ASTRONOMY QUALIFYING EXAM

August, 2015

Possibly Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \times 10^{10}$ cm
1 AU=1.5 $\times$ 10^{13} cm
1 pc= 3.26 $Ly =$ 3.1 $\times$ 10^{18} cm
a=7.56 $\times$ 10^{-15} erg cm^{-3} K^{-4}
c=3 $\times$ 10^{10} cm s^{-1}
σ =ac/4= 5.7 $\times$ 10^{-5} erg cm^{-2} K^{-4} s^{-1}
k=1.38 $\times$ 10^{-16} erg K^{-1}
e=4.8 $\times$ 10^{-10} esu
1 fermi=10^{-13} cm
N$_A$ = 6.02 $\times$ 10^{23} moles g^{-1}
G=6.67 $\times$ 10^{-8}g^{-1} cm^3 s^{-2}
m$_e$ = 9.1 $\times$ 10^{-28} g
h= 6.63 $\times$ 10^{-27} erg s
1 amu= 1.66053886 $\times$ 10^{-24} g
r_e = 2.8179 $\times$ 10^{-13} cm; (electron radius)

PROBLEM 1

1. (a) (2 points) Write or derive an equation for hydrostatic equilibrium in a form that is suitable for the interior of the sun, i.e., express dP/dr in terms of G, m, ρ, and r, where m is the mass interior to radius r and ρ is the mass density.

   (b) (1 point) Rewrite the equation with m as the independent variable, i.e, dP/dm = ...

   (c) (1 point) Use the dP/dm equation to obtain an approximate expression for the pressure at the center of the sun, in terms of G, M, and R, where M is the total mass of the sun and R is the solar radius.

   (d) (1 point) To the nearest powers of ten, what are the temperature and the density at the center of the sun?

   (e) (1 point) Write the "bottleneck" reaction (the least probable of the major reactions) for fusing hydrogen to helium in the core of the sun.

   (f) (2 points) At the middle of the solar photosphere, where the optical depth at 5000 Å is about 1, what (to the nearest 1000 K) is the temperature? Is the mass density at this depth much greater than, much less than, or about equal to the density of air at sea level? Is hydrogen mostly ionized, mostly neutral, mostly locked up in diatomic molecules, or in some other form? What is the dominant source of opacity at 5000 Å? Identify the atomic process as specifically as you can.

   (g) (2 points) In the approximation of local thermodynamic equilibrium (LTE), estimate the fraction of *all* hydrogen (ionized, neutral, molecular) that is in the Balmer (n = 2) level of neutral hydrogen.

PROBLEM 2

2. (a) (6 points) A star located 19.6 pc from the Sun has a temperature T = 8000 K and radius R = 0.5 Rsun. There is a planet orbiting the star with a semi-major axis a = 5 AU in a circular orbit. The planet has a radius of 2 Rj (Rj = Jupiters radius). Assume the only source of energy for the planet is the star, all light falling on the planet is absorbed, and the star+planet are perfect blackbodies.

   Estimate the temperature of the planet.

   (b) (2 points) Assume the planet described in part (1) transits its host star. What would be the observed transit depth?

   (c) (2 points) Assume aliens live on the planet, and an alien observes the Earth/Sun system from his/her planet. Whats the angular separation (in units of milli-arcseconds) he/she would measure for the Earth and Sun?

PROBLEM 3

3. (a) (1 point) Write down the general radiative transfer equation (RTE) in plane-parallel geometry and define all the terms including units.

   (b) (1 point) Define the 3 Eddington moments.

   (c) (1 point) Explain what the grey approximation is.

   (d) (1 point) Make the grey approximation and derive the 2 ordinary differential equations for the moments from the RTE.

   (e) (1 point) What does radiative equilibrium tell you about H in this case?

   (f) (1 point) Make the "two-stream" approximation

$$I = \begin{cases} I^+ & \mu \geq 0 \\ I^- & \mu < 0 \end{cases}$$

   and obtain the Eddington moments in this case.

   (g) (1 point) From the two-stream case above, find the Eddington factors $f = K/J$, and $h(0) = H(0)/J(0)$, where $H(0)$ is the value of H at the surface $\tau = 0$. Assume no external illumination.

   (h) (2 points) Using the Eddington factors found above, solve the moment equations for J.

   (i) (1 point) Assume $S = B$, where B is the grey planck function and find the temperature as a function of τ.

Aug 2015

#3

a) $$\mu \frac{dI}{dz} = \eta_\nu - \kappa_\nu I_\nu$$

$\mu = \cos\theta$
η_ν = emissivity
κ_ν = opacity
I_ν = radiation intensity

b) $$J_\nu = \frac{1}{2}\int_{-1}^{1} I_\nu \, d\mu$$

$$H_\nu = \frac{1}{2}\int_{-1}^{1} \mu I_\nu \, d\mu$$

$$K_\nu = \frac{1}{2}\int_{-1}^{1} \mu^2 I_\nu \, d\mu$$

c) The grey approximation says that χ is independent of ν

d) * In the grey approximation

$$\mu \frac{dI}{dz} = \eta - \kappa I$$

$$\cancel{\mu \frac{dI}{dt} = S - I}$$

$$\frac{1}{2}\int_{-1}^{1} \mu \frac{dI}{dz} = \frac{dH}{dz} = \frac{1}{2}\int_{-1}^{1} \chi_\nu (I_\nu - S_\nu)\, d\mu$$

$$= -\chi(J - S)$$

$$\frac{1}{2}\int_{-1}^{1} \mu^2 \frac{dI}{dz} = \frac{dK}{dz} = \frac{1}{2}\int_{-1}^{1} \chi(I - S)\,\mu\, d\mu$$

$$= \chi H$$

e) * In the case of radiative equilibrium (energy only carried by radiation

$$\int_0^\infty \frac{dH_\nu}{dz} d\nu = \frac{dH}{dz} = -\int \chi(J - S)$$

$$= 0 \quad \text{b/c absorption = emission}$$

PROBLEM 4

4. (a) (7 points) Assume that the gas component of a galaxy, with a mass fractionf_g, is virialized and follows the overall density profile of the galaxy, a singular isothermal sphere mass profile,

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2}$$

where σ is the velocity dispersion of the galaxy. The galaxy has a central AGN, which is radiating at the Eddington luminosity,

$$L_{Edd} = \frac{4\pi G c m_p M_{BH}}{\sigma_T}$$

A fraction, f_w, of the energy radiated by the central AGN is deposited into the gas in the form of kinetic energy. This kinetic "feedback" energy from the AGN can drive the gas in the host galaxy to flow outward. Assume that the final gas outflow is in a spherical shell with a constant velocity, v, and half of the kinetic feedback energy is converted to the gravitational potential of the gas and the other half to the kinetic energy of the gas during the outflowing process. Use the conservation or transfer of energy to show that the final gas wind speed is

$$v^3 = \frac{G L_{Edd} f_w}{2\sigma^2 f_g}$$

(b) (3 points) If the wind speed is large enough to escape the potential well of the galaxy, ($v = \sigma$), the central AGN will blow out the majority of gas in the galaxy and terminate the formation of stars. Show that this gives us the $M_{BH} - \sigma$ relation,

$$M_{BH} = \frac{1}{2\pi} \frac{\sigma_T}{G^2 c m_p} \frac{f_g}{f_w} \sigma^5$$

where G is the gravitational cnstant, σ_T is the Thomson cross section, c is the speed of light, and m_p is the mass of a proton.

PROBLEM 5

5. (a) (3 points) Describe the pre-MS to AGB evolution of a solar-metallicity 1 $M_\odot$ star. Plot the evolution in $\log T_{\rm central}$ vs. $\log \rho_{\rm central}$ and H-R diagrams. Also plot a Kippenhahn diagram showing the interior structure of the star (including burning and convective regions) as a function of time. Describe each phase of evolution, including the radiative properties and nuclear burning reactions and regions.

   (b) (3 points) Describe the pre-MS to AGB evolution of a solar-metallicity 5 $M_\odot$ star. Plot the evolution in $\log T_central$ vs. $\log \rho_central$, H-R, and Kippenhahn diagrams. Describe each phase of evolution, including the radiative properties and nuclear burning reactions and regions, emphasizing the differences between this star and a 1 $M_\odot$ star.

   (c) (2 points) Describe the evolution of a 40 $M_\odot$ star. Will this star create a Red Supergiant and why?

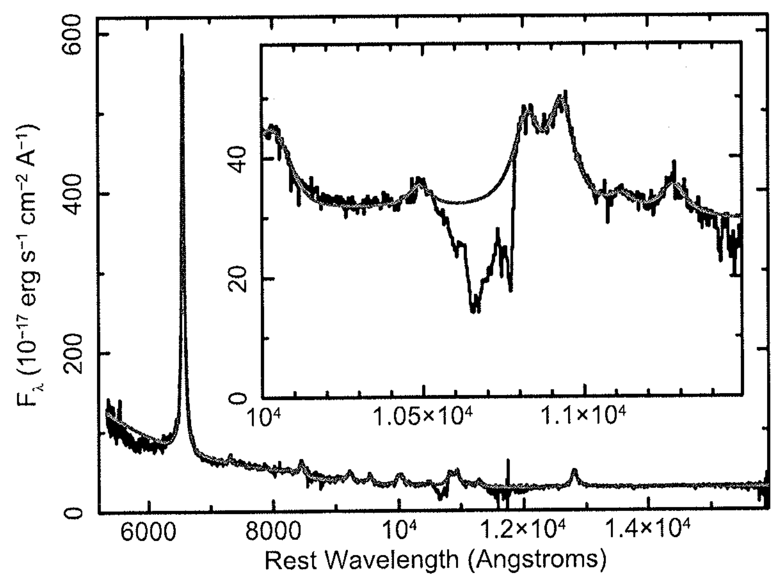
   (d) (2 points) Describe WNL, WNE, WC, and WO stars. These stars form an evolutionary sequence. Explain the connection between them.

PROBLEM 6

6. A fraction of quasars have broad, blueshifted absorption lines that indicate high-velocity outflows emerging from the central engine. Generally, an absorption profile can be described as:

$$\frac{I}{I_0} = exp(-\tau(\lambda))$$

where I is the observed flux density in the spectrum, I_0 is the intrinsic continuum (without absorption) and $\tau(\lambda)$ is the optical depth of the absorption trough originating from absorption by a single ion. An example of a broad absorption spectrum is shown below. The absorption is occurring in metastable helium in the 10830Å transition. A range of gas outflow velocities causes the absorption line to be broad.



Analysis of broad absorption lines is complicated by *partial covering*: the absorbing outflow does not cover all of the continuum emitting source, but rather covers only a fraction of it, C_f. Then, the absorption line looks shallower than it would be if the absorber covered the whole thing, and the inferred *apparent* optical depth is lower. However, this situation can be resolved, and the true optical depth and covering fraction can be determined if there are two lines in the spectrum that arise from the same lower level, because their true optical depth ratio is fixed by atomic physics. Specifically, the true optical depth ratio will be proportional to the ratio of $f_{ik}\lambda$, where f_{ik} and λ are the oscillator strength and wavelength of the transition. In that case, the intensity ratio can be expressed in these two equations:

$$I_s = (1 - C_f) + C_f e^{-\tau_s}$$

$$I_w = (1 - C_f) + C_f e^{-\tau_w}$$

where the subscripts w and s stand for weaker and stronger lines, respectively, and τ_s/τ_w is related by the ratio of their respective $f_{ik}\lambda$ values.

(a) (5 points) Consider a doublet, e.g., C IV. The first excited state has fine structure, so there are two possible transitions from the ground state to the first excited state, at 1548.2 and 1550.8Å (a doublet). The oscillator strengths for these two transtions are 0.190 and 0.0952 respectively. This means that the ratio of the optical depths τ_s/τ_w for these two transitions is effectively 2.

For the case of this doublet, solve the equations above for the covering fraction C_f and τ_s.

(b) (1 point) When scientists analyze an absorption line, they are often interested in measuring the column density N (in particles per cm^2) of the ion responsible for it. The optical depth and column density are related by the following equation:

$$\tau(\lambda) = \frac{\pi e^2}{m_e c^2} f_{ik} \lambda^2 N(\lambda),$$

where e is the charge on an electron, m_e is the mass of an electron, and c is the speed of light. Both τ and N are functions of lambda because as the absorption line is spread of a range of wavelengths due to the range of velocities over which the outflow is distributed.

Show that

$$\tau(v) = \frac{\pi e^2}{m_e c} f_{ik} \lambda N(v)$$

where v is velocity, λ is in Angstroms, and $N(v)$ is in atoms $\mathrm{cm}^{-2}(\mathrm{km\,s}^{-1})^{-1}$.

(c) (4 points) Further, show that

$$\tau(v) = 2.654 \times 10^{-15} f_{ik} \lambda N(v).$$