ASTRONOMY QUALIFYING EXAM

January, 2015

Possibly Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \times 10^{10}$ cm
1 AU=1.5×10^{13} cm
1 pc= 3.26 Ly = 3.1×10^{18} cm
a=7.56×10^{-15} erg cm^{-3} K^{-4}
c=3×10^{10} cm s^{-1}
σ =ac/4= 5.7×10^{-5} erg cm^{-2} K^{-4} s^{-1}
k=1.38×10^{-16} erg K^{-1}
e=4.8×10^{-10} esu
1 fermi=10^{-13} cm
N$_A$ = 6.02×10^{23} moles g^{-1}
G=6.67×10^{-8}g^{-1} cm^3 s^{-2}
m$_e$ = 9.1×10^{-28} g
h= 6.63×10^{-27} erg s
1 amu= $1.66053886 \times 10^{-24}$ g
$r_e = 2.8179 \times 10^{-13}$ cm; (electron radius)

Eddie? / Kilic?

PROBLEM 1

1. For this problem recall the thermodynamic identity $P = -\left.\frac{\partial E}{\partial V}\right|_S$

   (a) (1 point) Write down the polytropic equation of state.

   (b) (1 point) For a polytrope, the constant is a function of what thermodynamic quantity?

   (c) (2 points) For a polytropic equation of state derive the relationship between pressure and energy density. *Hint: Define the energy per unit mass $u = E/M$ and the specific volume per unit mass $v = V/M$ and then relate u to the energy density $\varepsilon = E/V$*

   (d) (3 points) Multiply the equation of hydrostatic equilibrium by $4\pi r^3$ and derive the Virial Theorem.

   (e) (3 points) Use the Virial Theorem to find the total energy of a star with a polytropic equation of state. Show that $\gamma = 4/3$ gives zero total energy and that $\gamma = 5/3$ corresponds to the classic case that the internal energy is half the gravitational energy in magnitude.

Kilic

PROBLEM 2

2. Consider a $3M_\odot$ main sequence star with $L = 80L_\odot$, $X = 0.7, Y = 0.28, Z = 0.02$ and

$$\epsilon_{\rm nuc} = \epsilon_c\left(1 - \frac{m}{0.1M}\right) \tag{1}$$

for $m < 0.1M$; m is the mass variable and M is the stellar mass.

(a) (2 points) Calculate and draw the luminosity profile, l, as a function of mass, m.

(b) (2 points) What is the numerical value of ϵ_c in erg/g/s?

(c) (3 points) Assuming radiative energy transport, calculate the H mass fraction as a function of mass and time, $X = X(m,t)$. What is the central value of X after 100 Myr? Draw X as a function of m at 100 Myr. (Assume that the energy generation per unit mass from hydrogen is 6×10^{18} erg g^{-1}).

(d) (2 points) Assuming that the inner 20% of the mass is convective, draw the new X profile as a function of m.

(e) (1 point) What is the H-burning lifetime for the star in (c) and (d)? How much is the lifetime extended due to convection?

John?

PROBLEM 3

3. A star has a temperature T = 6700 K, mass M = 1.4 $M_\odot$, and radius R = 1.25 $R_\odot$. There is a super-Earth exoplanet orbiting the star with a semi-major axis a = 5 AU in a circular orbit. The planet has a radius of 2 R_j (R_j = Jupiters radius). Assume the only source of energy for the planet is the star, all light falling on the planet is absorbed, and the star+planet are perfect blackbodies.

   (a) (8 points) Derive the temperature of the planet, in units of Kelvin. Assume that the temperature is uniform over the entire planet.

   (b) (2 points) Derive the period of the planet.

Henry

PROBLEM 4

4. The following two questions refer to the Milky Way Galaxy.

   (a) (3 points) List at least seven components of the Milky Way, which must include the most massive component.

   (b) (7 points) What are the observational evidences for these components?

John?

PROBLEM 5

5. The star beta Pic was observed to have a parallax of 51.44 milli-arcsec from the Hipparcos satellite.

   (a) (3 points) Given the apparent K-band magnitude of beta Pic, $m_K = 3.53$, what is the absolute K-band magnitude of beta pic, M_K ?

   (b) (2 points) A planet (beta Pic b) has recently been directly imaged around the star beta Pic. The planet has an apparent K-band magnitude of $m_K = 12.73$. How much less flux is the planet emitting in the K-band compared to its host star?

   (c) (3 points) The beta Pic b planet is oberved to be separated from its host star by 0.5 arcseconds. How far away is the planet located from its host star, in units of AU?

   (d) (2 points) Assume aliens live on the planet beta Pic b, and an alien observes the Earth/Sun system from his/her planet. Whats the angular separation (in units of milli-arcseconds) he/she would measure for the Earth and Sun?

Dai? / Henry?

PROBLEM 6

6. The following questions refer to the Milky Way Galaxy and its chemical evolution.

   (a) (1 point) Contrast thin disk and halo stars in terms of their kinematics, and metallicity.

   (b) (1 point) Give a plausible model for the formation of the Milky Way which explains the differences discussed in part a.

   (c) (2 points) According to chemical evolution theory, why does the value of [Fe/O] in any one location in the Galaxy tend to increase with time? If the initial mass function were flatter (higher fraction of massive stars), how would you expect that to affect the evolution of the local value of [Fe/O]? Explain.

   (d) (1 point) Sketch a plot of the rotation curve of the Milky Way and describe its behavior for both the bulge and disk. Make sure to include axis titles.

   (e) (3 points) Derive a functional relation between surface density σ (mass/pc^2), tangential (circular) velocity v, and galactocentric distance r for the disk. What is implied about the surface density in the disk? Explain.

   (f) (2 points) Given the rotation properties of the bulge, what is implied about the behavior of its volume density as a function of galactocentric distance? Use simple algebra to prove your point.