu$$

$$C_6^{13} + H_1^1 \rightarrow N_7^{14} + \gamma$$

$$N_7^{14} + H_1^1 \rightarrow O_8^{15} + \gamma$$

$$O_8^{15} \rightarrow N_7^{15} + e^+ + \nu$$

$$N_7^{15} + H_1^1 \rightarrow C_6^{12} + He_2^4$$

b) He-burning is done via the triple-$\alpha$ rxn in stars, and produces carbon ash. In low mass stars, the He-core contracts to the point of degeneracy before He fusion begins, resulting
$<2 M_\odot$ in He flashes lifting the degeneracy before stable fusion starts. In higher mass stars, this is not necessary as the required central temperature can be reached simply via core $<8 M_\odot$ contraction. He burning lasts for ~10% of time of H-burning (~120 Myr v 10 billion yr for 1 $M_\odot$ star).

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#3 (cont.)

c) Carbon burning: $C^{12}_{6} + He^{4}_{2} \rightarrow O^{16}_{8} + \gamma$

$T \approx 5 \cdot 10^{8}\,K$

$2C^{12}_{6} \rightarrow Mg^{24*}_{12} \rightarrow Ne^{20}_{10} + \alpha$

$\rightarrow Na^{23}_{11} + p$

Neon burning: $Ne^{20} + \gamma \leftrightarrow O^{16} + \alpha$

$Ne^{20} + \alpha \leftrightarrow Mg^{24} + \gamma$ $\qquad T \approx 1.5 \cdot 10^{9}\,K$

$\Rightarrow 2Ne^{20} \rightarrow Mg^{24} + O^{16}$

Oxygen burning: $2O^{16}_{8} \rightarrow S^{32} \rightarrow Si^{28} + \alpha$

$T \approx 2 \cdot 10^{9}\,K$

Silicon burning: * combination of photodisintigration chain to generate $\alpha$ particles and successive fusions w/ $\alpha$-particles from $Si^{32}$ to $Ni^{56}$ which decays to $Fe^{56}$

$T \approx 3 \cdot 10^{9}\,K$

Eddie? / Herry?

# PROBLEM 4

a) **(3 points)** Imagine a large cloud of pure interstellar hydrogen having density n atoms/$cm^3$. $\Phi$ is the number of photons emitted by a star per second which are capable of photoionizing neutral hydrogen ($\lambda < 912$Å), while $\alpha n^2$ is the number of recombinations per second per $cm^3$. If each photon results in a photoionization and the rate of photoionization equals the rate of recombination, find an expression for the Strömgren sphere $R_s$, i.e. the radius of the ionized gas cloud, in terms of n, $\Phi$, and $\alpha$.

b) **(2 points)** Find $R_s$ in parsecs for an O star if $\Phi = 10^{49}$photons/s, n = 10 atoms/$cm^3$, and $\alpha = 2 \times 10^{-13}$.

c) **(2 points)** Find $R_s$ in parsecs for the sun if $\Phi = 5 \times 10^{23}$ photons/s, while n and $\alpha$ remain the same.

d) **(3 points)** Could the cloud around the sun be seen by an astronomer on $\alpha$-Centauri (distance=1.31pc) using a telescope which can just barely resolve objects which are 1" in angular size?

Constants:

1 parsec=$3.086 \times 10^{18}$cm

1 radian = 206265 arcsec

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# Astro #4

a) $[n] = \frac{\text{atoms}}{cm^3}$

$[\Phi] = \frac{\text{photons}}{s}$

* Assuming # of ionizations and recombinations are the same over a 1s interval

$$\Phi = \int \alpha n^2 dV$$

$$\Phi = \alpha n^2 \frac{4}{3}\pi r^3$$

$$\left(\frac{3\Phi}{4\alpha n^2 \pi}\right)^{1/3} = r$$

b) $$r = \left(\frac{3 \cdot 10^{49} \frac{1}{\cancel{s}}}{\pi \cdot 4 \cdot 2 \cdot 10^{-13} (10)^2 \frac{1}{cm^3}}\right)^{1/3}$$

$$= 4.92 \cdot 10^{19}\ cm$$

$$\approx 15.96\ pc$$

c) $$r = \left(\frac{5 \cdot 10^{23}}{\pi \cdot 4 \cdot 2 \cdot 10^{-13} \cdot 10^2 \frac{1}{cm^3}}\right)^{1/3}$$

$$= 1.258 \cdot 10^{11}\ cm$$

$$= 4.08 \cdot 10^{-8}\ pc$$

Wang?/Dai?

# PROBLEM 5

**a) (5 points)** There are four commonly used distances in extra-galactic astronomy, the co-moving line-of-sight distance, the co-moving transverse distance, the luminosity distance, and the angular diameter distance.

Given a cosmological object at a redshift z ($z > 1$), describe how to calculate these four distances. The cosmological parameters, $H_0$, $\Omega_m$, and $\Omega_\Lambda$ are all given.

**b) (1 point)** What is the definition of the surface brightness of an astronomical object?

**c) (3 points)** For a cosmological extended object at z ($z > 1$) with a constant emissivity per unit area, show that the surface brightness, $\sigma$, of the object scales as $\sigma \propto (1+z)^{-4}$.

**d) (1 point)** If the object is nearby, show that the surface brightness is roughly a constant as a function of distance.

Lerghly?

# PROBLEM 6

a) **(3 points)** A sequence of radio images from the quasar 3C 273 shows a blob of radio emission moving away from the nucleus with an angular velocity of 0.0008 arcsec $yr^{-1}$. Assuming that the radio knot is moving in the plane of the sky, and using the distance of $d = 440h^{-1}$ Mpc for 3C 273, derive the apparent transverse velocity $v_{app}$ away from the nucleus. What is the value, in units of c, for normalized Hubble constant $h = 0.71$? Is this physically reasonable?

b) **(4 points)** Next, assuming that instead of moving in the plane of the sky, the blob is moving at an angle $\phi$ to our line of sight with an actual speed v (as distinguished from the apparent velocity $v_{app}$). Derive an equation for v/c in terms of the apparent transverse velocity and $\phi$.

c) **(3 points)** Show that $v/c < 1$ for angles satisfying

$$\frac{v_{app}^2/c^2 - 1}{v_{app}^2/c^2 + 1} < \cos\phi < 1$$