ASTRONOMY QUALIFYING EXAM
January 2013

Possibly Useful Quantities

$L_\odot = 3.9 \times 10^{33}$ erg s^{-1}
$M_\odot = 2 \times 10^{33}$ g
$M_{bol\odot} = 4.74$
$R_\odot = 7 \times 10^{10}$ cm
1 AU = 1.5×10^{13} cm
1 pc = 3.26 Ly. = 3.1×10^{18} cm
$a = 7.56 \times 10^{-15}$ erg cm^{-3} K^{-4}
$c = 3 \times 10^{10}$ cm s^{-1}
$\sigma = ac/4 = 5.7 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
$k = 1.38 \times 10^{-16}$ erg K^{-1}
$e = 4.8 \times 10^{-10}$ esu
1 fermi = 10^{-13} cm
$N_A = 6.02 \times 10^{23}$ moles g^{-1}
$G = 6.67 \times 10^{-8}$ g^{-1} cm^3 s^{-2}
$m_e = 9.1 \times 10^{-28}$ g
$h = 6.63 \times 10^{-27}$ erg s
1 amu = $1.66053886 \times 10^{-24}$ g

Dai? / Leighly?

PROBLEM 1

a. (7 points) Assume that the gas component of a galaxy, with a mass fraction f_g, is virialized and follow the overall density profile of the galaxy, a singular isothermal sphere mass profile,

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2}, \tag{1}$$

where σ is the velocity dispersion of the galaxy. The galaxy has a central AGN, which is radiating at the Eddington luminosity,

$$L_{Edd} = 4\pi G c m_p M_{BH}/\sigma_T. \tag{2}$$

A fraction, f_w, of the energy radiated by the central AGN is deposited to the gas in the form of kinetic energy. This kinetic "feedback" energy from the AGN can drive the gas in the host galaxy to flow outward. Assume that the final gas outflow is in a spherical shell with a constant velocity, v, and half of the kinetic feedback energy is converted to the gravitational potential of the gas and the other half to the kinetic energy of the gas during the outflowing process. Use the conservation or transfer of energy to show that the final gas wind speed is

$$v^3 = \frac{G L_{Edd} f_w}{2\sigma^2}. \tag{3}$$

b. (3 points) If the wind speed is large enough to escape the potential well of the galaxy ($v = \sigma$), the central AGN will blow out the majority of gas in the galaxy and terminate the formation of stars. Show that this gives us the M_{BH}–σ relation,

$$M_{BH} = \frac{1}{2\pi} \frac{\sigma_T}{G^2 c m_p} \frac{1}{f_w} \sigma^5, \tag{4}$$

where G is the gravitational constant, σ_T is the Thomson cross section, c is the speed of light, and m_p is the mass of a proton.

Jan 2013

Astro #1

a) $\rho(r) = \frac{\sigma^2}{2\pi G r^2}$

$$M_G = \int \rho(r)\, dV$$

$$= \int_0^R \int_0^{2\pi} \int_0^{\pi} \frac{\sigma}{2\pi G r^2} r^2 \sin\theta \, dr\, d\theta\, d\varphi$$

$$= \int_0^R dr \int_0^{2\pi} d\varphi \int_0^{\pi} \frac{\sigma^2 \sin\theta}{2\pi G} d\theta$$

$$= \int_0^R dr \int_0^{2\pi} d\varphi \frac{\sigma^2}{2\pi G} \left(-\cos\theta \Big|_0^{\pi} \right)$$

$$-(-1) - -1$$

$$= \int_0^R dr \int_0^{2\pi} \frac{\sigma^2}{\pi G} d\varphi$$

$$= \int_0^R \frac{2\sigma^2}{G} dr$$

$$M_G = \frac{2\sigma^2 R}{G}$$

$$\Rightarrow \frac{1}{2} m_g v^2 = \frac{1}{2} f w L_{eda} t$$

Jan 2013

#1 (cont.)

b) $$v^3 = \frac{G L_{edd} f_w}{2\sigma^2}$$

$$L_{edd} = \frac{4\pi G c m_p M_{BH}}{\sigma_T}$$

* Assuming $v = \sigma$

$$\sigma^3 = \frac{G f_w}{2\sigma^2}\left(\frac{4\pi G c m_p M_{BH}}{\sigma_T}\right)$$

$$\frac{2\sigma^5 \sigma_T}{4\pi G^2 c m_p f_w} = M_{BH}$$

$$\hookrightarrow M_{BH} = \frac{\sigma_T \sigma^5}{2\pi G^2 c m_p f_w}$$

Wang

PROBLEM 2

The Universe is dominated by dark energy today, but for a rough estimate of the age of the Universe at $2 < z < 100$, we can assume a matter dominated universe.

The Friedman Equation is

$$\dot{R}^2 + k = \frac{8\pi G}{3}\rho R^2, \qquad (5)$$

where

$$\frac{8\pi G}{3}\rho = \Omega H^2. \qquad (6)$$

(1) Derive the formula for the age of a matter-dominated universe at redshift z, assuming that we know t_0 (the age of the Universe today). (5 pts)

(2) What is the current measurement of t_0? Estimate the age of the Universe at $z = 10$ using this information. (2 pts)

(3) How does dark energy change the age of the Universe today, compared to a flat universe with matter only? (3 pts)

Leighly?

PROBLEM 3

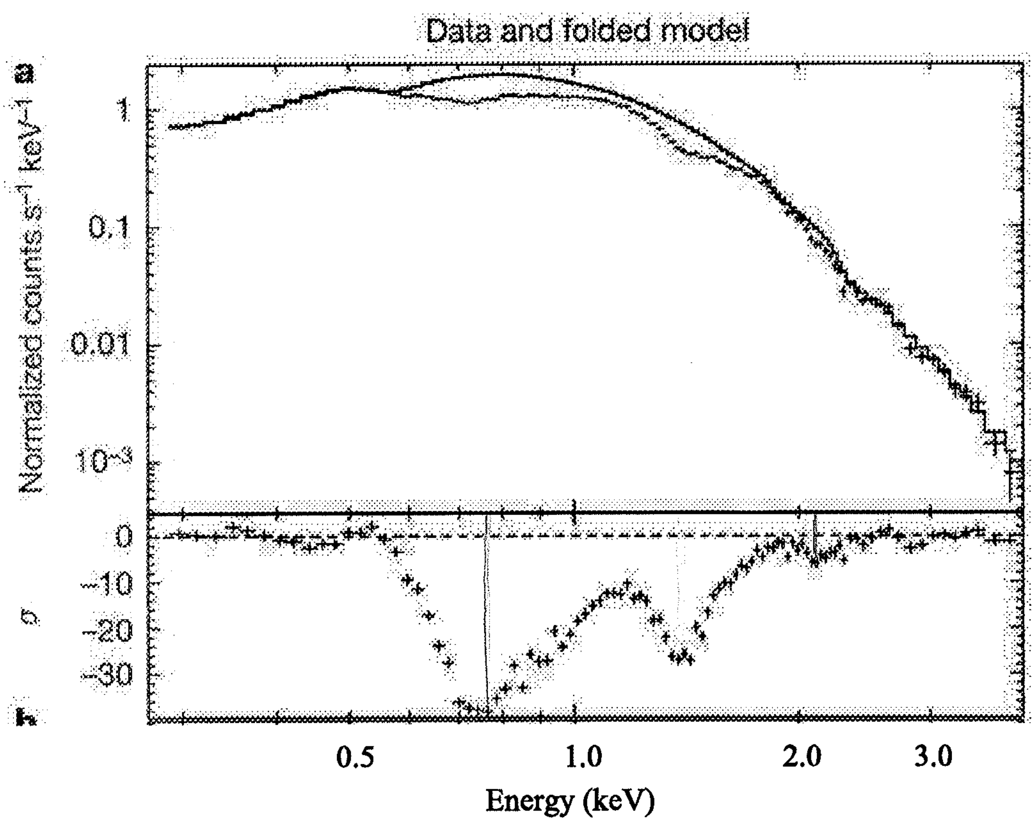
a. An electron in an electromagnetic field will experience a Lorentz force. Write down the equation for the Lorentz force. (2 points).

b. Consider an electron in a uniform magnetic field with a velcocity v. What is the frequency of light emitted by this electron if the velocity vector is oriented perpendicular to the magnetic field lines? (2 points)

c. The figure below shows the X-ray spectrum of an isolated neutron star. Direct your attention to the lower panel, which shows the difference between the spectrum and a blackbody continuum model. Three (possibly 4) absorption lines are seen. Please estimate the frequencies (in Hz) of these absorption lines. Which one is the fundamental frequency and which are harmonics? (2 points)

d. Estimate the magnetic field strength, in gauss, of the neutron star, ignoring general relativistic effects. (2 points)

e. Neutron stars are very compact, and general relativity should not be ignored. GR will affect the frequency of the absorption feature. Will the real feature have a higher frequency or lower frequency than estimated in part (d)? Explain. (2 points)



Jan 2013

Astro #3

a) $F = q\left(E + \frac{v}{c} \times B\right)$

b)

Henry

PROBLEM 4

Briefly define and discuss the relevance of the following terms to modern astronomy. 1 point per question

1. Cepheid variable star
2. Initial mass function
3. tunneling in the context of the PPI chain reaction
4. age-metallicity relation
5. damped Lyα system (DLA)
6. s-process
7. G dwarf problem
8. Tully-Fisher relation
9. Galactic thin disk
10. isophotal radius

Jan 2013

Astro #4

a) A Cepheid Variable star is a type of pulsating star that follows the period-luminosity relation, which states that the longer the star's period of pulsation, the more luminous the Cepheid is. By calibrating this relationship using parallax techniques to determine the distances to nearby Cepheids, the period-luminosity relation allows us to determine the distance to these sphecial class of stars simply by knowing how bright they are. Cepheids make up one rung of the astronomical distance ladder.

b) The initial mass function (IMF) of a galaxy is an attempt to estimate the amount of stars that will form in a galaxy of a certain mass, within a certain volume

c) In the context of the PPI chain, tunnelling is the ability of charged particles to overcome the coulomb barrier b/w them and begin the fusion process. Without the ability of the particles to tunnel, the hydrogen atoms would not be able to this barrier (at any velocity) and fusion would not occur. This quantum mechanical property of these H atoms is what allows stars to exist, as w/o fusion, the stars would only illuminate themselves by conversion of gravitational potential energy + would be unable to sustain themselves for long periods of time.

d) The age-metallicity relation is the correlation b/w the ages of stars in a galaxy and their chemical compositions. As early, massive stars die, they chemically enrich the surrounding medium. Therefore, as newer stars form from these gas clouds they contain a higher percentage of fusion byproducts like carbon and iron (metals) than stars that formed earlier. Therefore, when we look at stars of similar types, stars with higher metallicities must be younger than stars w/ lower metallicities. This provides us w/ a rough estimate of stellar ages w/in a galaxy.

Jan 2013

#4 (cont.)

e) Damped Lyman-α systems are concentrations of neutral hydrogen gas associated w/ quasars at high-z. It is believed that these systems contain most of the neutral hydrogen in the universe, and that they are correlated w/ the early stages of galaxy formation. Therefore study of the dynamics of these systems may allow us to better understand galaxy formation mechanisms.

f) The s-process is a nuclear fusion process by which elements heavier than Fe can be formed. In regions w/ a low neutron flux, like stars undergoing neutron cooling or SN remnants, the atom will fuse with the neutron and create an unstable particle, where the neutron will eventually β-decay, increasing the # of protons in the nucleus of the atom. Non-radioactive heavy elements can be formed by this process.

g) The G-dwarf problem arises from a discrepancy b/w theory + observation of stars in the solar neighborhood. Current models of chemical enrichment in the galaxy suggest that we should see many more G/F class stars w/ metallicities close to 0 than we do. This suggests that there was another method of chemical enrichment that occured earlier in the galaxy formation process

h) The Tulley-Fisher Relation is a relationship that exists b/w the luminosity and maximal rotation velocity of a spiral galaxy. Because of this relationship, it can be used as a rung on the distance ladder for nearby galaxy's.

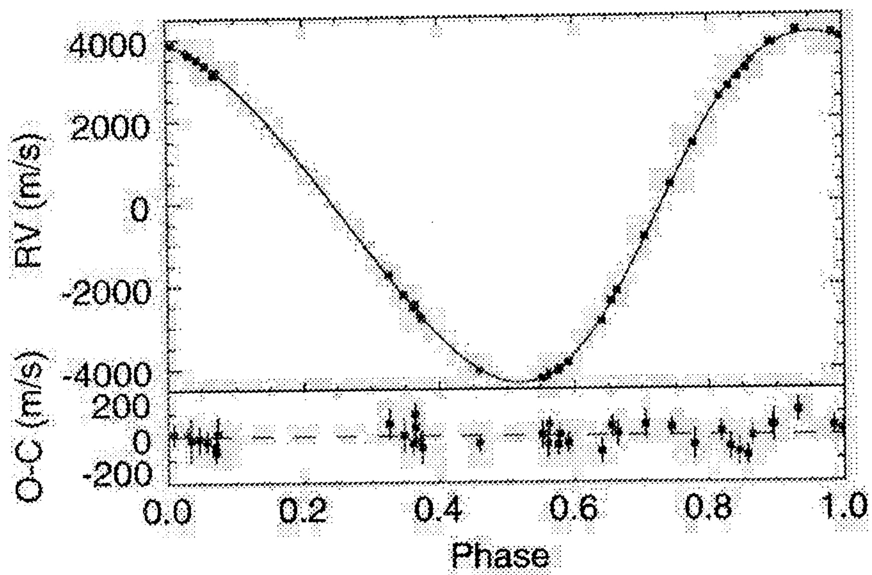
i) The galactic thin disk is the region w/in the disk of a spiral galaxy close to the mid-plane of the disk where most star formation occurs. Stars w/in the thin disk are younger and will over time drift away as their specific velocities carry them away from the mid-plane. Stars w/in the thin disk typically have higher metallicities according to the age metallicity relation.

j) The isophotal radius is a measurement of the approximate size of a galaxy. Near the edges of a galaxy, its luminosity becomes low, making an exact measurement of its size hard. Typically, the isophotal radius are measured by a % of the night-sky background brightness

John

PROBLEM 5

The following radial velocity phase curve is observed for a companion orbiting a star. Assume e=0 and P=79 days:



a. (7 pts) Derive a general expression for the companion mass.

b. (1 pt) What is the minimum mass of the companion, assuming the host star is a Solar analog?

c. (1 pt) What is the semi-major axis, a, of the companion in AU? Assume sini = 1 and the host star is a Solar analog.

d. (1 pt) The companion is observed to transit the primary star, producing a 2% drop in flux. Assuming the primary is a Solar analog, what is the radius of the companion in R_{Sun}?

Jan 2013

Astro #5

* See pg 184-188 B.O.B

a) $e = 0 \rightarrow$ circular orbit

$P = 79$ days

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3$$

* m_1 is mass of star, m_2 mass of companion

$$\left(\frac{GP^2}{4\pi^2 a^3}\right)^{-1} - m_1 = m_2$$

$$P = \frac{2\pi a}{V} \rightarrow \frac{PV}{2\pi} = a$$

* but $V = \frac{V_r}{\sin i}$

$$\Rightarrow a = \frac{PV_r}{2\pi \sin i}$$

$$\Rightarrow \frac{4\pi^2 \left(\frac{P^3 V_r^3}{(2\pi)^3 \sin^3 i}\right)}{G P^2} - m_1 = m_2$$

$$\frac{V_r^3 P}{2\pi G \sin^3 i} - m_1 = m_2$$

b) * for minimum mass $\rightarrow \sin^3 i = 1$

$$\frac{V_r^3 P}{2\pi G} - m_1 = m_2$$

$$\frac{(4000 \frac{m}{s})^3 (3.15 \cdot 10^7 \frac{s}{yr} \cdot 79 \, yr)}{2\pi (6.67 \cdot 10^{-8}) \frac{cm^3}{g s^2}} - 2 \cdot 10^{33} g =$$

Eddie

PROBLEM 6

1. (4 pts) Show that the formal solution of the plane-parallel radiative transfer equation can be written:

$$I_\nu(\tau_1, \mu) = I_\nu(\tau_2, \mu)e^{-(\tau_2-\tau_1)/\mu} + \frac{1}{\mu}\int_{\tau_1}^{\tau_2} S_\nu(t)e^{-(t-\tau_1)/\mu}\, d\cancel{\mu}\tau \tag{7}$$

where $S_\nu(t)$ is the source function, $\tau_{1,2}$ are optical depth points in the atmosphere, and μ is the cosine of the angle of the ray.

2. (2 pts) Apply Eqn. 7 to an arbitrary point in the atmosphere of a semi-infinite slab to find:

$$I_\nu(\tau, \mu) = \int_\tau^\infty S_\nu(t)e^{-(t-\tau)/\mu}\, dt/\mu \quad \text{for } 0 \le \mu \le 1 \tag{8}$$

$$I_\nu(\tau, \mu) = \int_0^\tau S_\nu(t)e^{-(\tau-t)/(+\mu)}\, dt/(+\mu) \quad \text{for} -1 \le \mu \le 0 \tag{9}$$

3. (2pts) Integrate Eqns 8 and 9 over angle to find

$$J_\nu(\tau) = 1/2\left[\int_\tau^\infty dt\, S_\nu(t)\int_1^\infty dw\, e^{-w(t-\tau)}/w + \int_0^\tau dt\, S_\nu(t)\int_1^\infty dw\, e^{-w(\tau-t)}/w\right] \tag{10}$$

These integrals are of standard form (the first exponential integral):

$$E_1(x) = \int_1^\infty e^{-xt}/t\, dt$$

4. (1 pt) Show that in terms of E_1, J may be written:

$$J_\nu(\tau) = 1/2\int_\tau^\infty dt\, S_\nu(t)E_1(|t-\tau|)$$

5. (1 pt) Explain the nature of this final operator.

Jan 2013

#6

a) $$\frac{dI}{ds} = \mu \frac{dI}{dz} = -\kappa_\nu I_\nu + \eta_\nu$$

$$\mu \frac{dI}{-\kappa_\nu dz} = + I_\nu - \frac{\eta_\nu}{\kappa_\nu}$$

$$\mu \frac{dI}{d\tau} = I - S$$

$$\mu \frac{dI}{d\tau} - I = -S$$

$$\frac{dI}{d\tau} - \frac{1}{\mu} I = -\frac{1}{\mu} S$$

$$\frac{dI}{d\tau} e^{-\tau/\mu} - \frac{I}{\mu} e^{-\tau/\mu} = -\frac{1}{\mu} S e^{-\tau/\mu}$$

$$\frac{d(I e^{-\tau/\mu})}{d\tau} = \frac{-1}{\mu} S e^{-\tau/\mu}$$

$$\int_{\tau_1}^{\tau_2} \frac{d(I e^{-\tau/\mu})}{d\tau} = \int_{\tau_1}^{\tau_2} \frac{-1}{\mu} S e^{-\tau/\mu}$$

$$I e^{-\tau/\mu} \Big|_{\tau_1}^{\tau_2} = \int_{\tau_1}^{\tau_2} \frac{-S}{\mu e^{\tau/\mu}}$$

$$I(\tau,\mu) e^{-\tau_2/\mu} - I(\tau,\mu) e^{-\tau_1/\mu} = \int_{\tau_1}^{\tau_2} \frac{-S}{\mu} e^{-\tau/\mu}$$

$$I(\tau_1,\mu) e^{-\tau_1/\mu} = I(\tau_2,\mu) e^{-\tau_2/\mu} + \int_{\tau_1}^{\tau_2} \frac{S}{\mu} e^{-\tau/\mu}$$

$$I(\tau_1,\mu) = I(\tau_2,\mu) \exp(-(\tau_2-\tau_1)/\mu) + \frac{1}{\mu} \int_{\tau_1}^{\tau_2} S \exp[-(t-\tau_1)/\mu \, dt$$

b) * At an arbitrary point, incoming rays ($\mu < 0$); outgoing rays ($\mu > 0$)

⇒ Outgoing rays $\tau_1 = \tau$, $\tau_2 = \infty$

$$I(\tau,\mu) = I \exp[-\cancel{(\infty-\tau)}^{\,0}/\mu] + \frac{1}{\mu} \int_{\tau}^{\infty} S \exp[-(t-\tau)/\mu] dt$$

$$= \frac{1}{\mu} \int_{\tau}^{\infty} S \exp[-(t-\tau)/\mu] d\tau$$

#6 (cont.)

b) $\Rightarrow$ for incomming rays $\tau_1 = 0 \quad \tau_2 = \tau$

$$I(\tau,\mu) = \int_0^{\tau} S(t)\exp[-(t-\tau)/-\mu]\frac{dt}{-\mu} + \cancel{I_\nu(\infty,-\mu)e^{-(\infty-0)/\mu}} \to 0$$

c) $$J_\nu(\tau) = \frac{1}{2}\int_{-1}^{1} I_\nu(\tau,\mu)\,d\mu$$

$$= \frac{1}{2}\left[\int_{-1}^{0} I_\nu(\tau,\mu)d\mu + \int_0^1 I_\nu d\mu\right]$$

$$= \frac{1}{2}\left[\int_{-1}^{0}\int_0^{\tau} S_\nu(t)e^{-(\tau-t)/(-\mu)}dt\cdot\frac{1}{-\mu}d\mu + \int_0^1\int_{\tau}^{\infty} S_\nu e^{-(t-\tau)/\mu}\frac{dt}{\mu}d\mu\right]$$

* assuming an isotropic source function

$$= \frac{1}{2}\left[\int_{\tau}^{\infty} dt\, S_\nu \int_0^1 \exp[-(t-\tau)/\mu]\frac{1}{\mu}d\mu + \int_0^{\tau} dt\, S_\nu \int_{-1}^{0}\exp[-(\tau-t)/+\mu]\frac{d\mu}{+\mu}\right]$$

* let $x = t-\tau \qquad x' = \tau - t$

$y = \frac{1}{\mu} \to dy = \frac{-1}{\mu^2}d\mu \to y^{-2}dy = d\mu \qquad y = \frac{-1}{\mu}$

d) $$= \frac{1}{2}\left[\int_{\tau}^{\infty} S_\nu dt \int_{\infty}^{1} -\exp[-xy]\frac{dy}{y} + \int_0^{\tau} dt\, S_\nu \int_{+1}^{+\infty}\exp[-xy]\frac{dy}{y}\right]$$

$$= \frac{1}{2}\left[\int_{\tau}^{\infty} dt\, S_\nu \int_1^{\infty}\exp[-xy]\frac{dy}{y} + \int_0^{\tau} dt\, S_\nu \int_1^{\infty}\exp[-xy]\frac{dy}{y}\right]$$

$$= \frac{1}{2}\left[\int_{\tau}^{\infty} dt\, S_\nu\, E_1(t-\tau) + \int_0^{\tau} dt\, S_\nu\, E_1(\tau-t)\right]$$

$$= \frac{1}{2}\int_0^{\tau_{max}} dt\, S_\nu\, E_1(|t-\tau|)$$

e) Known as Λ-operator