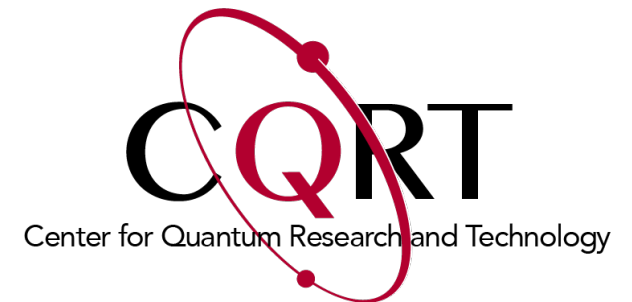


# Driven generalized quantum Rayleigh-van der Pol oscillators

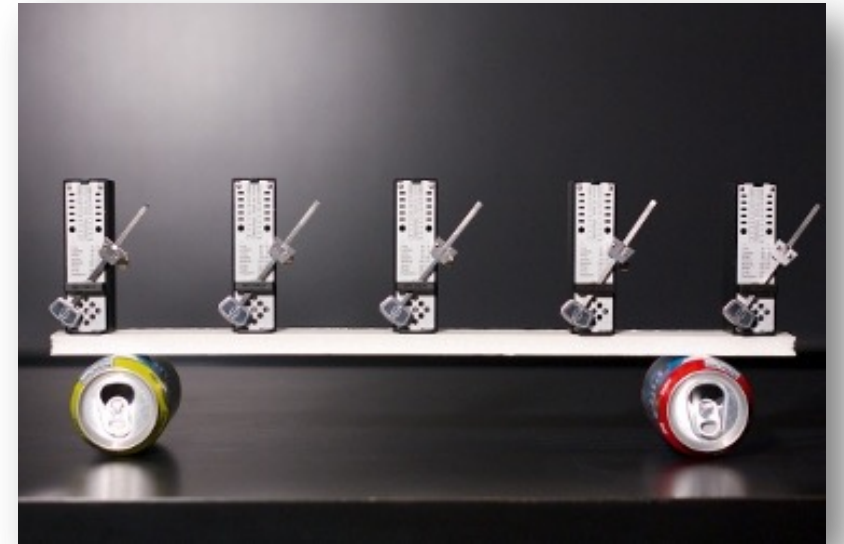


A. J. Sudler  
Blume Research Group

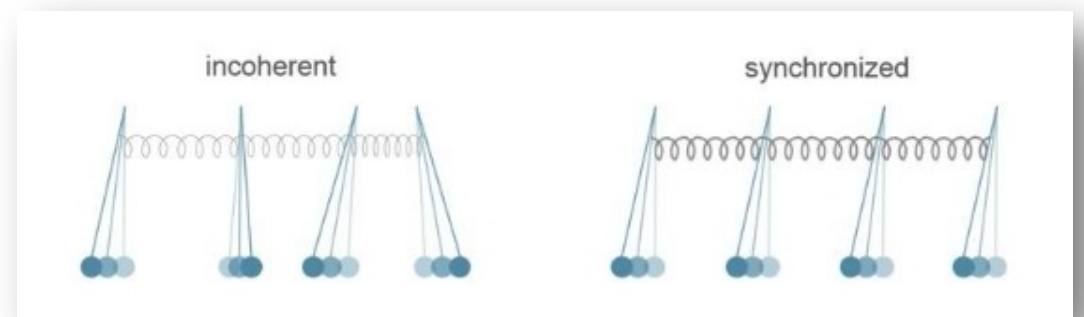


# Synchronization

- **phase locking** and **entrainment** of self-sustained oscillator systems
  - i.e. pacemakers, blinking fireflies
- **self-sustained**: supporting asymptotic finite amplitude oscillations
- well understood classically



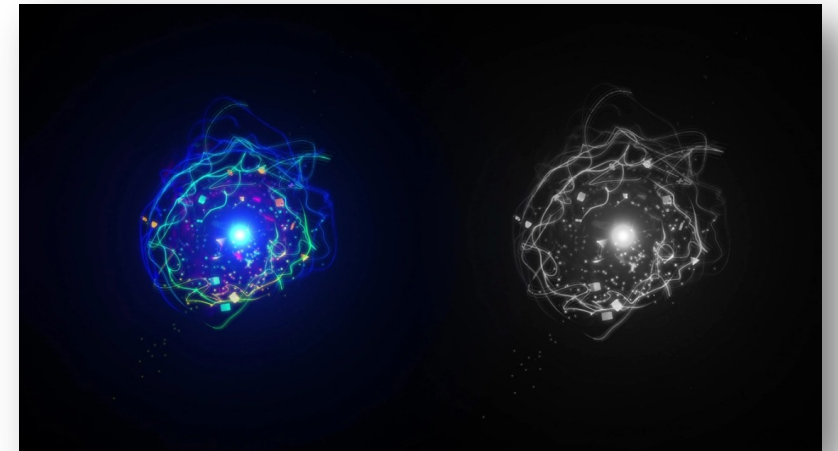
[1] Harvard Natural Sciences Lecture Demonstrations, <https://sciencedemonstrations.fas.harvard.edu/presentations/synchronization-metronomes>



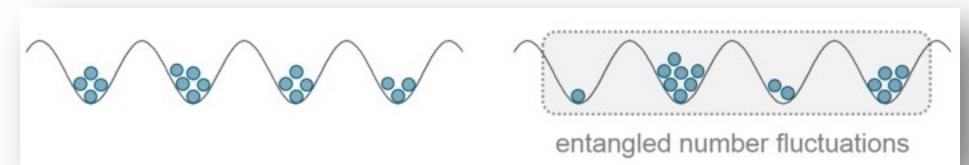
[2] Max Planck Society, "Classical synchronization indicates persistent entanglement in isolated quantum systems". <https://phys.org/news/2017-05-classical-synchronization-persistent-entanglement-isolated.html>

# Quantum Synchronization Motivation

- less understood in the quantum regime
- exciting area for quantum technology
  - biology, telecommunications, etc.
- big questions:
  1. differences in single-oscillator systems?
  2. what is coupled with what?
  3. how does the response change?



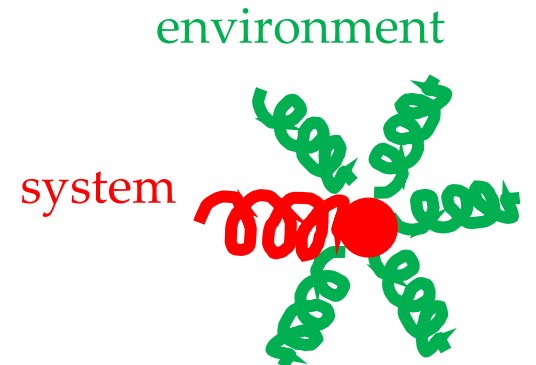
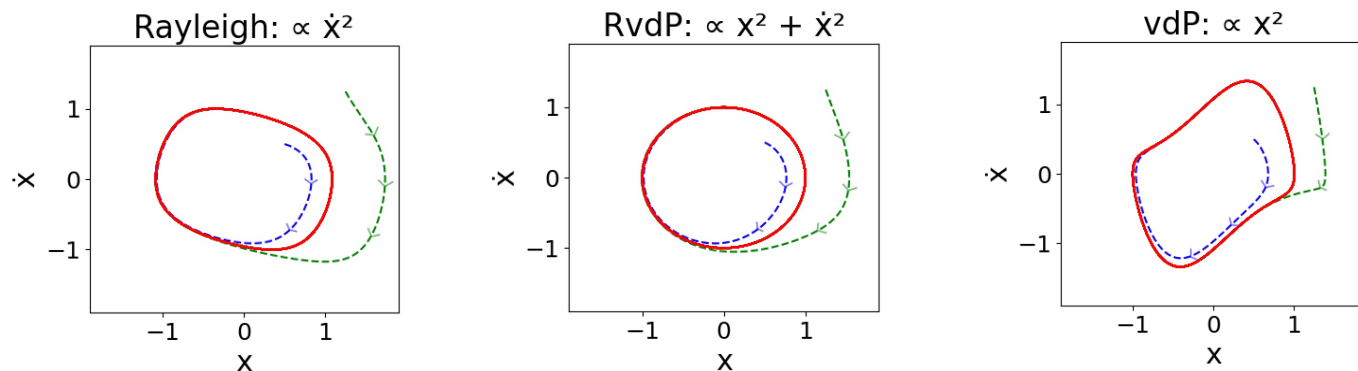
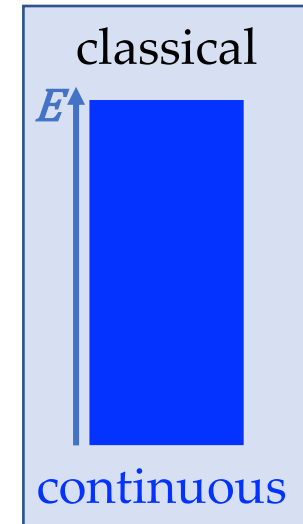
[3] Frank, Adam. "What is quantum mechanics trying to tell us?", 2 June 2022. <https://bigthink.com/13-8/quantum-mechanics-interpretation/>



[2] Max Planck Society. "Classical synchronization indicates persistent entanglement in isolated quantum systems". <https://phys.org/news/2017-05-classical-synchronization-persistent-entanglement-isolated.html>

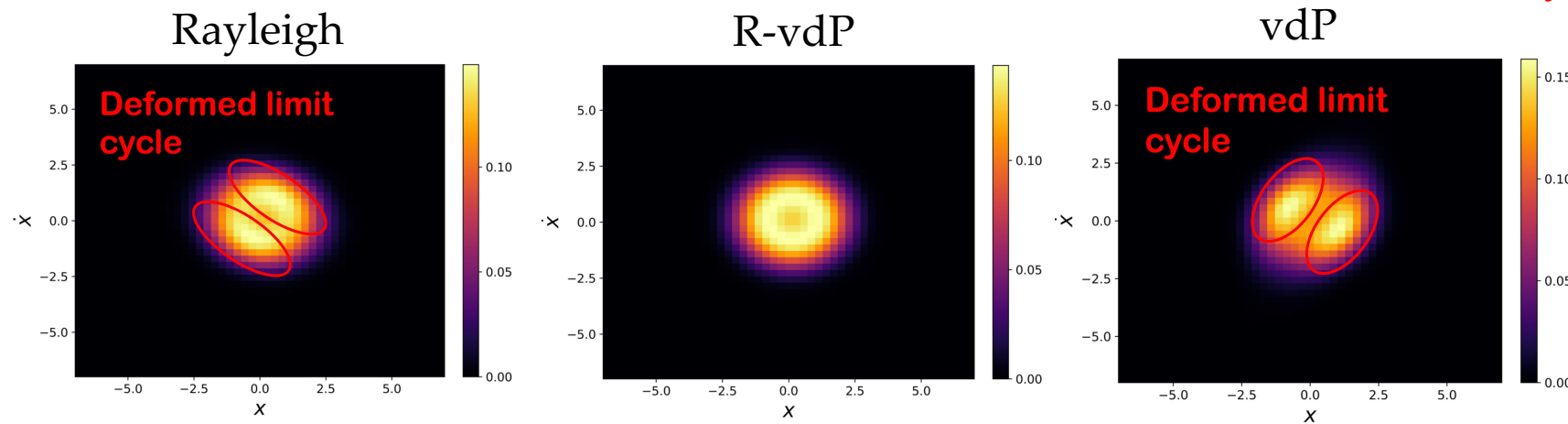
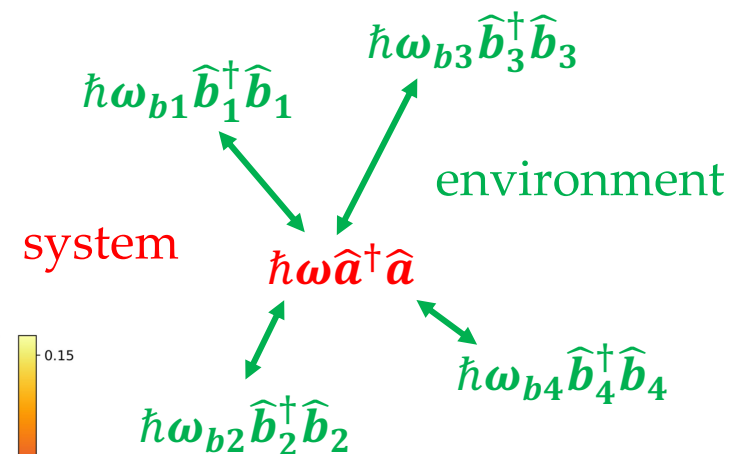
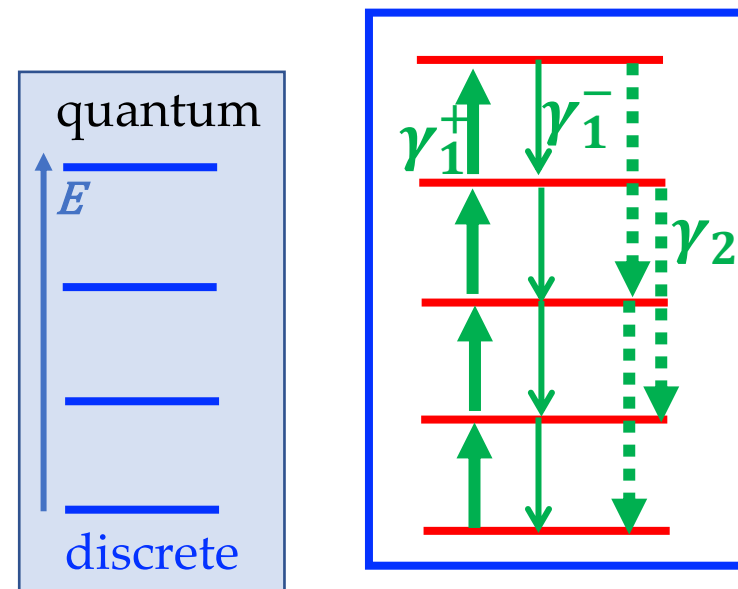
# Background: generalized Rayleigh-van der Pol (R-vdP) oscillators

- three gain/dissipative processes:
  1. linear energy gain, proportional to  $\dot{x}$
  2. **van der Pol (vdP) damping**: non-linear energy loss proportional to  $x^2$
  3. **Rayleigh (R) damping**: non-linear energy loss proportional to  $\dot{x}^2$
- self-sustained
  - **limit cycles**

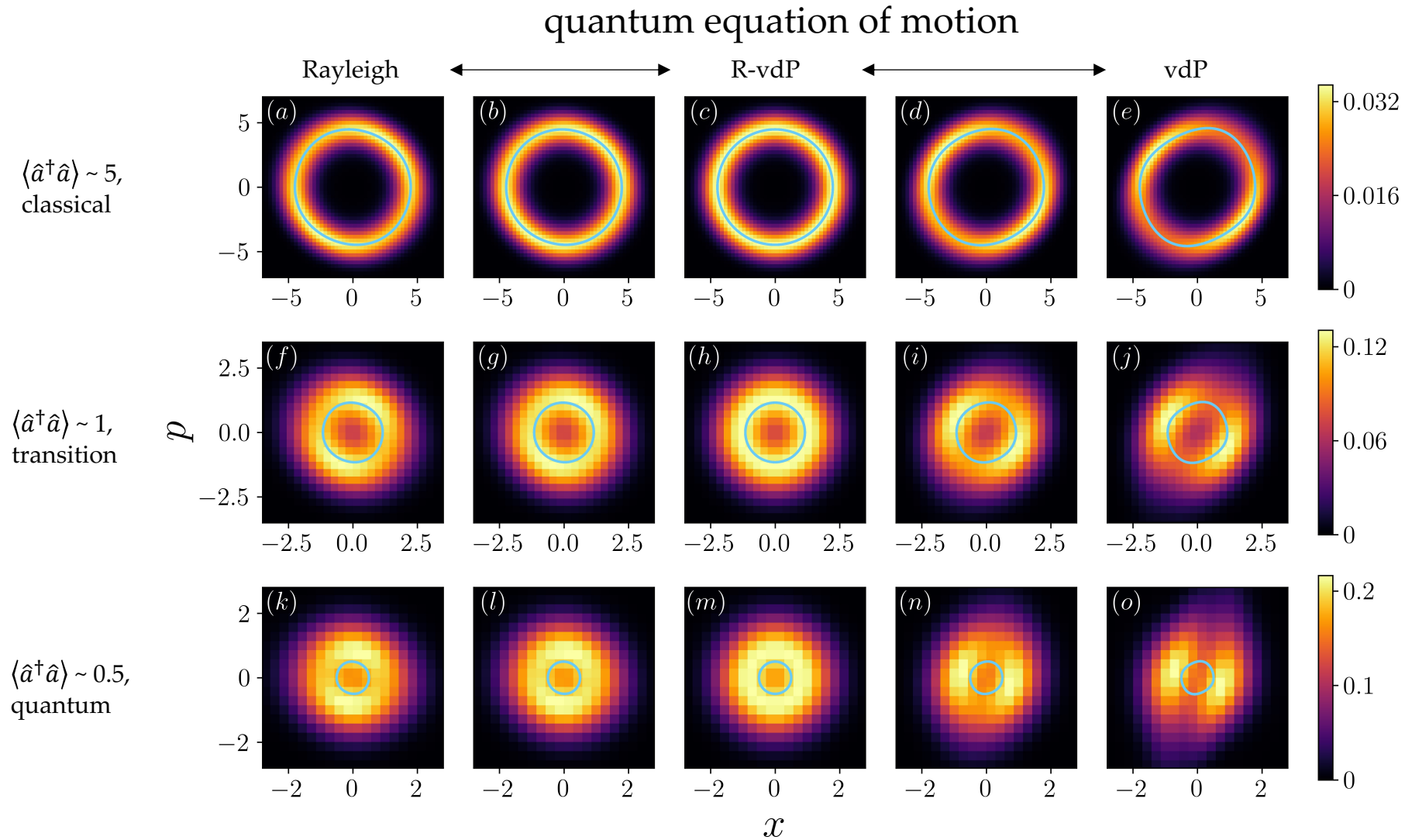


# Quantum Description

- quantum gain/dissipative processes:
  1. one-excitation gain ( $\gamma_1^+$ )
  2. one-excitation loss ( $\gamma_1^-$ )
  3. two-excitation loss ( $\gamma_2$ )
- $\langle \hat{a}^\dagger \hat{a} \rangle$  describes “quantumness”
- Wigner functions as a phase-space analog

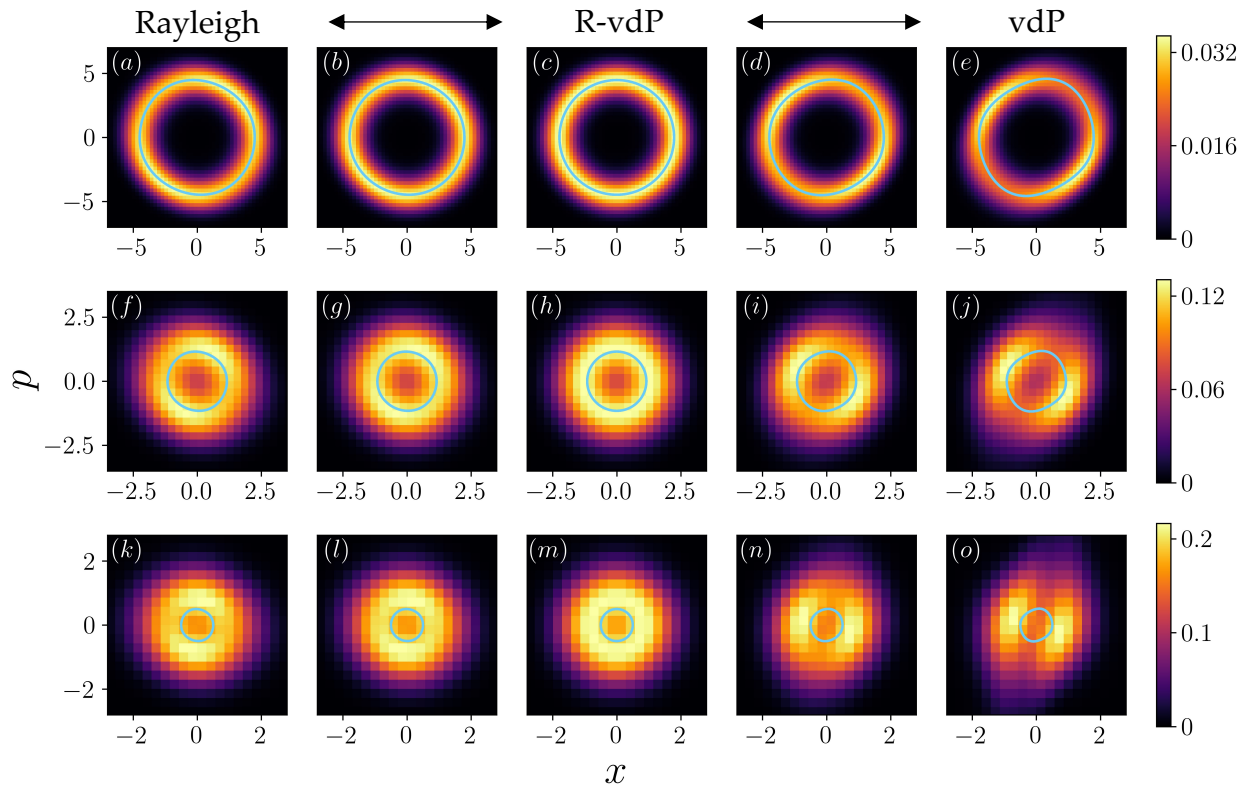


# Quantum Limit Cycles

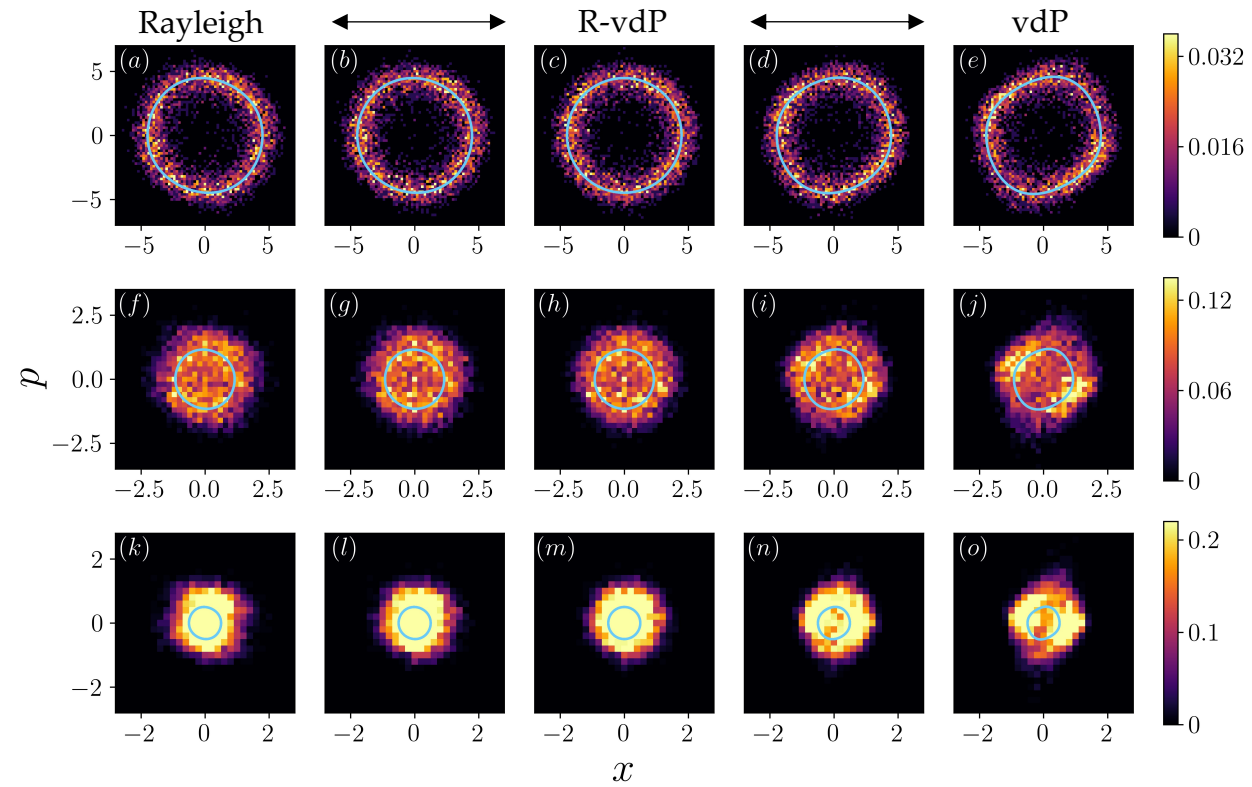


# Quantum vs. Classical Comparison

quantum equation of motion



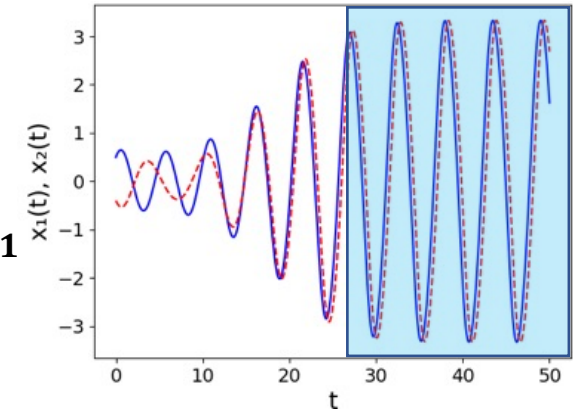
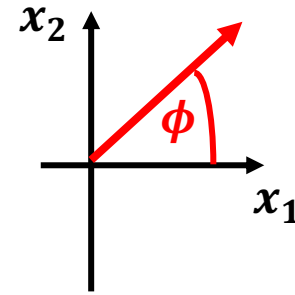
classical equation of motion + white-noise temperature



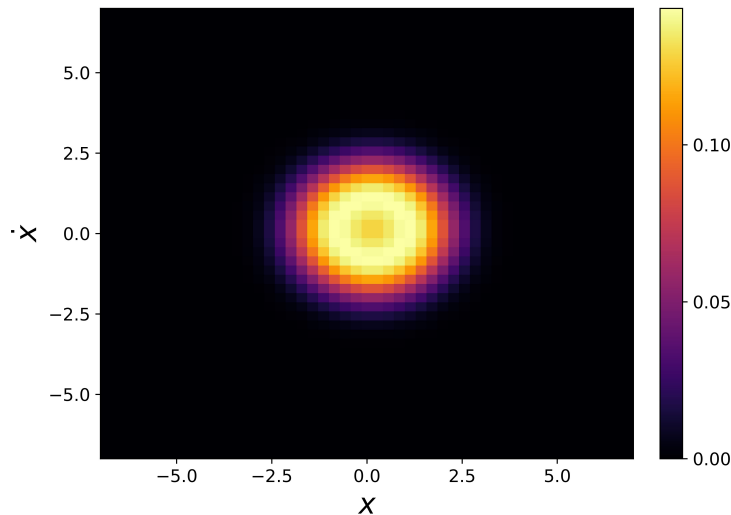


# Synchronizing Quantum Systems

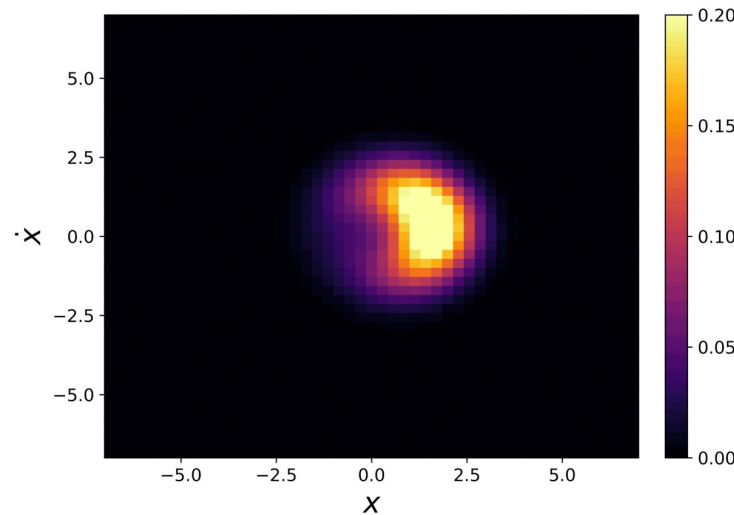
- systems synchronized by **external drive** ( $\omega_D, \Omega$ )
- angular preference apparent in phase space



no drive  $\rightarrow$  sync. = 0



drive  $\rightarrow$  finite sync.  $< 1$

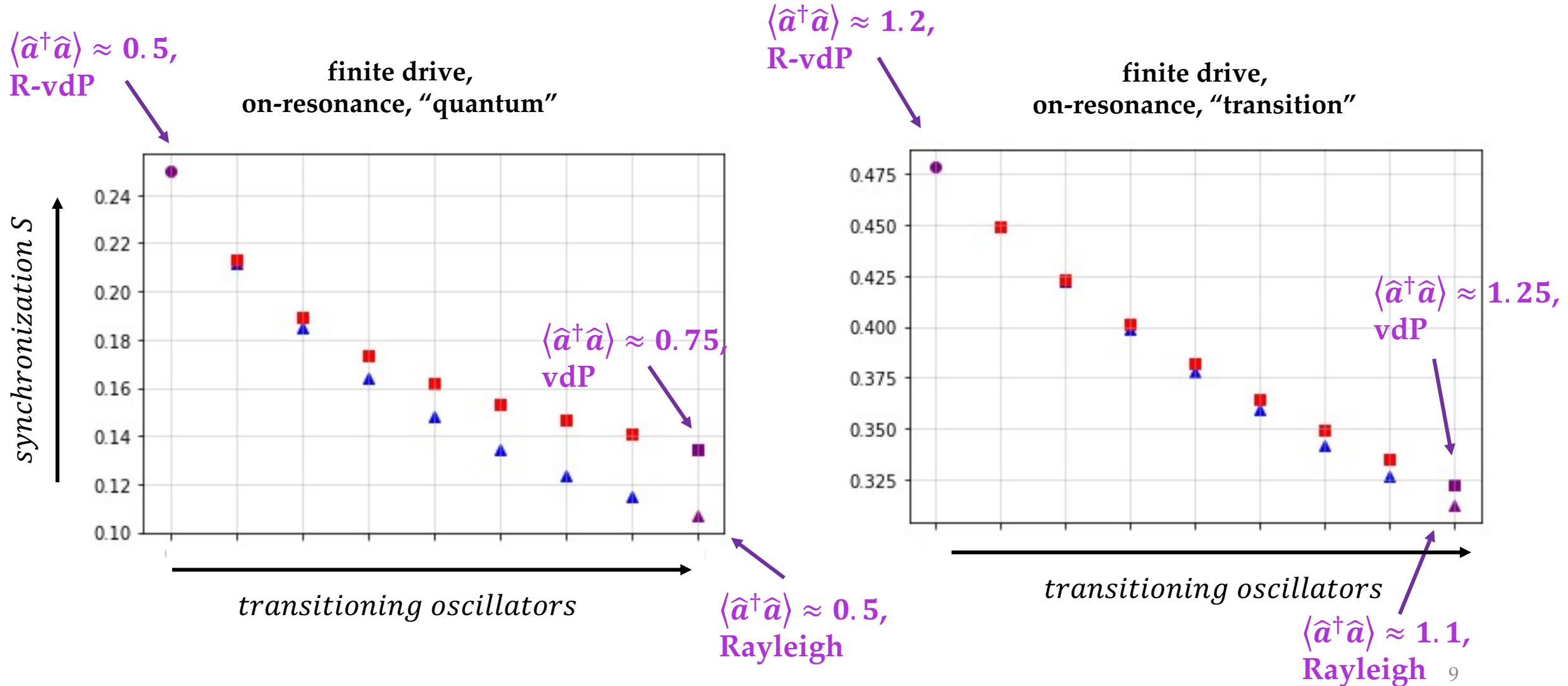


quantified by **synchronization (S)**

- defined by the mean resultant length calculated from  $\hat{\phi}$
- $0 < S < 1$

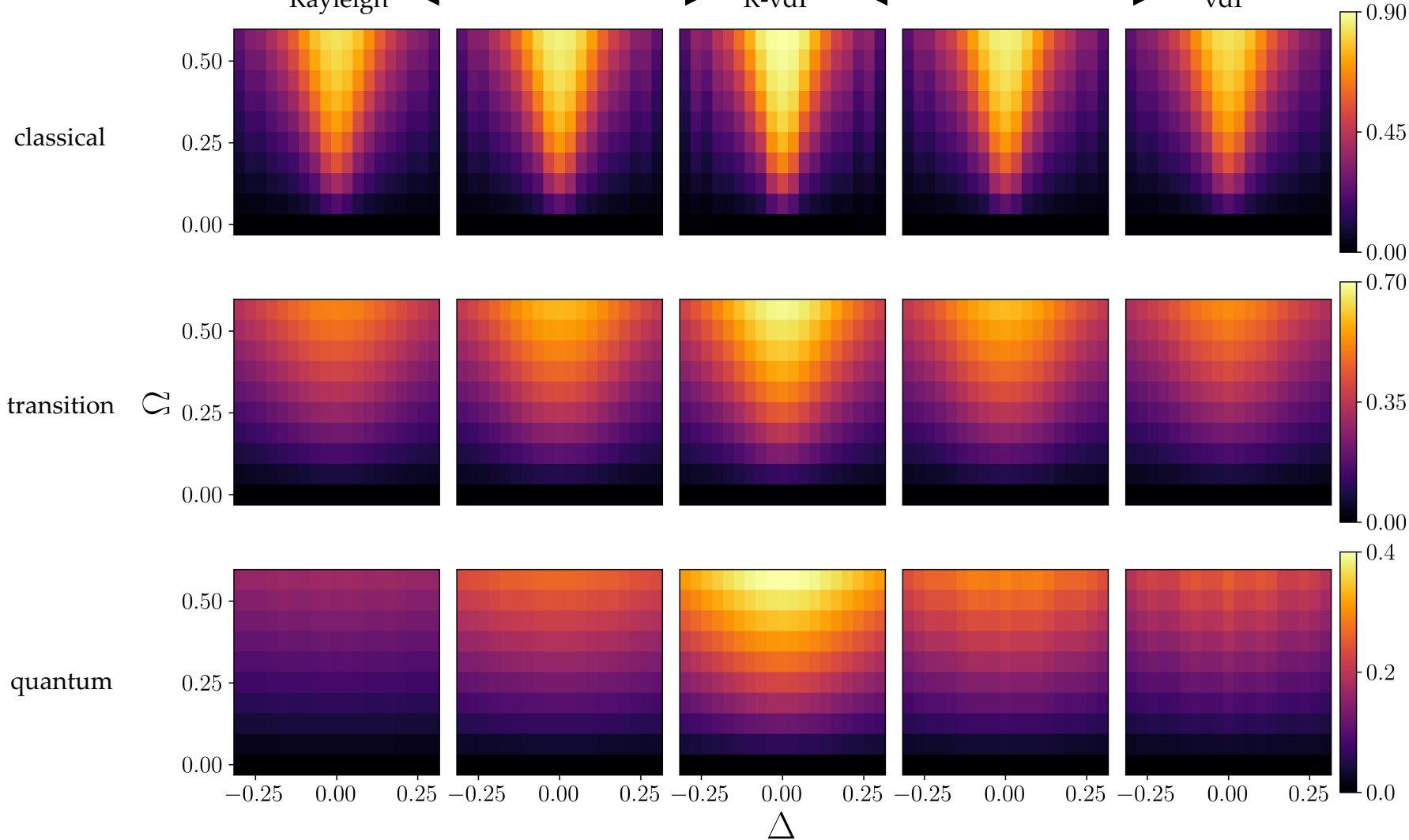


# Synchronization and Drive Strength



# Synchronization Arnold Tongues

Rayleigh ← R-vdP → vdP



$\Omega$  = drive strength

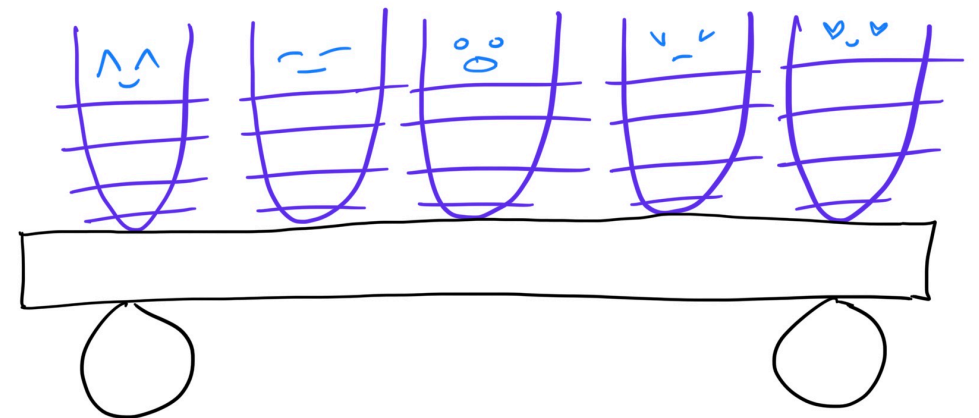
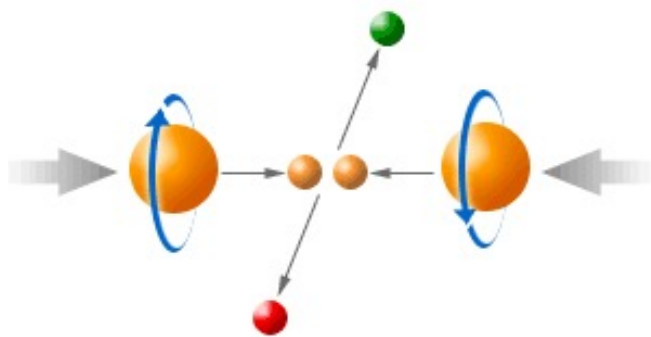
$\Delta = \omega_D - \omega_0$  = detuning

For a larger detuning, we need a higher drive strength to achieve the same synchronization.

The classical regime experiences the highest synchronization for no detuning.

# Future Direction with Synchronization

- Continue to monitor results as the higher-energy regimes approach convergence
  - Rotations and reference frames
- Couple several quantum oscillators to one another to explore synchronization effects
- Spin vs. Spatial degrees of freedom



# Acknowledgements and References

- Thank you to Dr. Blume!!!
  - Thanks to all the graduate students in the group who have helped me—Jugal, Dave, and Kevin.
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[1] Harvard Natural Sciences Lecture Demonstrations, <https://sciencedemonstrations.fas.harvard.edu/presentations/synchronization-metronomes>

[2] Max Planck Society, “Classical synchronization indicates persistent entanglement in isolated quantum systems”. <https://phys.org/news/2017-05-classical-synchronization-persistent-entanglement-isolated.html>

[3] Frank, Adam. “What is quantum mechanics trying to tell us?”, 2 June 2022. <https://bigthink.com/13-8/quantum-mechanics-interpretation/>

[4] Brookhaven National Laboratory. “Spin Physics”. <https://www.bnl.gov/rhic/spin.php>

## Questions?