ASTRONOMY QUALIFYING EXAM
August 2012

Possibly Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \times 10^{10}$ cm
1 AU = 1.5 $\times 10^{13}$ cm
1 pc = 3.26 Ly. = 3.1 $\times 10^{18}$ cm
a = 7.56 $\times 10^{-15}$ erg cm^{-3} K^{-4}
c = 3 $\times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
k = 1.38 $\times 10^{-16}$ erg K^{-1}
e = 4.8 $\times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
G = 6.67 $\times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
h = 6.63 $\times 10^{-27}$ erg s
1 amu = 1.66053886 $\times 10^{-24}$ g

John

PROBLEM 1

Suppose a main sequence star at a distance of d = 8 pc from the Earth has been observed to have a maximum radial velocity of 0.1 m/s, and the radial velocity varies periodically with a period P =1.5 years. From this we conclude that the star must have an unseen companion, a planet.

Assume that the star has mass of 0.8 solar masses, and T=5000 K. For simplicity, assume circular orbits and an inclination angle of 90°.

a. Calculate the average separation between the star and the planet. (4 points)

b. Calculate the mass of the planet. (4 points)

$$\frac{m_p^3}{(m_s + m_p)^2} \sin^3 i = \frac{P}{2\pi G} v_{1,r}^3$$

c. Can life exist on this planet? Explain. (2 points)

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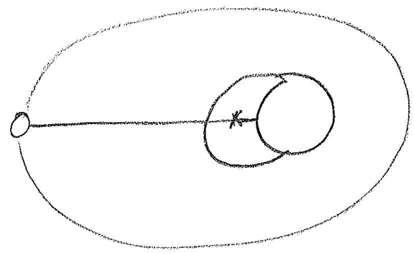
Astro #1

a) * Assuming circular orbits



$M = .8 M_\odot$
$= 1.6 \cdot 10^{33} g$

$T = 5000 K$

$$P^2 = \frac{4\pi^2}{G(M+m)} a^3$$

* rewrite in terms of reduced mass, $\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{m_s m_p}{m_s + m_p}$

* For star: $P^2 = \frac{4\pi^2}{G(m_s + \mu)} a^3$ $\qquad \frac{G m_s \mu}{a^2} = m_s \frac{V_s^2}{a} \rightarrow V_s = \sqrt{\frac{G\mu}{a}}$

$$\Rightarrow a = \left(\frac{G P^2 (m_s + \mu)}{4\pi^2}\right)^{1/3}$$

* For planet: $P^2 = \frac{4\pi^2}{G(\mu + m_p)} b^3$ $\qquad \frac{G m_p \mu}{b^2} = m_p \frac{V_p^2}{b}$

$$b = \left(\frac{G P^2 (\mu + m_p)}{4\pi^2}\right)^{1/3} \qquad V_p = \sqrt{\frac{G\mu}{b}}$$

* Separation always constant so center of mass is stationary

$$a + b = \left(\frac{GP^2}{4\pi^2}\right)^{1/3} \left((m_s + \mu)^{1/3} + (m_p + \mu)^{1/3}\right)$$

$$P^2 = \frac{4\pi^2}{G m_s} a^3 \qquad m_s = 0.8\, M_\odot$$

$$\Rightarrow \quad a = \left(\frac{G m_s P^2}{4\pi^2}\right)^{1/3}$$

$$= \left[\frac{6.67\cdot 10^{-8}\ \frac{cm^3}{g\cdot s^2}\cdot 0.8\cdot 2\cdot 10^{33} g\cdot (1.5\cdot\pi\cdot 10^7 s)^2}{4\pi^2}\right]^{1/3}$$

Correct

$$= 1.82\cdot 10^{13}\ cm$$
$$\approx 1.21\ AU$$

$$V_s = \sqrt{\frac{G\mu}{a}} \qquad V_s = \sqrt{\frac{G m_s}{a}}$$

$$a V_s^2 = G\left(\frac{m_s m_p}{m_s + m_p}\right)$$

$$\frac{a V_s^2}{G} = \alpha = \frac{m_s m_p}{m_s + m_p}$$

$$(m_s + m_p)\alpha = m_s m_p$$

$$m_s \alpha = m_p (m_s - \alpha)$$

$$\frac{m_s \alpha}{m_s - \alpha} = m_p = 2.72\cdot 10^{18} g$$

$$P^2 = \frac{4\pi^2}{G(m_s + m_p)} a^3$$

$$\frac{P^2}{a^3} = \frac{4\pi^2}{G m_s + m_p}$$

$$m_s + m_p = \frac{4\pi^2 a^3}{G P^2}$$

$$m_p = \frac{4\pi^2 a^3}{G P^2} - m_s$$

$$= \frac{4\cdot\pi^2\cdot(1.82\cdot 10^{13} cm)^3}{6.67\cdot 10^{-8}\ \frac{cm^3}{g s^2}\cdot(1.5\cdot\pi\cdot 10^7 s)^2} - 0.8\cdot 2\cdot 10^{33} g$$

$$= 0$$

$$P^2 = \frac{4\pi^2}{G(m_s + \mu)} a^3$$

$$P^2 G\left(m_s + \frac{m_s m_p}{m_s + m_p}\right) = 4\pi^2 a^3$$

$$\frac{m_s m_p}{m_s + m_p} = \frac{4\pi^2 a^3}{P^2 G} - m_s$$

$$m_s m_p = \left(\frac{4\pi^2 a^3}{P^2 G} - m_s\right)(m_s + m_p)$$

$$\left(m_s - \frac{4\pi^2 a^3}{P^2 G}\right) m_p = \left(\frac{4\pi^2 a^3}{P^2 G} - m_s\right) m_s$$

$$m_p = -m_s$$

$$\frac{a^3}{m_s + \mu} = \frac{b^3}{m_p + \mu}$$

$$a^3(m_p + \mu) = b^3(m_s + \mu)$$

$$a^3\left(m_p + \frac{m_s m_p}{m_s + m_p}\right) = b^3\left(m_s + \frac{m_s m_p}{m_s + m_p}\right)$$

$$\frac{a^3}{b^3} = \frac{m_s + \frac{m_s m_p}{m_s + m_p}}{m_p + \frac{m_s m_p}{m_s + m_p}}$$

$$= \frac{m_s(m_s + m_p) + m_s m_p}{m_p(m_s + m_p) + m_s m_p}$$

$$\frac{a}{b} \approx \frac{m_s}{m_p} \qquad = \frac{m_s^2 \cancel{+ 2 m_s m_p}}{\cancel{m_p^2} + 2 m_s m_p}$$

Dai? / Leighly?

PROBLEM 2

The Crab pulsar has a period of $P = 0.0333$ seconds, and a slow-down rate of $\dot{P} = 4.21 \times 10^{-13}$. The Crab nebula emits a total luminosity of 5×10^{31} W. A neutron star can be assumed to have a mass equal to $1.4\,M_\odot$ and a radius of 10km.

a. What is the size of the light cylinder for the crab pulsar? (2 points)

b. Show that the rate of rotational energy lost approximately equals the luminosity of the nebula. (3 points)

c. The energy per second emitted by a rotating magnetic dipole is

$$\frac{dE}{dt} = -\frac{32\pi^5 B^2 R^6 \sin^2\theta}{3\mu_0 c^3 P^4}.$$

Assuming that the rotational kinetic energy lost by the star is carried away by magnetic dipole radiation, derive an equation for the magnetic field at the pole of the neutron star. Use the parameters for the Crab pulsar to obtain a value of the magnetic field in Teslas. (2 points)

c. Discuss and explain the properties of various classes of pulsars. Sketch the $P\dot{P}$ diagram, show how it is populated by different classes of pulsars, and explain how we use this diagram to infer the magnetic field and age of these objects. (3 points)

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Astro #2

a) * See Fig 16.26 of B.O.B (pg 600) for description of light cylinder

$$R_c = \frac{cP}{2\pi}$$

$$= \frac{3\cdot 10^{10}\ \frac{cm}{s} \cdot (.0333\ s)}{2\pi}$$

$$= 1.59 \cdot 10^{8}\ cm$$

b) $\dot{P} = 4.21 \cdot 10^{-13}\ s$

$$RE = \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} I \left(\frac{P}{2\pi}\right)^{-2}$$

$$= \frac{2}{5} MR^2 \left(\frac{P}{2\pi}\right)^{-2}$$

$$= \frac{8\pi^2 MR^2}{5P^2}$$

$$[E] = \frac{g\,cm^2}{s^2} \quad [\dot{E}] = \frac{g\,cm^2}{s^3}$$

$$\dot{E} = \frac{8\pi^2 MR^2}{5}\left(-2\frac{\dot{P}}{P^3}\right)$$

$$= \frac{-16\pi^2 MR^2 \dot{P}}{5P^3}$$

$$= \frac{-16\pi^2 (1.4\cdot 2\cdot 10^{33} g)(1\cdot 10^6 cm)^2 (4.21\cdot 10^{-13})}{5\ (0.0333\ s)^3}$$

$$= -1.01\cdot 10^{39}\ \frac{g\,cm^2}{s^3}$$

$$L = 5\cdot 10^{31}\ W$$

$$= 5\cdot 10^{31}\ \frac{kg\,m^2}{s^3}$$

$$= 5\cdot 10^{38}\ \frac{g\,cm^2}{s^3}$$

$\dot{E} \approx 2L \Rightarrow$ close enough, off by factor of 2 somewhere

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#2 (cont.)

$$[T] = \frac{N}{A\,m} \leftarrow \frac{kg\,m}{s^2}$$

$$[\mu_0] = 4\pi \cdot 10^{-7}\ \frac{N}{A^2}$$

c) $$\frac{dE}{dt} = \frac{-32\pi^5 B^2 R^6 \sin^2\theta}{3\mu_0 c^3 P^4}$$

$$\Rightarrow B = \left[\frac{3\dot{E}\mu_0 c^3 P^4}{-32\pi^5 R^6 \sin^2\theta}\right]^{1/2} \leftarrow \text{B.O.B says assume } \theta = 90^\circ$$

↳ "light house assumption"

$$B_{pole} = \left[\frac{3\left(-1.01\cdot 10^{32}\ \frac{N\,m}{s}\right)\left(4\pi\cdot 10^{-7}\ \frac{N}{A^2}\right)\left(3\cdot 10^8\ \frac{m}{s}\right)^3 (0.0333\ s)^4}{-32(\pi)^5 (10\cdot 10^3\ m)^6 \sin^2(90^\circ)}\right]$$

$$= \frac{\frac{N^2 m^4}{A^2}}{m^6} \rightarrow \frac{N^2}{m^2 A^2}$$

$$= 1.14 \cdot 10^9\ T$$ ($8\cdot 10^8\ T$ to account for extra factor of 2 error)

d) See pg 601 of B.O.B for diagram; info sparse

Kilic

PROBLEM 3

In this problem you are asked to discuss and compare two types of H fusion which occur in stars along with the chemical evolution of nitrogen.

a. (1 point) Write down the three reaction steps in the PPI reaction. Show all isotopes and bi-products involved.

b. (1 point) Write down the six reactions in the CN cycle. Show all isotopes and bi-products involved. Identify the two relatively fast reactions and the slowest reaction of the six. Why is carbon referred to as a catalyst?

c. (2 points) Make a qualitative comparison of PPI and the CN cycle in terms of the threshold temperature and temperature sensitivity of the energy generation coefficient ϵ, i.e., $d\epsilon/dT$. Discuss the relative amount that each cycle contributes to the total energy generation in the Sun's core.

d. (3 points) Explain the relevance of the CN cycle to the evolution of the total nitrogen abundance in a galaxy. Explain what stellar types (mass ranges) are thought to produce significant amounts of N.

e. (3 points) The nearby figure shows the universal behavior of the N/O abundance ratio as a function of metallicity, as measured by O/H. Note the flat behavior at low metallicities and the upward turn starting at around solar metallicity of about 8.7. Explain this change in slope.

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Astro #3

a) P-P I Chain

$$\left.\begin{array}{l} H_1^1 + H_1^1 \rightarrow H_1^2 + \nu + e^+ + \gamma \\ H_1^2 + H_1^1 \rightarrow He_2^3 + \gamma \end{array}\right\} \times 2$$

$$He_2^3 + He_2^3 \rightarrow He_2^4 + 2H_1^1$$

b) CN cycle

$$C_6^{12} + H_1^1 \rightarrow N_7^{13} + \gamma$$

* Decay rxns are quickest

$$N_7^{13} \rightarrow N_6^{13} + e^+ + \nu$$

$$C_6^{13} + H_1^1 \rightarrow N_7^{14} + \gamma$$

$$N_7^{14} + H_1^1 \rightarrow O_8^{15} + \gamma$$

← limiting rxn

$$O_8^{15} \rightarrow N_7^{15} + e^+ + \nu$$

$$N_7^{15} + H_1^1 \rightarrow C_6^{12} + He_2^4$$

Carbon is referred to as a catalyst b/c while it is part of the rxn chain, the abundance of it is uneffected by the rxn itself (1 C_6^{12} at both beginning and end)

c) P-P I chain: $\sim 1 \cdot 10^7$ K needed to start rxn, $E_{pp} \propto T^4$

CN cycle: $\sim 1.5 \cdot 10^7$ K needed to dominate over PP I chain, $E_{CN} \propto T^{18}$

W/in the sun, most energy produced via PP chain

Dai? / Leighly?

PROBLEM 4

a. (3 points) The hot gaseous halo of galaxy clusters is pressure supported, and thus it follows the hydrostatic equilibrium equation. Write down the hydrostatic equilibrium equation and the ideal gas law.

b. (4 points) First derive the following equation

$$\frac{dP}{dr} = \frac{k}{\mu m_H}\left(T\frac{\partial \rho}{\partial r} + \rho\frac{\partial T}{\partial r}\right), \tag{1}$$

assuming that the mean molecular weight is a constant for the gas. Then derive the expression for the total gravitational mass

$$M = -\frac{kTr}{\mu m_H G}\left(\frac{\partial \ln \rho}{\partial \ln r} + \frac{\partial \ln T}{\partial \ln r}\right). \tag{2}$$

c. (3 points) Using a so-called β model for the gas density and assuming that the gas is isothermal, the expression for M can be written as

$$M = \frac{3\beta kTr}{\mu m_H G}\left(\frac{r^2}{r^2 + r_c^2}\right). \tag{3}$$

For a cluster with a temperature of T = 5 keV, a core radius r_c=400 kpc, mean molecular weight $\mu = 0.61$, and $\beta = 0.7$, find the total mass of the cluster within 2 Mpc.

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Astro #4

a) $PV = nRT$ (Ideal Gas Law)

$= NkT$

$$\frac{dP}{dr} = -\frac{GM}{r^2}\rho \quad \text{(HSE eqn)}$$

b) Rewrite Ideal gas law as

$$P = \frac{\rho}{m_H \mu} kT$$

$$\frac{dP}{dr} = \frac{k}{\mu m_H} \frac{d}{dr}(\rho T)$$

$$= \frac{k}{\mu m_H}\left(\rho \frac{\partial T}{\partial r} + T\frac{\partial \rho}{\partial r}\right)$$

* Rewriting HSE eqn

$$\frac{r^2 dP}{-G\rho dr} = m$$

$$\Rightarrow m = \frac{r^2}{-G\rho}\left(\frac{k}{\mu m_H}\left[\rho\frac{\partial T}{\partial r} + T\frac{\partial \rho}{\partial r}\right]\right)$$

$$= -\frac{kTr}{\mu m_H G}\left(\frac{r}{T}\frac{\partial T}{\partial r} + \frac{r\partial \rho}{\rho \partial r}\right)$$

$$= -\frac{kTr}{\mu m_H G}\left(\frac{\partial \ln \rho}{\partial \ln r} + \frac{\partial \ln(T)}{\partial \ln r}\right)$$

c) * β assumes $\beta = \frac{P_{gas}}{P_{tot}} \Rightarrow P_{gas} = \beta P_{tot}$

$$P = \frac{1}{\beta}\frac{\rho}{m_H \mu} kT, \quad T = \text{const.}$$

$$\Rightarrow m = \frac{r^2}{-G\rho}\frac{dP}{dr}$$

$$= \frac{r^2}{-G\rho}\frac{d}{dr}\left[\frac{1}{\beta}\frac{\rho}{m_H \mu}kT\right] ??$$

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#4 (cont.)

* Note: 1eV = 11,600 K

c) * Calculation only

$$M = \frac{3\beta k T r}{\mu m_H G}\left(\frac{r^2}{r^2 + r_c^2}\right)$$

$$= \frac{3(0.7)(1.38\cdot10^{-7}\ \frac{erg}{K})(5\ keV)(2\ Mpc)}{(0.61)(1.66\cdot16^{-24}\ g)(6.67\cdot10^{-8}\ \frac{cm^3}{g s^2})}\left[\frac{(2\ Mpc)^2}{(2\ Mpc)^2 + (400\ kpc)^2}\right]$$

$$= \frac{2.1(1.38\cdot10^{-7}\ \frac{g\,\cancel{cm^2}}{\cancel{s^2} K})(5000\cdot11{,}600\ \cancel{K})(2\cdot10^{6}\cdot3.1\cdot10^{18}\ \cancel{cm})}{(0.61)(1.66\cdot10^{-24}\ g)(6.67\cdot10^{-8}\ \frac{\cancel{cm^3}}{\cancel{g}\,\cancel{s^2}})}\left[\frac{(2\cdot10^{6})^2}{(2\cdot10^{6})^2 + (4\cdot10^{5})^2}\right]$$

$$= 1.48\cdot10^{57}\ g$$

Wang

PROBLEM 5

Inflationary paradigm is an integral part of modern cosmology.

a. Why do we need inflation in the extremely early universe? (2 pts)

b. What condition on the equation of state must be satisfied for inflation to occur? Explain. (2 pts)

c. Use the conservation of energy and momentum to derive the condition in (b). (6 pts)

Eddie

PROBLEM 6

a. (2 pts) Write down the equation of radiative transfer for a plane-parallel atmosphere and define all the terms.

b. (3 pts) Assuming that there is no external irradiation at the surface, show that

$$I_\lambda = S_\lambda(1 - e^{-\tau_\lambda})$$

c. (5 pts) What is I_λ in terms of S_λ for the optically thin and optically thick cases? Do you expect to see emission or absorption lines at the wavelengths of large opacity, κ_λ?

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#6

a) Write down equation of radiative transfer in plane-parallel + define all terms

$$\mu \frac{dI_\nu}{dz} = \eta_\nu - \chi_\nu I_\nu$$

$\mu = \cos \Theta$

η_ν = scattering coefficient

χ_ν = opacity

I_ν = intensity

b) Assuming there is no external irradiation at the surface, show: $I_\lambda = S_\lambda(1-e^{\tau_\lambda})$

$$\frac{dI_\lambda}{dz} = \eta_\lambda - \chi_\lambda I_\lambda$$

$$\frac{dI_\lambda}{\chi_\lambda dz} = \frac{\eta_\lambda}{\chi_\lambda} - I_\lambda$$

* let $d\tau = \chi_\lambda dz$, $\frac{\eta_\lambda}{\chi_\lambda} = S_\lambda$

$$\frac{dI_\lambda}{d\tau} = S_\lambda - I_\lambda$$

$$\left(\frac{dI_\lambda}{d\tau} + I_\lambda = S_\lambda\right)e^\tau$$

$$\frac{dI_\lambda}{d\tau} e^{+\tau} + I e^{+\tau} = S_\lambda e^\tau$$

$$\frac{d I e^\tau}{d\tau} = S_\lambda e^\tau$$

$$-I_0 + I e^\tau = S_\lambda\left[e^{\tau_\lambda} - 1\right]$$

$$I_\lambda = S_\lambda\left(1 - e^{-\tau}\right)$$

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#6 (cont.)

c) Optically Thin $\rightarrow$ $\tau \ll 1$

$$I_\lambda = S_\lambda (1 - e^{-\tau_\lambda})$$

$$= S_\lambda (1 - [1 - \tau_\lambda + \tfrac{1}{2}\tau_\lambda^2 + \ldots])$$

$$= [+\tau_\lambda - \tfrac{1}{2}\tau_\lambda^2 + \tfrac{1}{6}\tau_\lambda^3 - \ldots] S_\lambda$$

$$= S_\lambda \tau_\lambda - \tfrac{1}{2}\tau_\lambda^2 S_\lambda$$

Optically Thick $\rightarrow$ $\tau \gg 1$

$$I_\lambda = S_\lambda (1 - \cancelto{0}{e^{-\tau_\lambda}})$$

$$I_\lambda = S_\lambda$$

In the optically thick case, you would expect to only see the source function. In the optically thin case, you would expect to see emission lines.

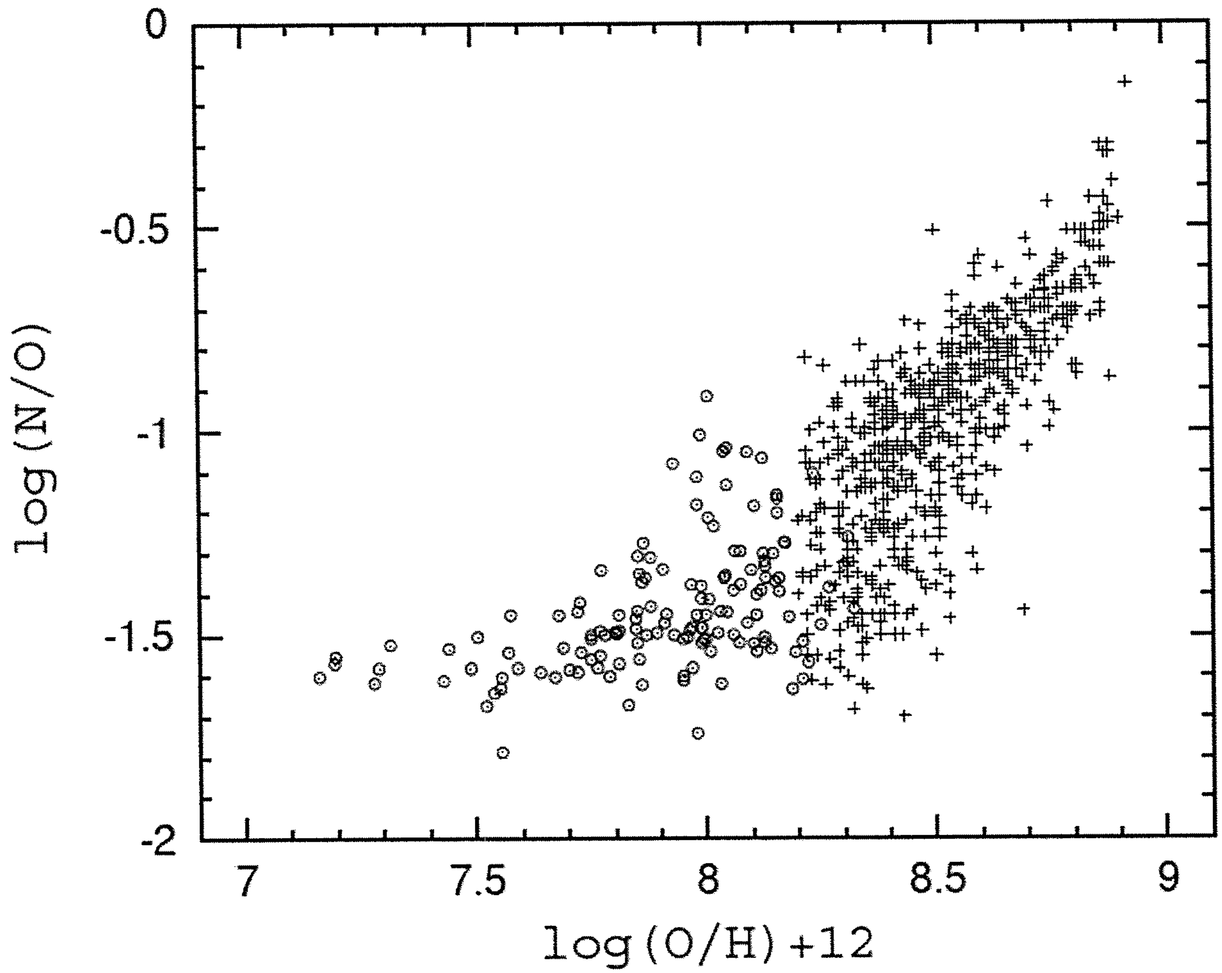

Fig. 4. The N/O–O/H for H II regions in spiral (pluses) and irregular (circles) galaxies.