driven generalized quantum Rayleigh-van der Pol oscillators

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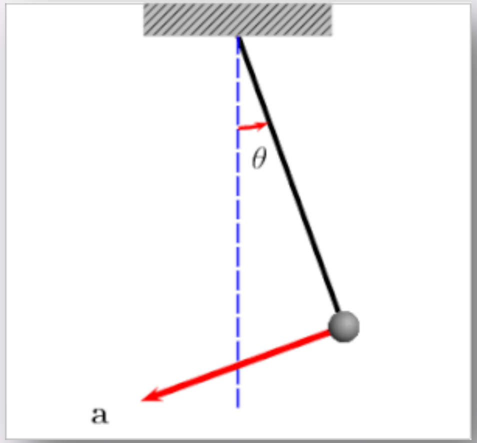
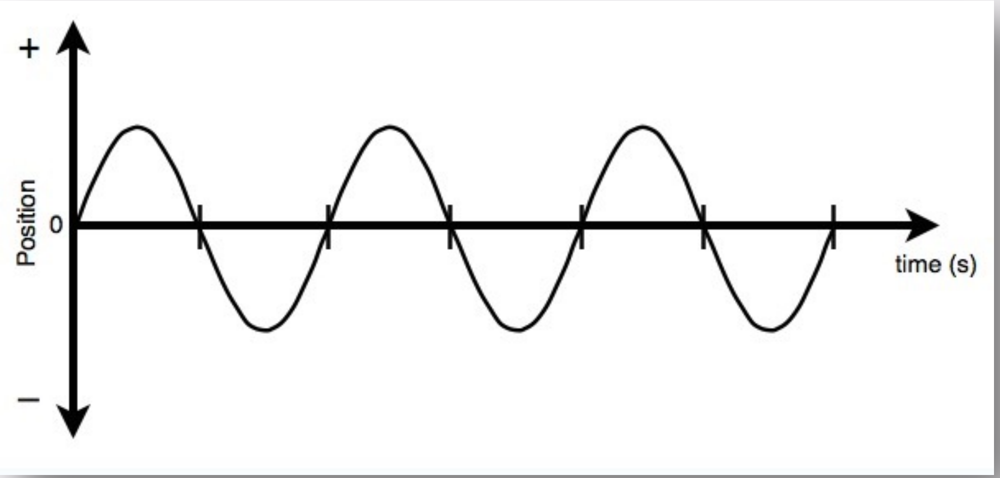
😇classical oscillators😇

- **simple harmonic motion:** repetitive back-and-forth movement through an equilibrium position
  - i.e. springs, pendulums



- classical equation-of–motion:

$$\ddot{x} = -\omega^2 x$$



position, velocity, and acceleration oscillate with time

🤔"self-sustained?"🤔

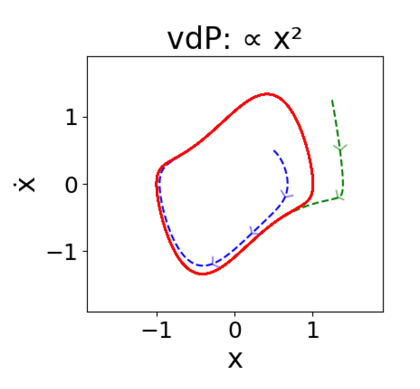
- **self-sustained oscillations** occur in systems where a periodic response is generated without the need for an external force to the system

$$\ddot{x} = -\omega^2 x + \dot{x}\left[(\gamma_1^+ - \gamma_1^-) - \gamma_{2,vdP}x^2 - \gamma_{2,ray}\dot{x}^2\right]$$

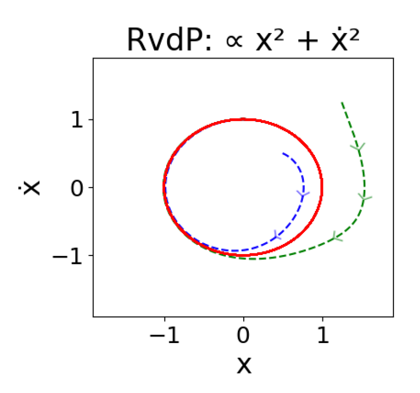
linear energy gain (if positive)   non-linear energy loss

- visualize in **phase space** to view **limit cycles**

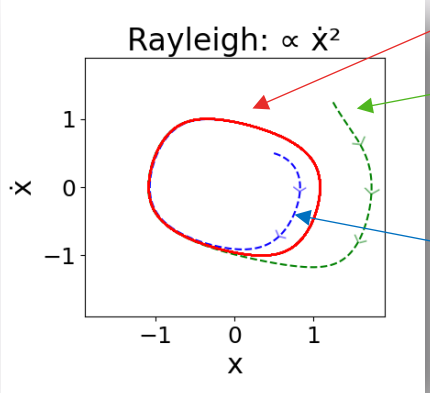
van der Pol damping



Rayleigh-van der Pol damping



Rayleigh damping



limit cycle

outside trajectory:
energy loss
initially dominates

inside trajectory:
energy gain
initially dominates

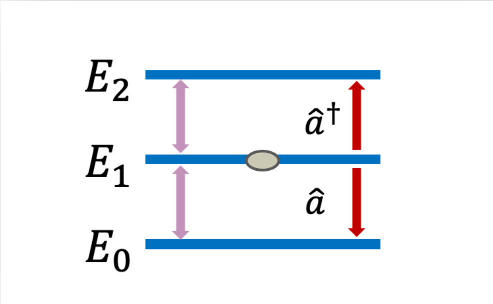
quantum oscillators

- **energy discretization** becomes apparent in the quantum regime
  - we must consider a different framework to describe self-sustained oscillations in the quantum regime, <a†a> < 1
- the quantum master equation (Ben Arosh et al., PRR 3, 013130 (2021)) contains linear and non-linear damping terms as well

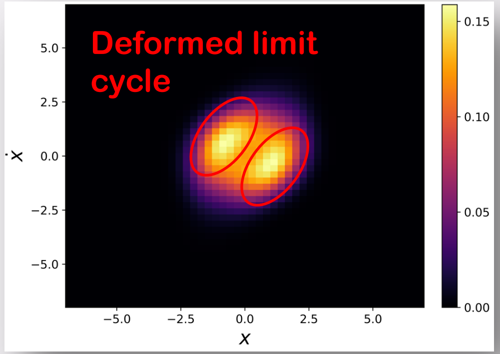
$$x(t) \rightarrow \hat{\rho}(t)$$

$$\dot{\hat{\rho}} = -\imath[\hat{H}, \hat{\rho}] + \gamma_1^{+} D[\hat{a}^{\dagger}]\hat{\rho} + \gamma_1^{-} D[\hat{a}]\hat{\rho} + \alpha D[\hat{a}^2]\hat{\rho} + \beta D[\hat{x}\hat{a}](\hat{\rho}) + \delta D[\hat{p}\hat{a}](\hat{\rho})$$

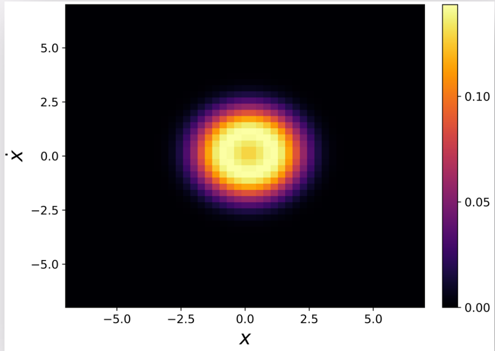
$$\ddot{x} = -\omega^2 x + \dot{x}\left[(\gamma_1^{+} - \gamma_1^{-}) - \gamma_{2,vdP} x^2 - \gamma_{2,ray} \dot{x}^2\right]$$



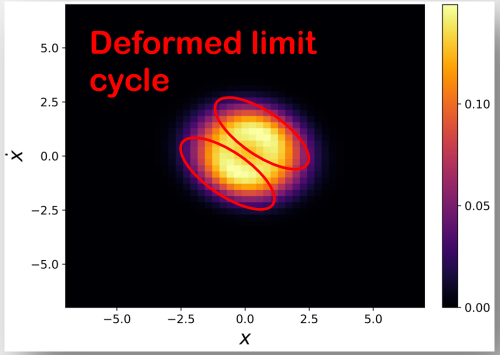
vdP



R-vdP

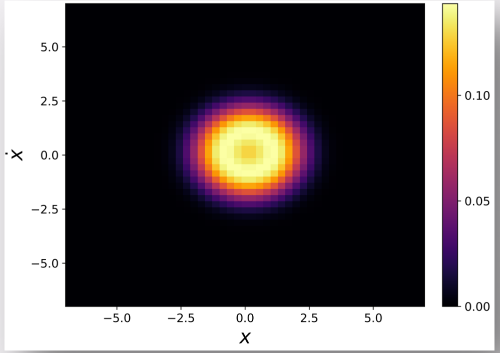


ray

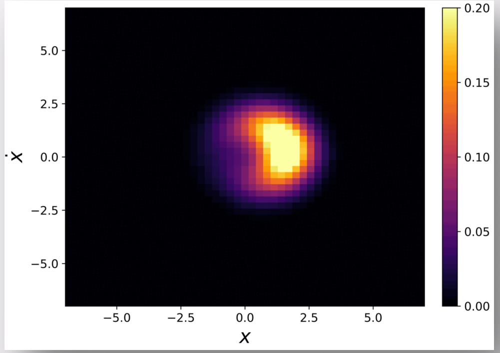


our work

no synchronization



some synchronization



- in the classical regime:
  - limit cycle is key ingredient for **synchronization**
  - classical synchronization allows us to accomplish certain tasks (like efficiently running the power grid)
- we seek to:
  1. understand oscillator dynamics in both the laboratory frame and a frame rotating with the oscillator
  2. study the response of systems with an external drive where the damping terms are not equal to each other

     i.e. investigate synchronization, susceptibility
  3. clarify the generality in the deviation in response from the quantum and classical systems

tools

- C code for the trajectories, using fourth-order Runge-Kutta methods to solve the differential equations
- **python**, gnuplot, and Mathematica for analyzing and visualizing the results

questions?