Rings and Holes and Knots, Oh My!

Topology in 1D quantum systems

Colin Riggert, University of Oklahoma

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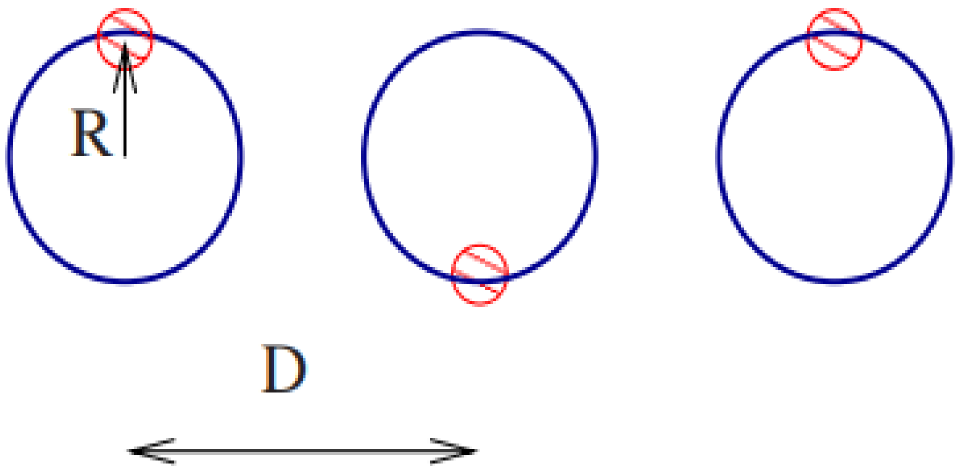
Advisor: Prof. Kieran Mullen

Outline of festivities

- I. Introduction to the problem
- II. Preliminary quantum mech.
- III. Two Ring Cases
- IV. Three Ring Cases
- V. Rings in Magnetic Fields
- VI. Future Directions

Why are we here?

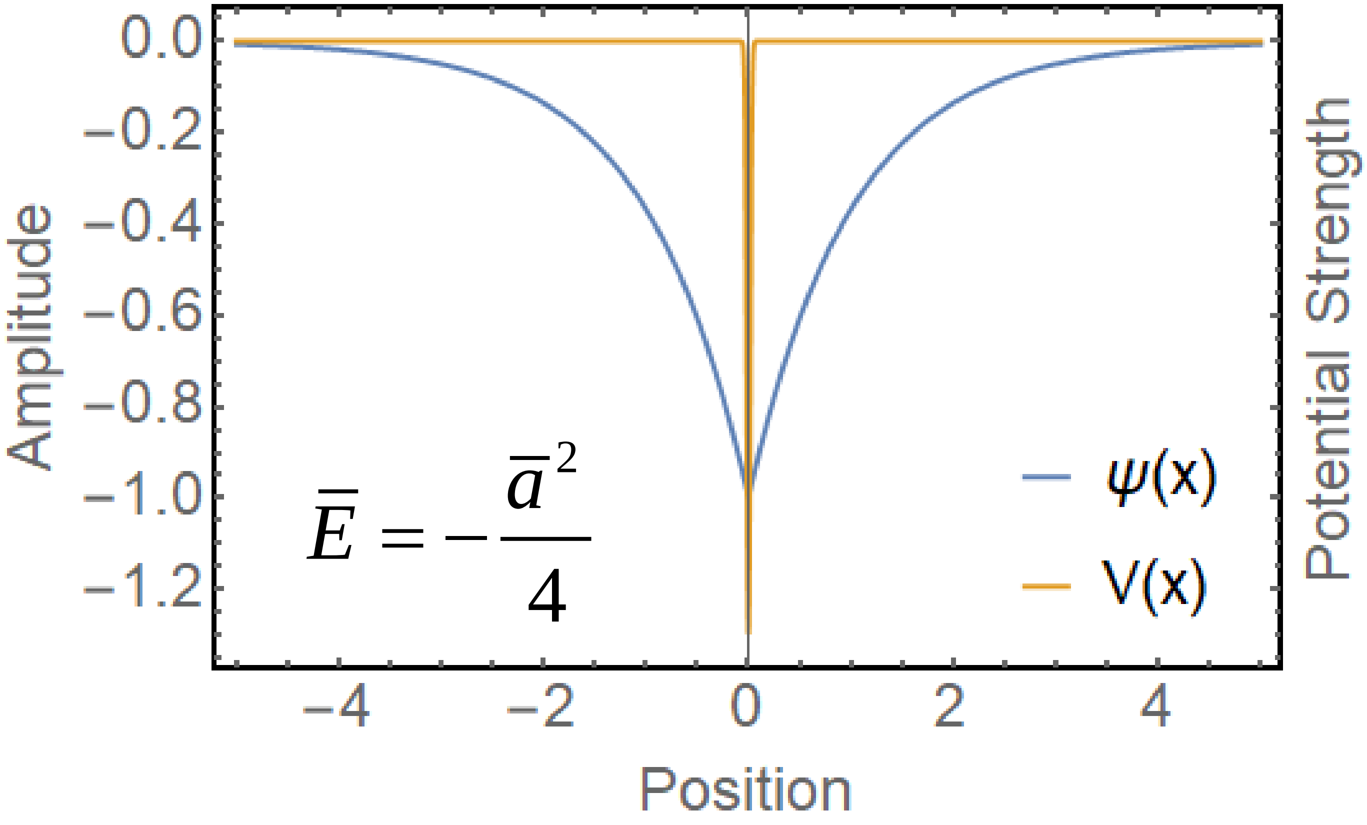
- What is topology?
  - Connectedness
  - Boundary conditions
  - Holes
- Why do we care?
  - 1D quantum rings: topological polarization
- Where do we go from here?
  - Tunneling



Preliminaries – δ-function potentials

$$-\frac{\partial^2}{\partial x^2}\psi(x) - \bar{a}\delta(x)\psi(x) = \bar{E}\psi(x)$$

- Intuition: Attractive δ-function potential in 1D creates a bound state
  - Lower energy than propagating states
  - Exponential localization of wavefunction



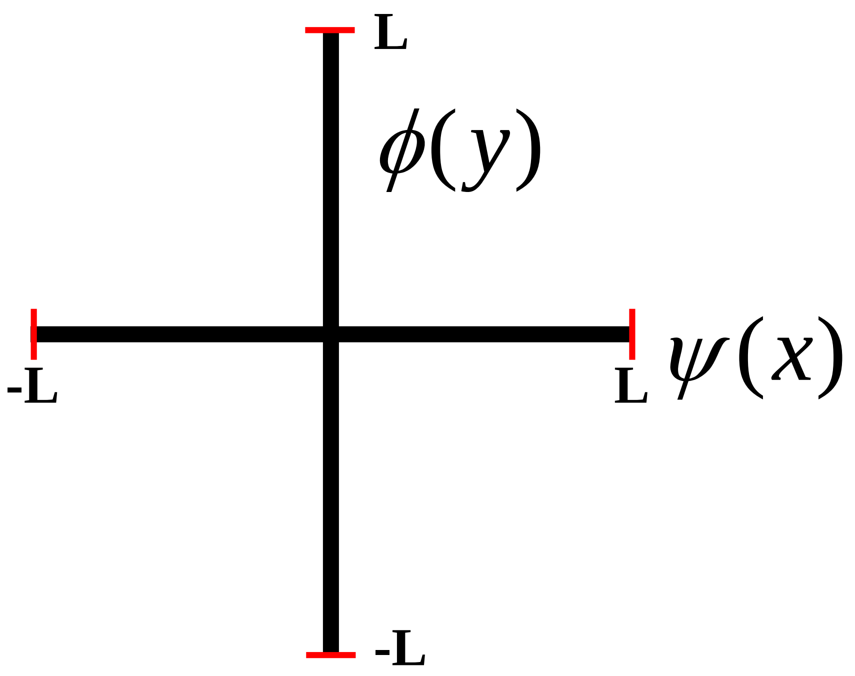
Preliminaries – quantum wires

- Crossed quantum wires with point of contact tunneling

$$-\frac{\partial^2}{\partial x^2}\psi(x)+\bar{a}\delta(x)\phi(0)=\bar{E}\psi(x)$$

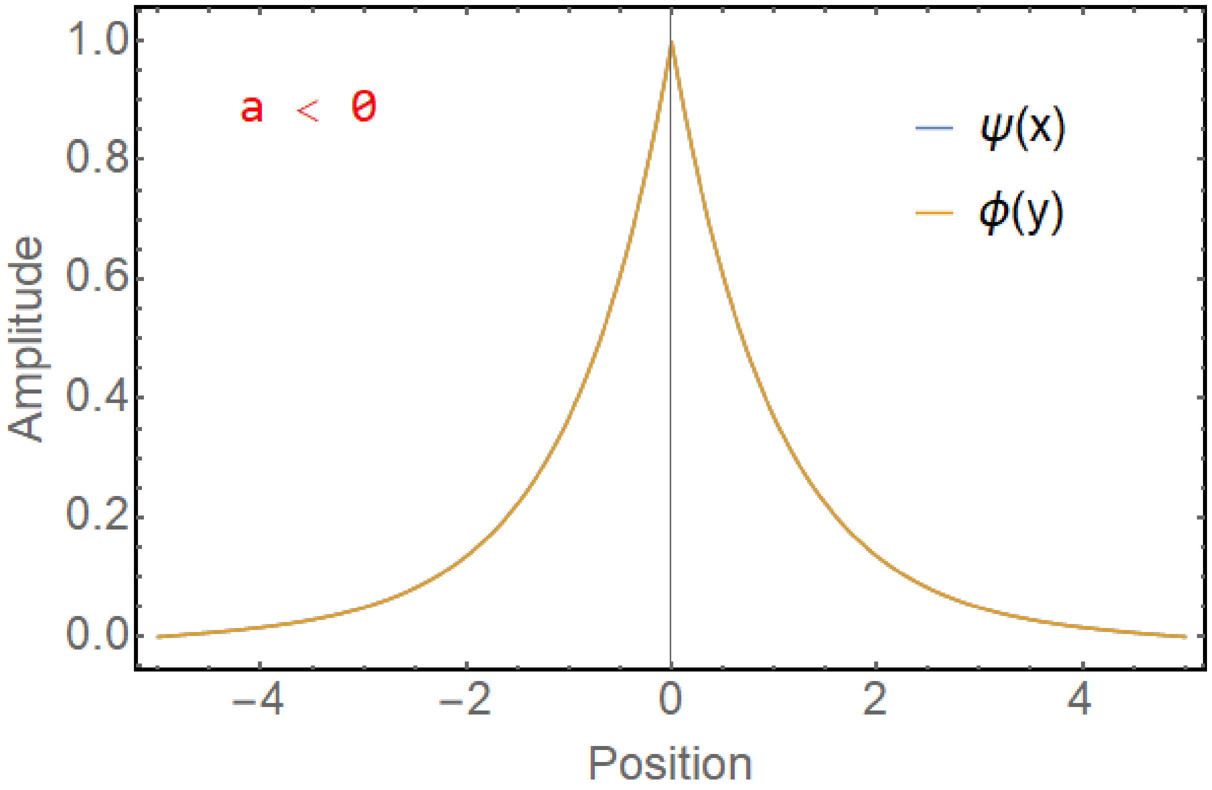
$$-\frac{\partial^2}{\partial y^2}\phi(y)+\bar{a}\delta(y)\psi(0)=\bar{E}\phi(y)$$

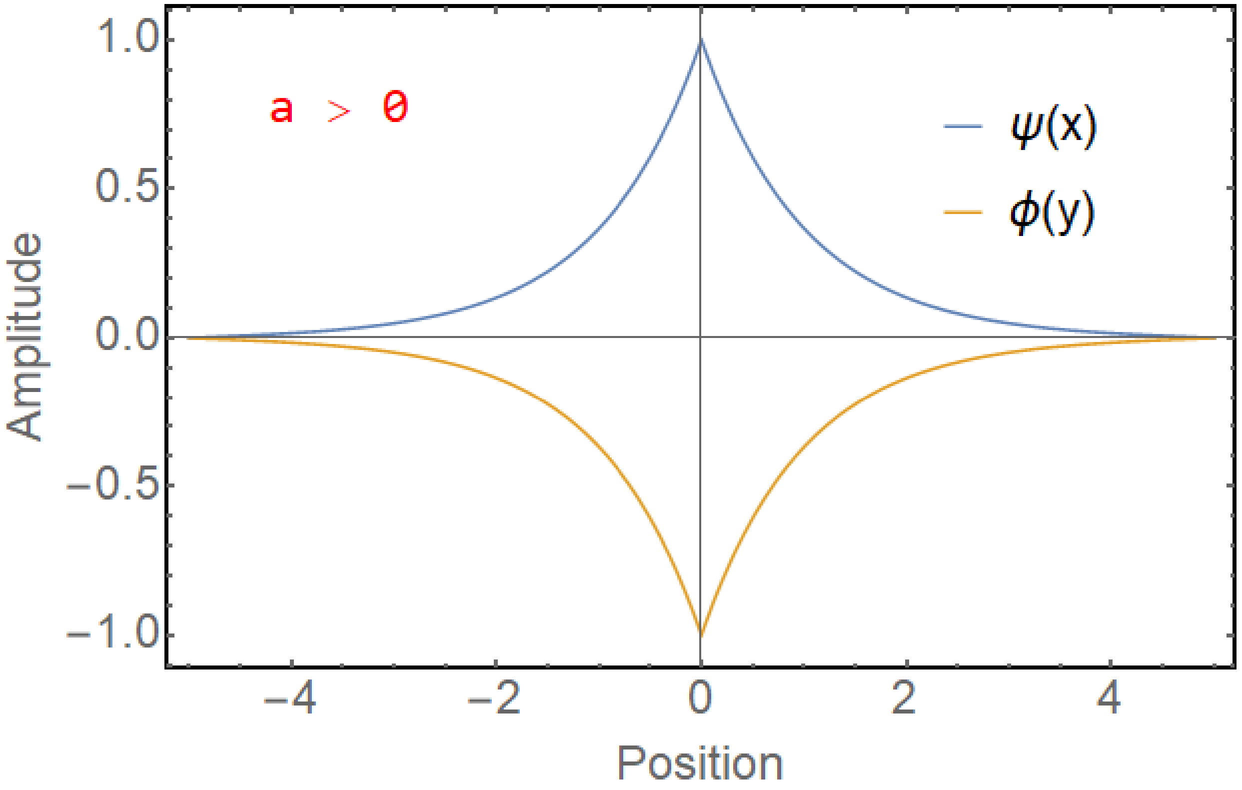
- Zero Boundary Conditions (ZBC) at ± *L*



Preliminaries – ZBC Results

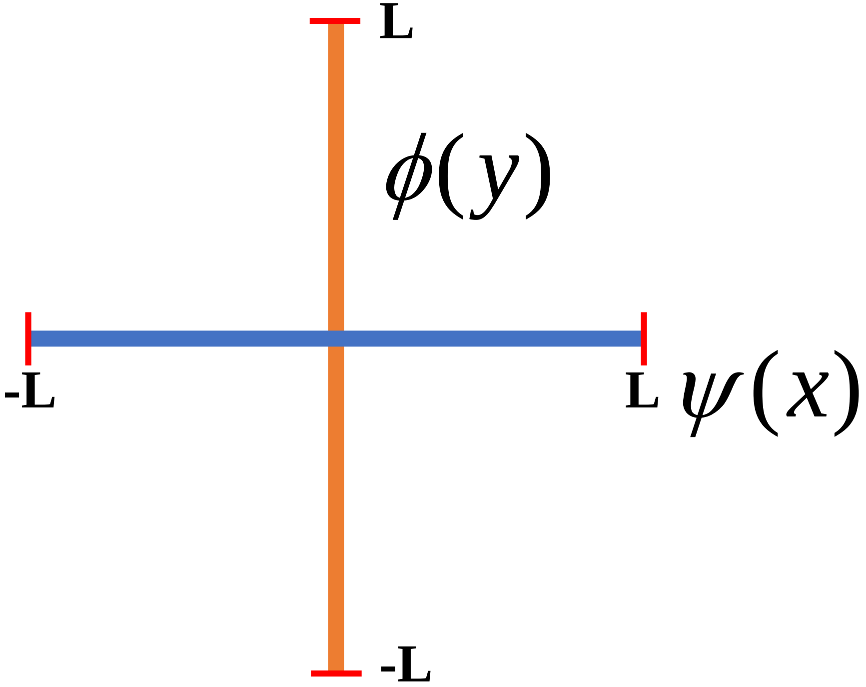
- Exponentially localized bound state!
- Positive coupling constant causes a phase shift of π between the two wires

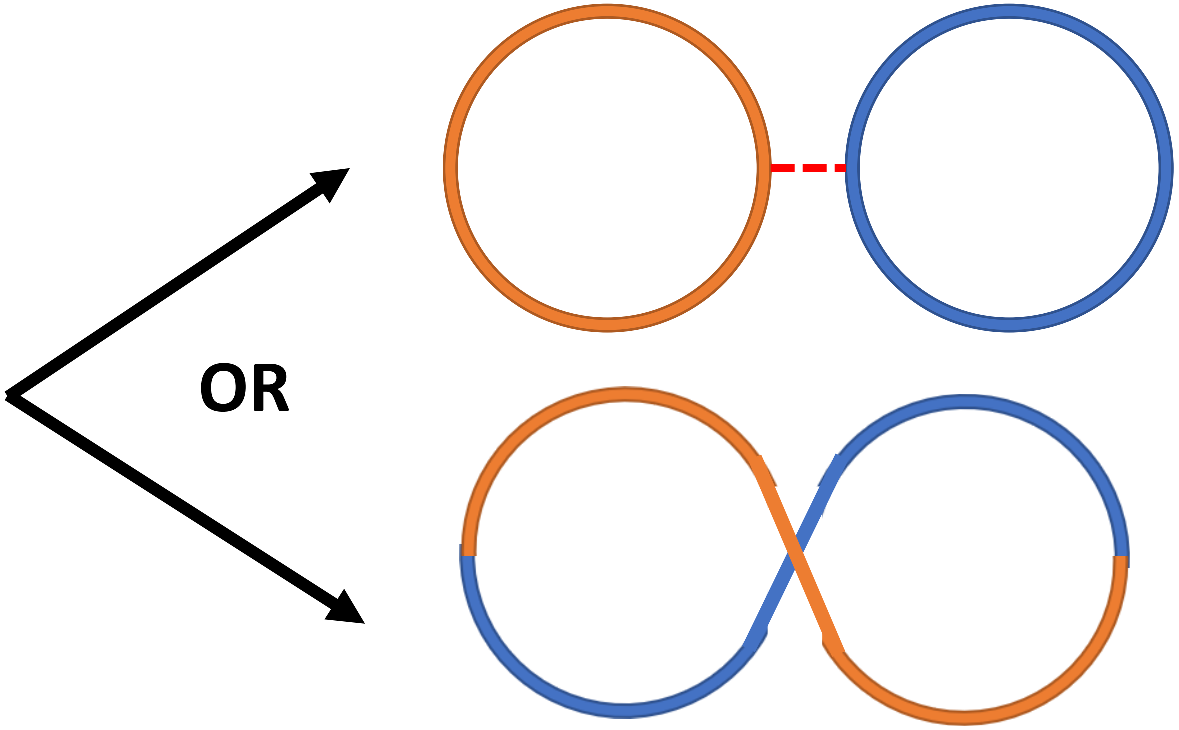




Setting up the rings

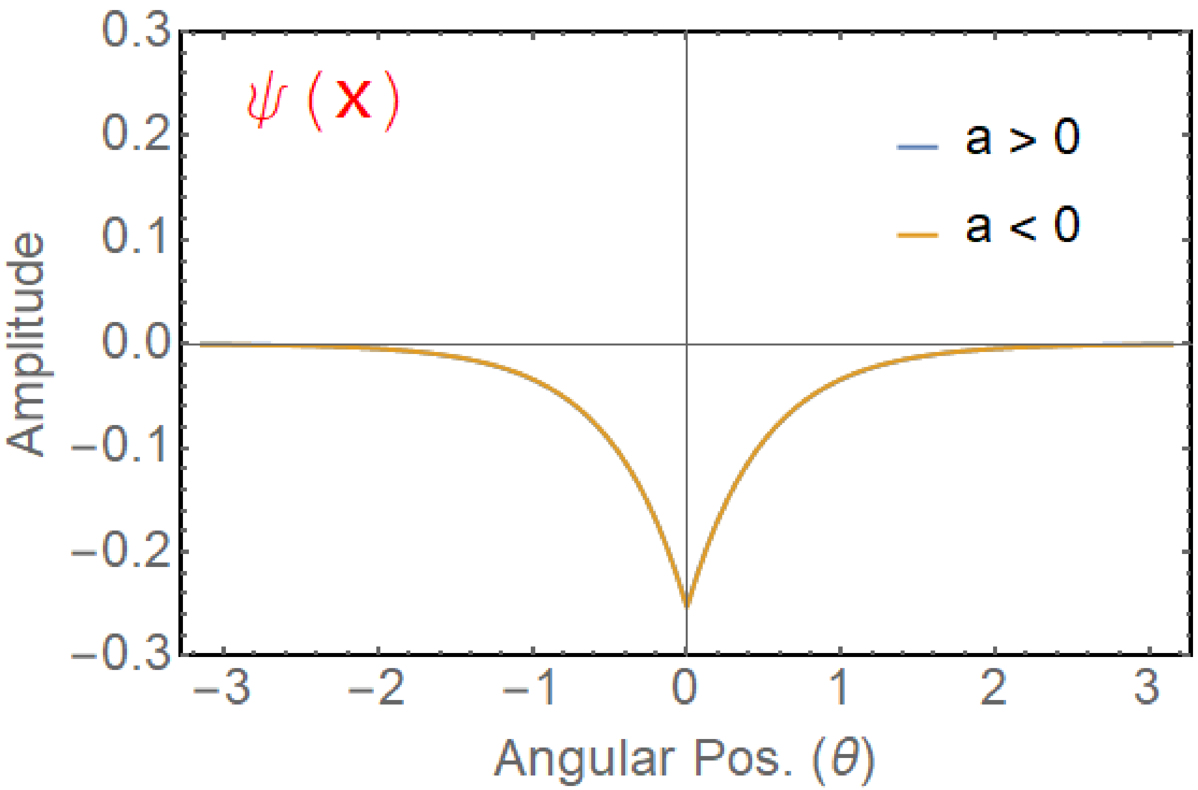
- How do we make the wires into rings? We have two choices for connecting the edges

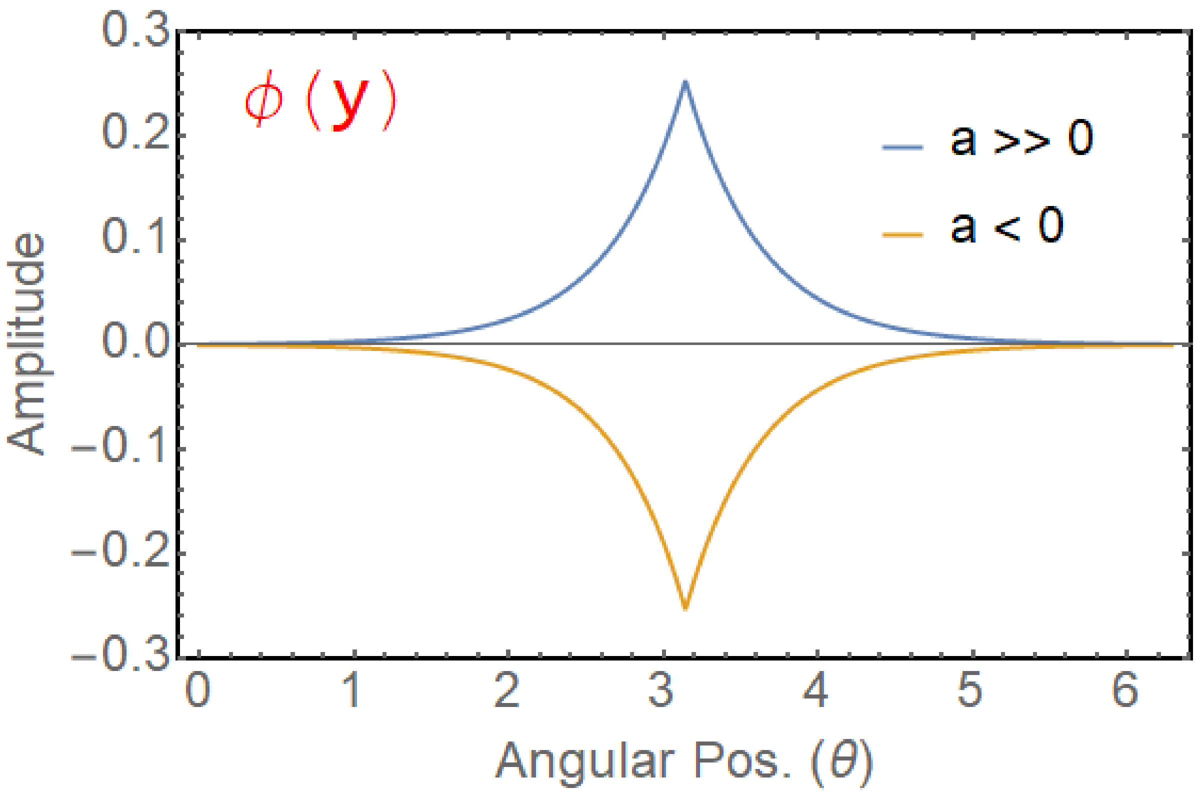




- These options are topologically distinct!

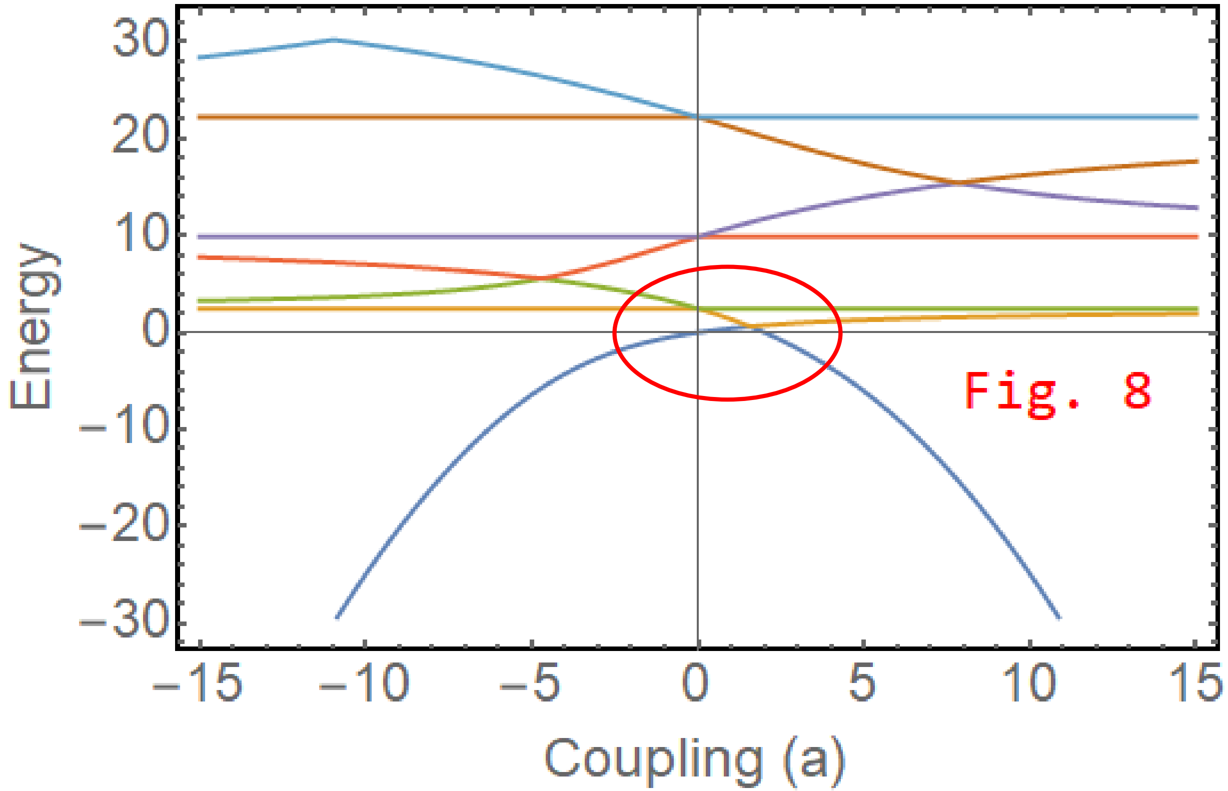
Two ring results - Wavefunctions

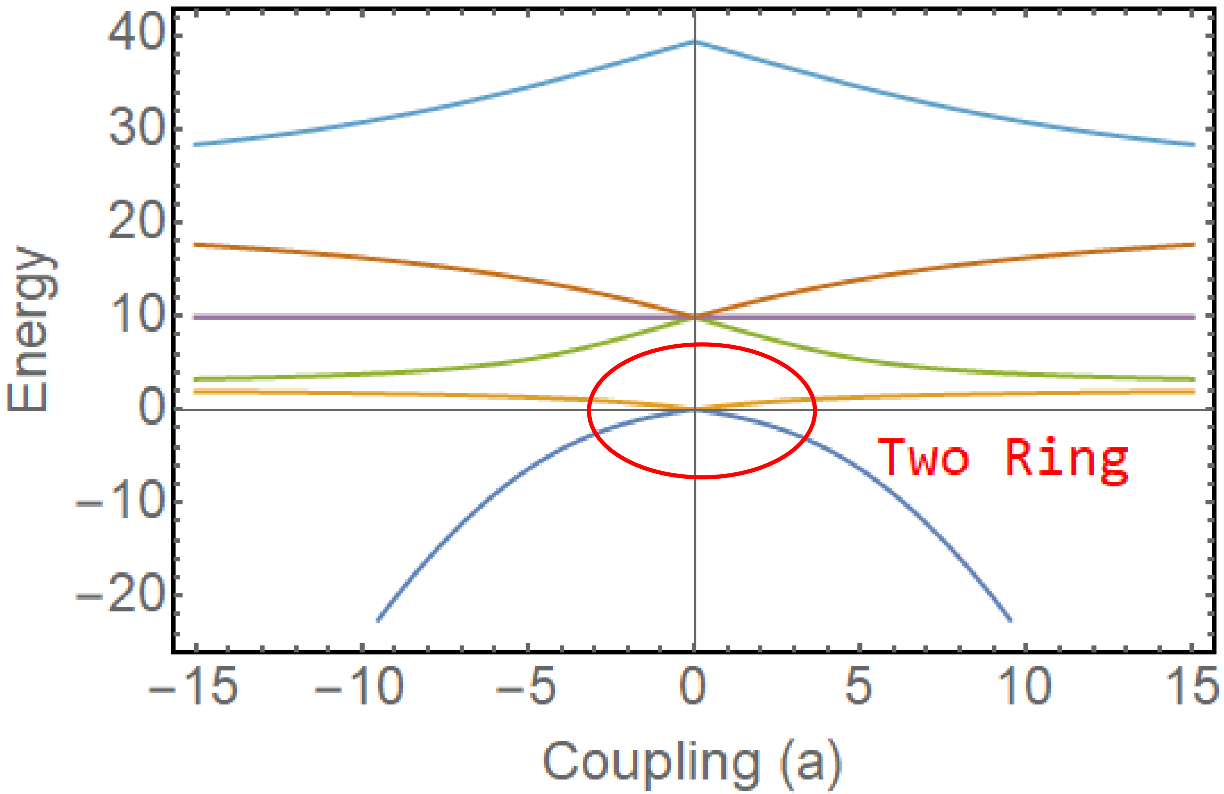




Two ring results - Spectra

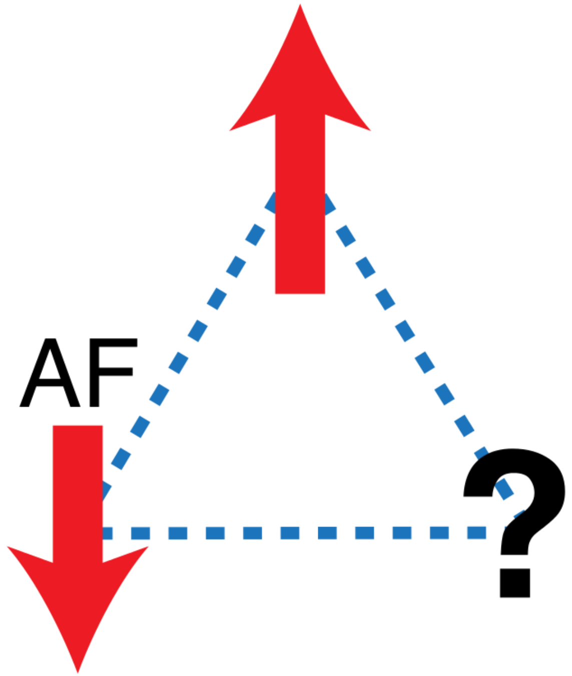
- Different topologies lead to different degeneracies and different spectra

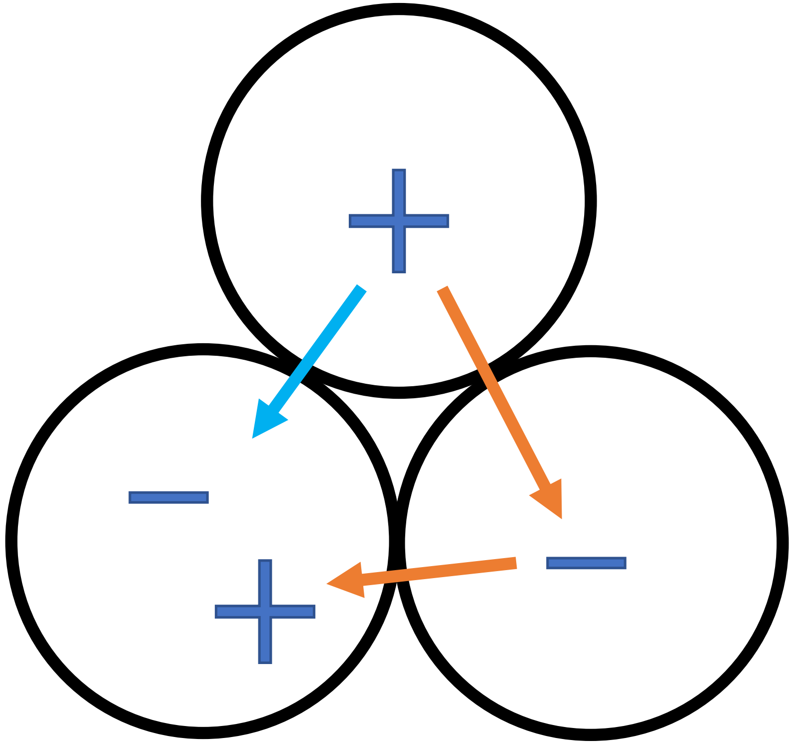




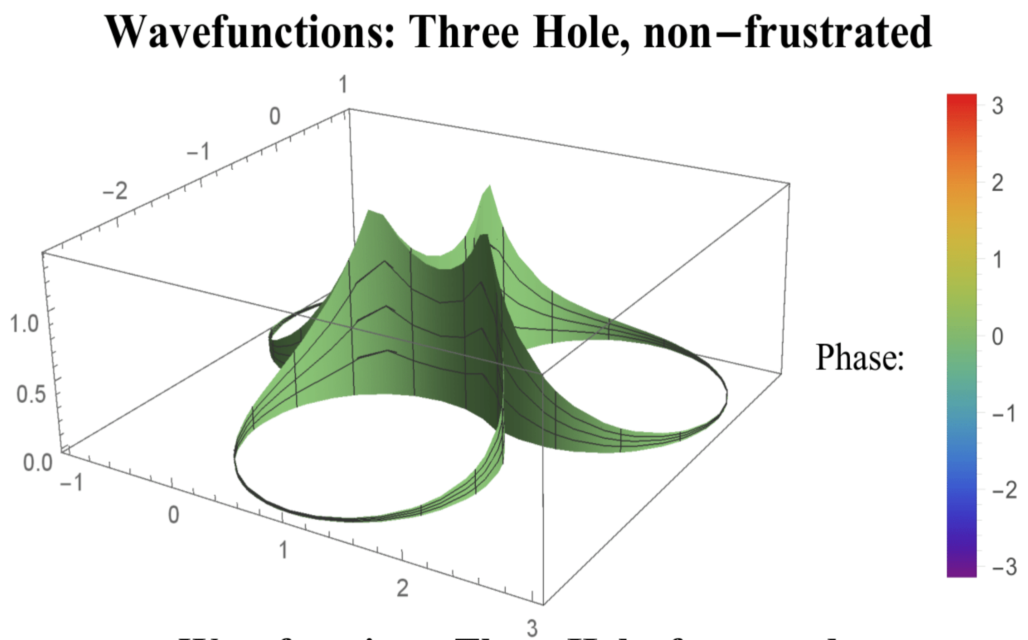
Three rings and geometrical frustration

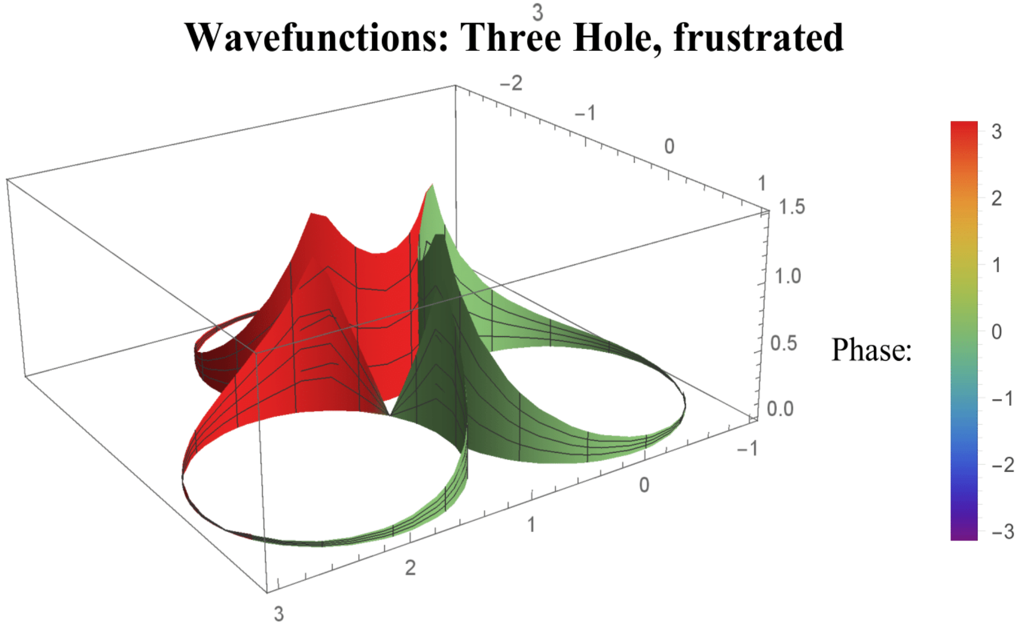
- Triangular lattices can create *geometrical frustration*
- Alternating systems not self consistent, adopt complicated forms
  - Ising Model Spin Lattices
  - Three Rings with a > 0?



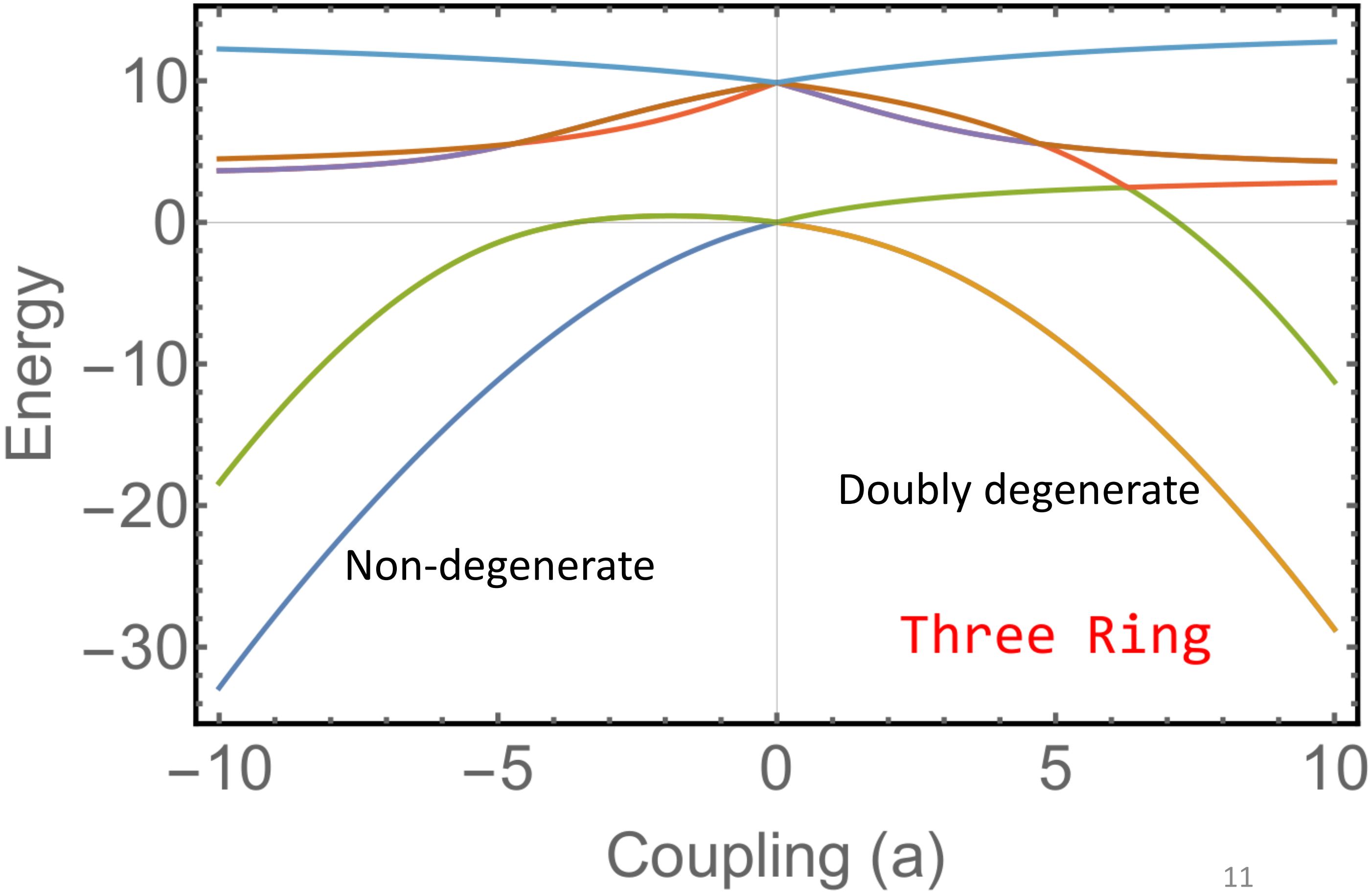


Three ring results

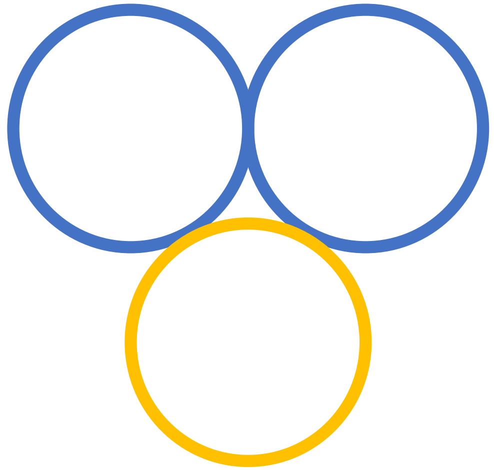




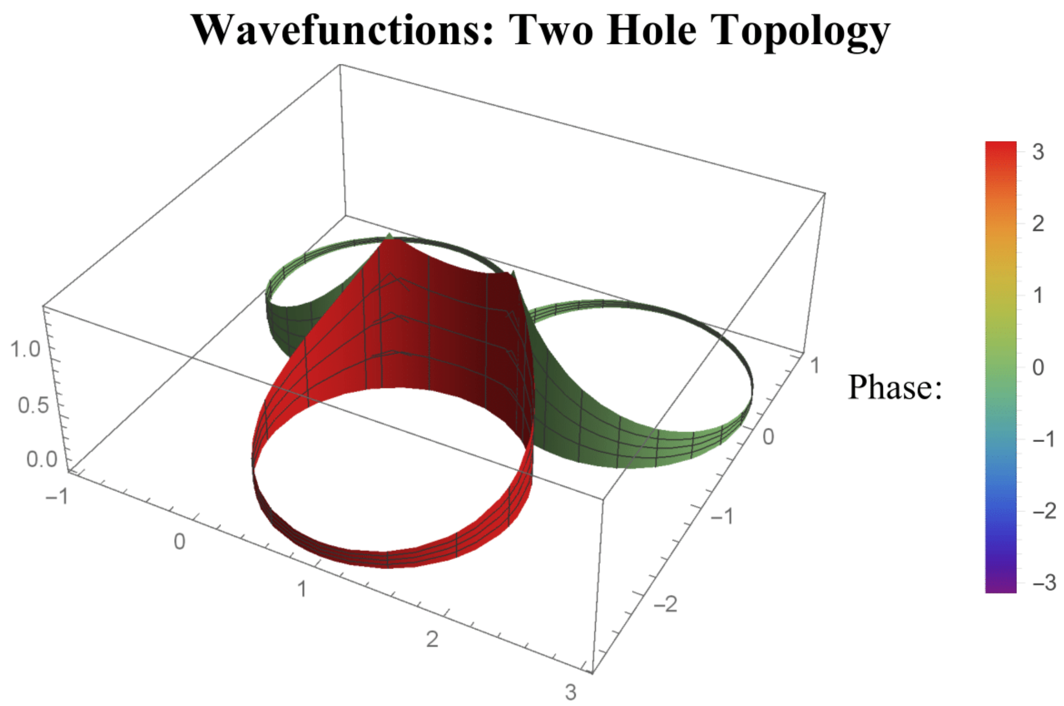
- Successfully see frustration!
- Triply degenerate at a = 0

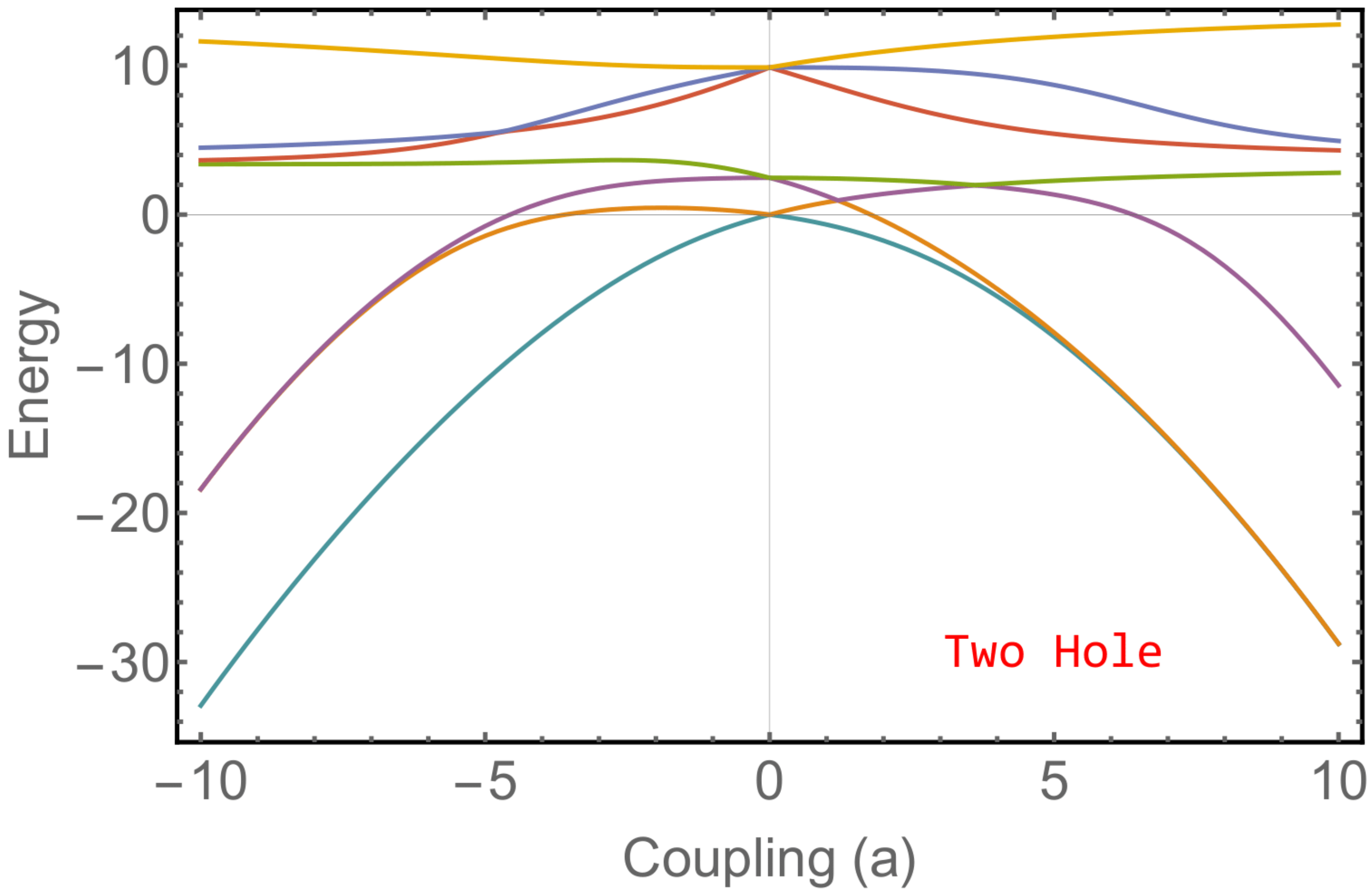


Three ring topology – two hole



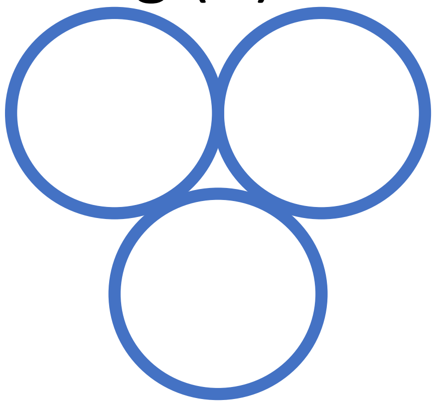
- Doubly degenerate at a = 0

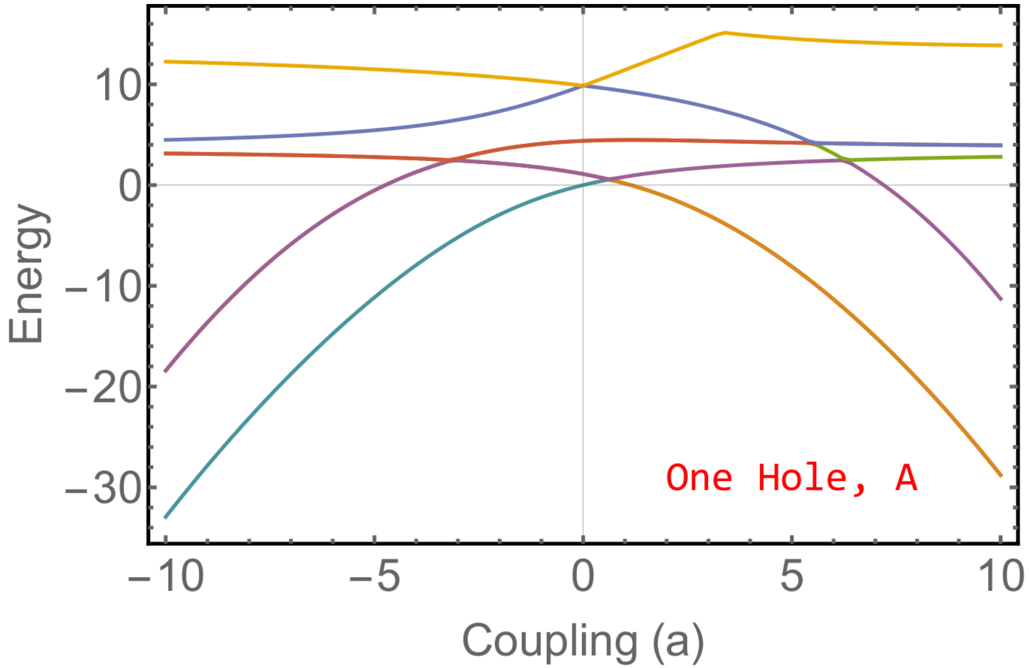




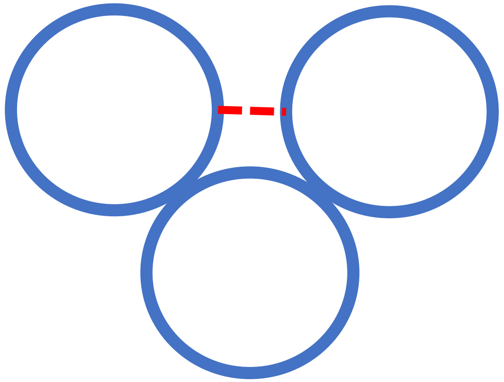
Three ring topology – one hole

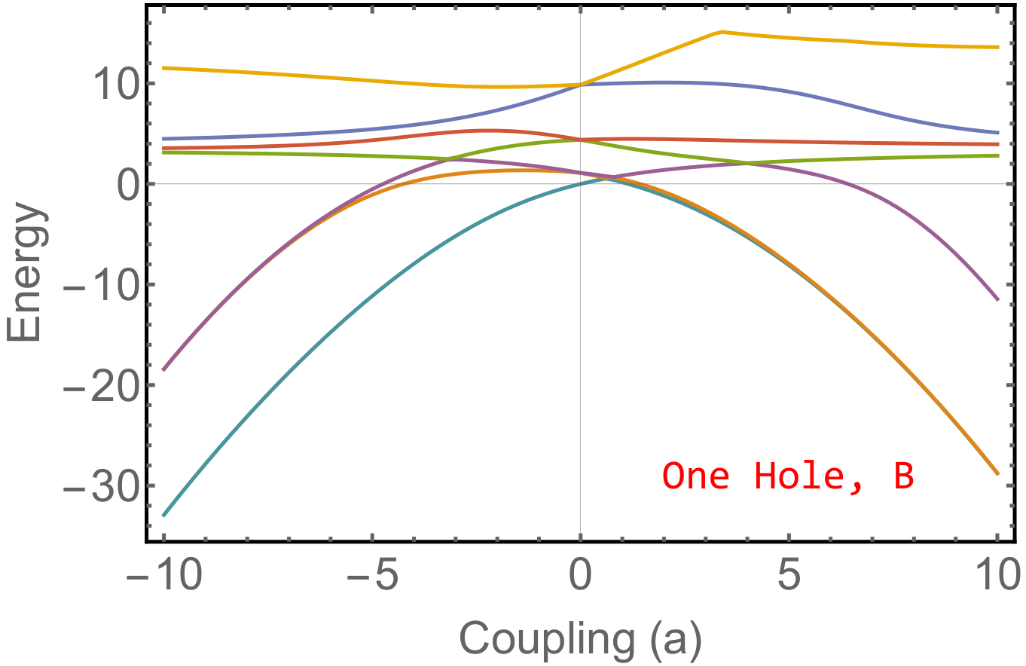
- Two Embeddings:
  - Triple Crossing (A):





- Double Crossing (B):





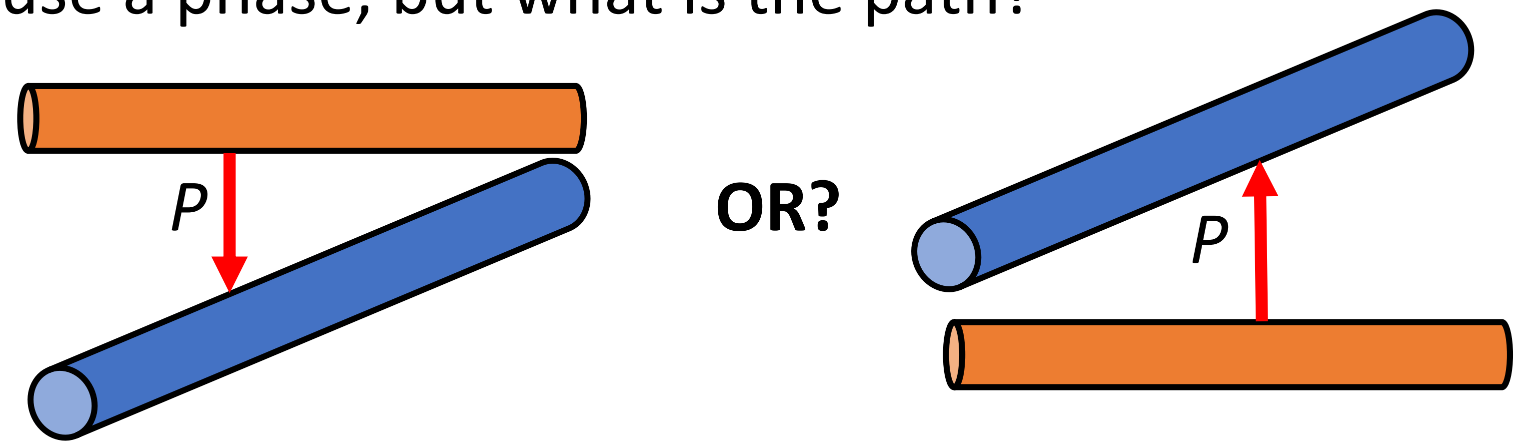
Preliminaries – AB Effect

- 1959: Y. Aharonov and D. Bohm show an e⁻ moving along $\vec{A}$ a path P in picks up a phase:

$$\varphi = \frac{e}{\hbar c}\int_{P} \vec{A}\cdot d\vec{l}$$

$\vec{A}$

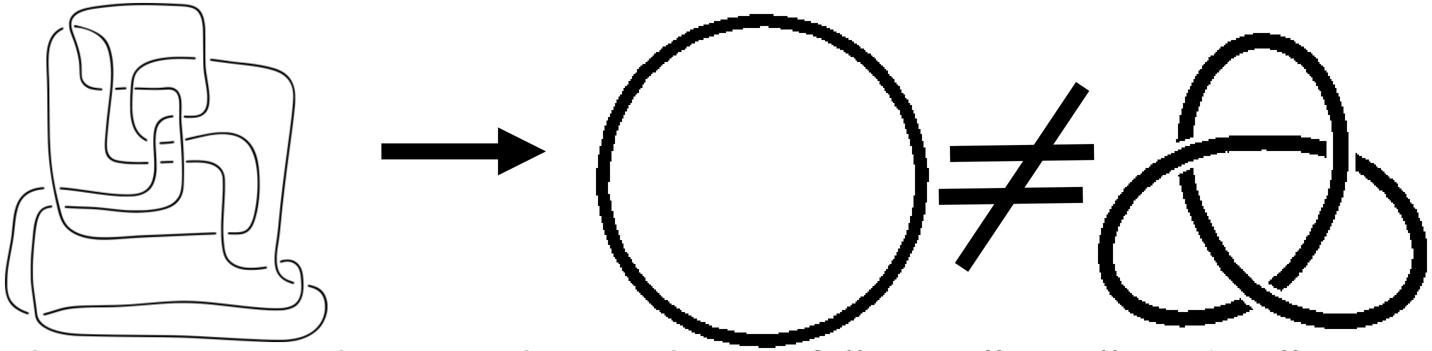
- Jumps cause a phase, but what is the path?



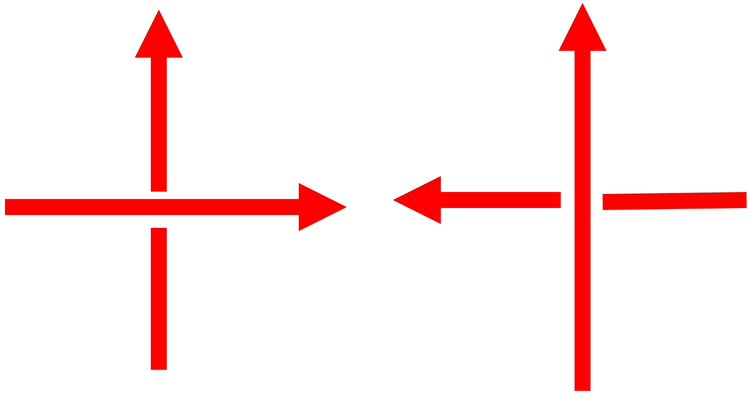
- “Over” and “under”-ness of crossings matters!

Preliminaries – Knot Theory

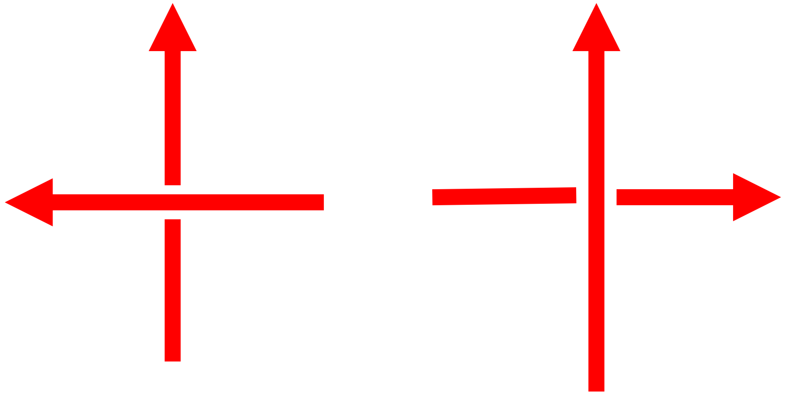
- Our tool for this description of systems and crossings
- Two useful ideas:
  - Knottedness: can you deform one embedding into another?



  - Writhe: counts the total number of "over" or "under" crossings



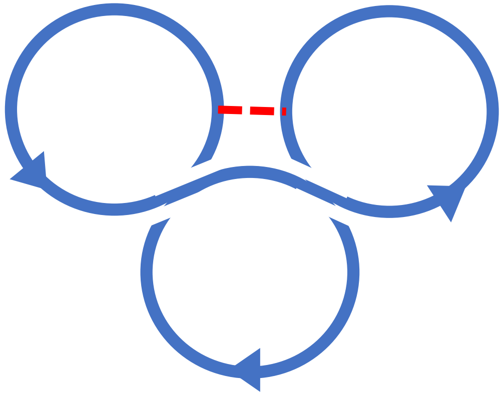
Writhe: +1



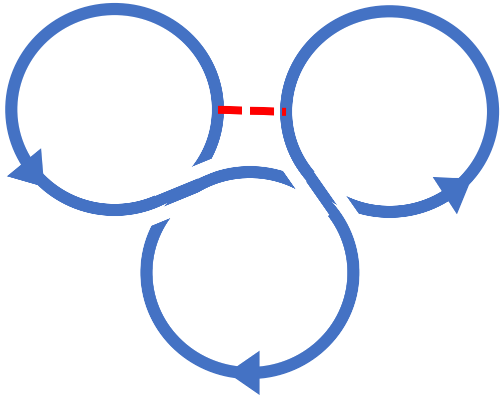
Writhe: -1

Mickey Mouse Ears (Double Crossings)

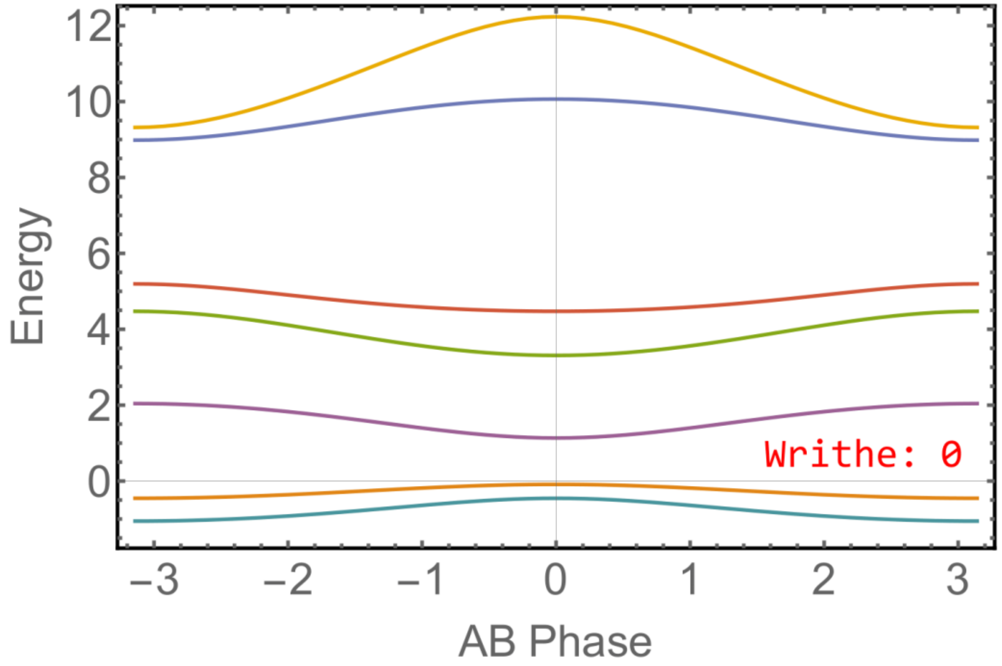
- Equivalent to the unknot, but two possible writhes

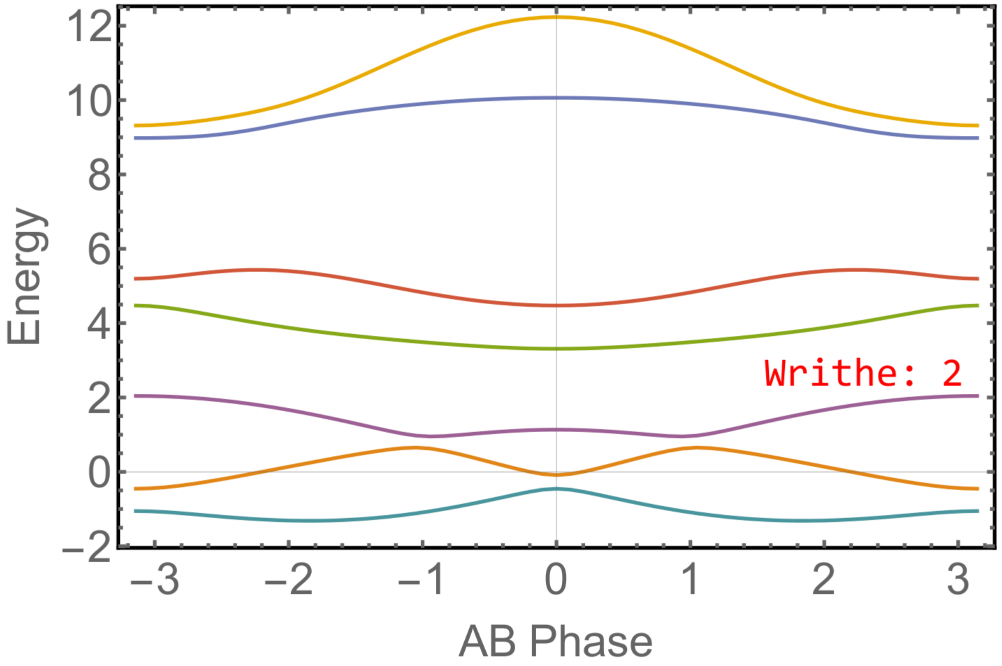


Writhe: 0



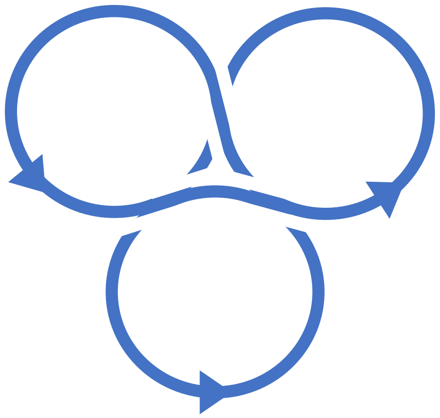
Writhe: 2



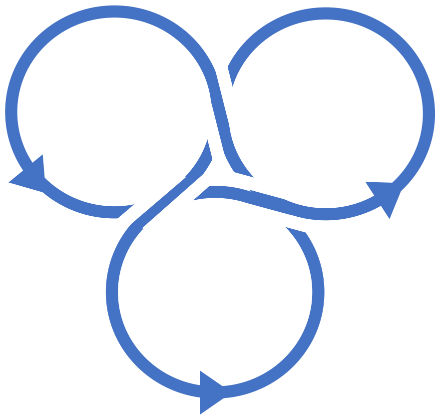


Trefoils (triple crossings)

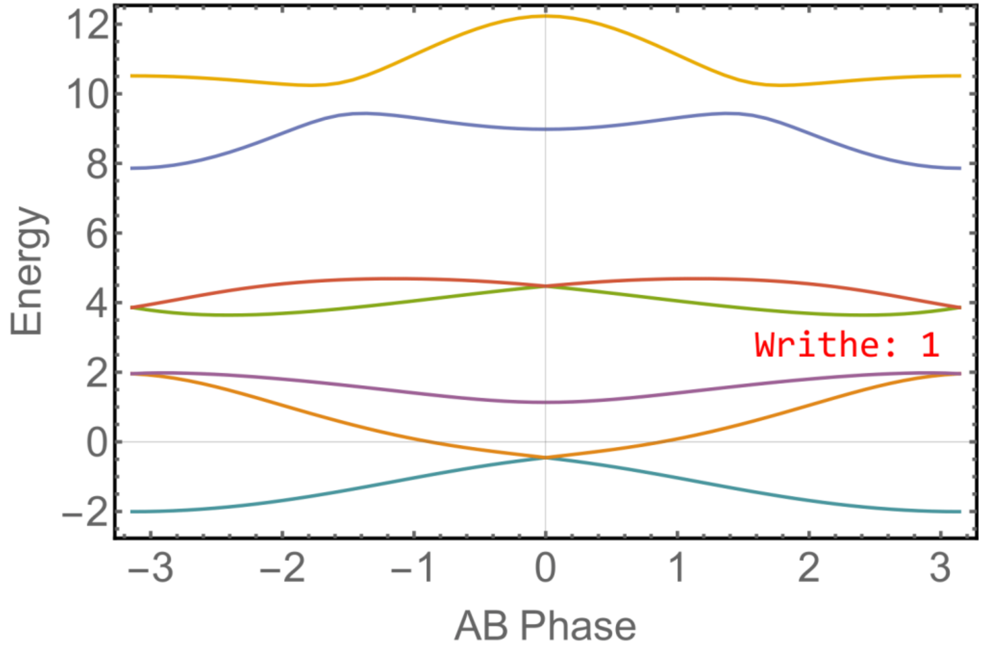
- Can be equivalent to trefoil knot or unknot: distinguished by writhe

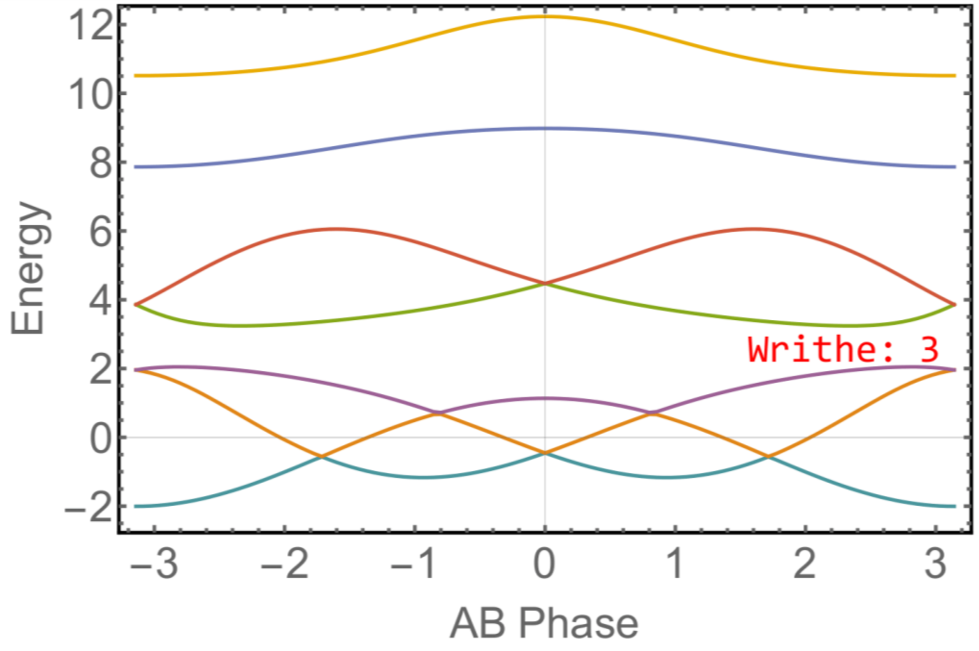


Writhe: 1
Unknot



Writhe: 3
Trefoil





Conclusions and next steps

- Systems of quantum rings offer the ability to see both topological and pseudo-topological effects in an experimentally viable setting
  - Effects observed in both the presence and absence of external fields

- Currently conducting research on these effects on larger systems of rings, and crystal lattices of rings

- Future research to investigate magnetic phase commensuration

Acknowledgements

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Two ring results - Gradients

- Different topologies lead to introduction of unbound state in figure eight