Dynamical Phases of a Spin-1 BEC

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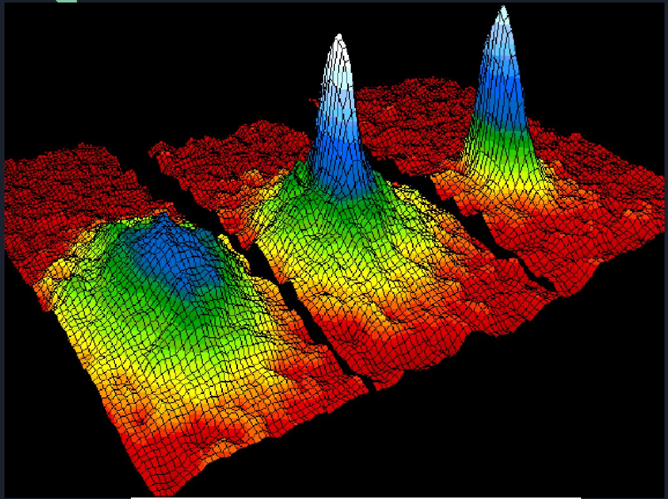
What is a Spin-1 BEC?



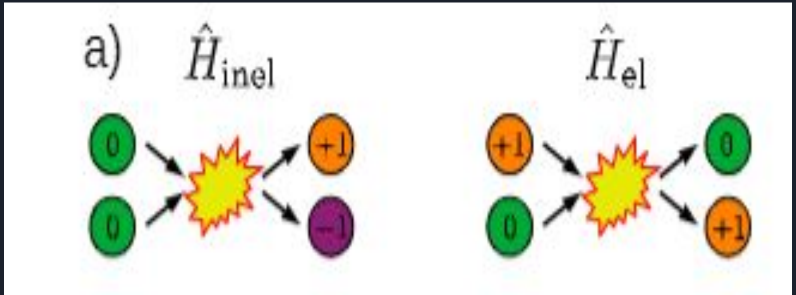
$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$v_{RMS} = \sqrt{kT}$$

- BEC (Bose-Einstein Condensate) is a gas of atoms (often Na or Rb) cooled to a few nano-Kelvin
- Atoms' wavefunctions begin to overlap
- Spin-1 BEC means the atoms can have the spin states of $m = 0, \pm 1$

What happens inside a Spin-1 BEC?

- These atoms of different spin states can collide
- These collisions can create quantum entangled states
- Applications in quantum sensing, like gravimetry
- These collisions can be elastic (spin conserving) or inelastic (spin-changing)



$m_1 + m_2 = m_1' + m_2'$ $\Sigma m_i = \Sigma m_f$

Elastic: $0 + 0 \rightarrow 0 + 0$

Inelastic: $0 + 0 \rightarrow -1 + +1$

Understanding Collisions

- This quantum system becomes very difficult to manage due to both spin and spatial degrees of freedom

- However, we can “freeze” out the spatial degree of freedom. The energy scale between spatial and spin is very large, spatial being high-energy, spin being low-energy, and we are only interested in the lower energy spin degree of freedom

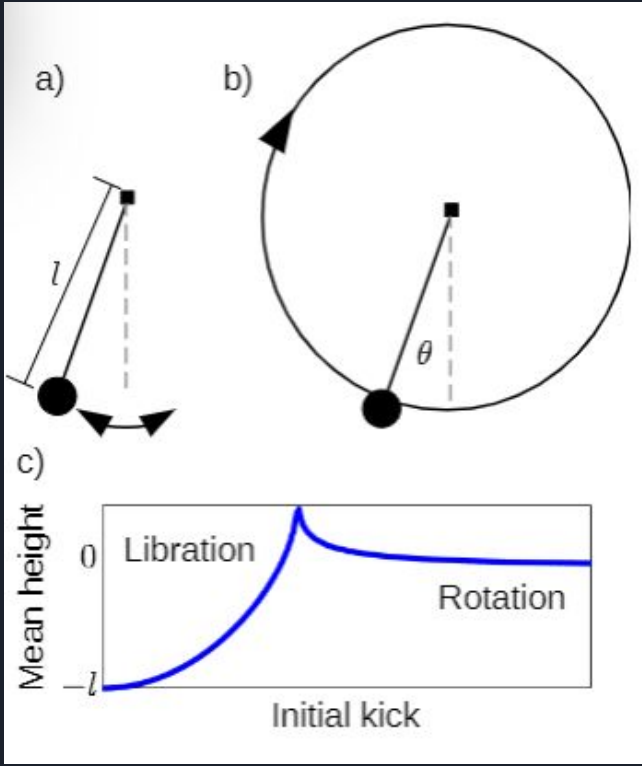
- In certain limits we can simplify the problem by ignoring quantum fluctuations, which gives us a classical model that can be understood as a non-rigid pendulum

- By studying the pendulum dynamics, we can understand the dynamics of the spin 1 BEC

What are Dynamical Phases?

- Dynamical phases: Qualitatively distinct regimes of behavior separated by a sharp transition

- Ex: A pendulum swinging back and forth
- a). Libration
- b). Rotation
- Role of Order Parameter: a measure of order in a system that allows delineation of different phases
- Order parameter: Mean height of the pendulum



Non-rigid Pendulum

- By ignoring quantum noise in the BEC system and treating it classically, we find the Hamiltonian of the system to be:

$$H = c\rho_0[(1-\rho_0) + \sqrt{(1-\rho_0)^2 - m^2} cos\theta] + \delta(1-\rho_0)$$

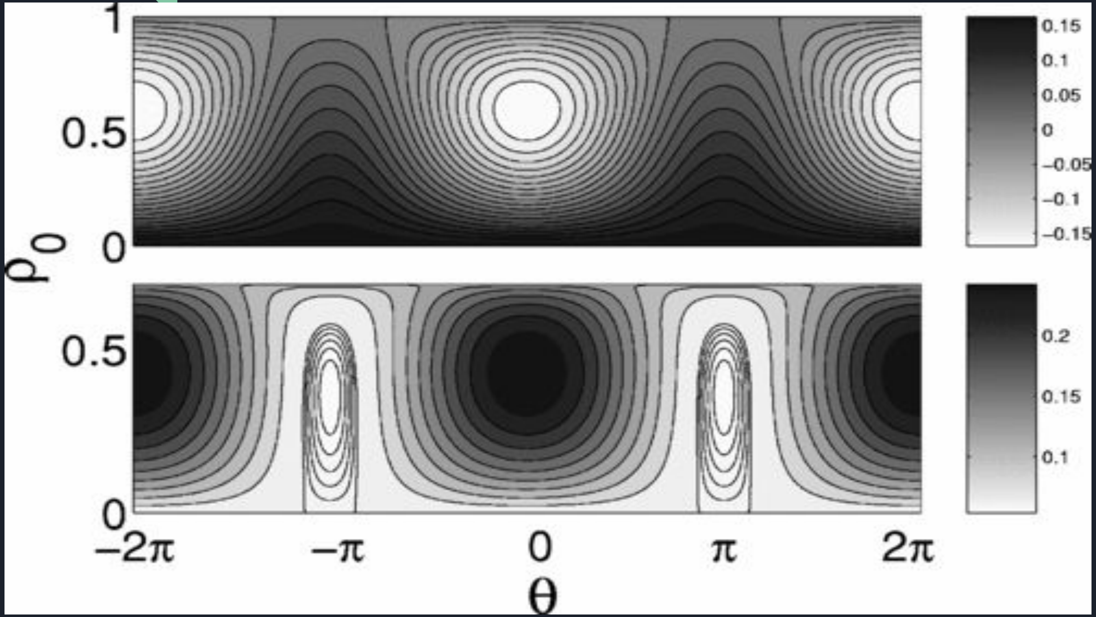
$$\rho_0 = \frac{n_0}{N}$$

θ = Spinor phase (relative phase of different spin components)

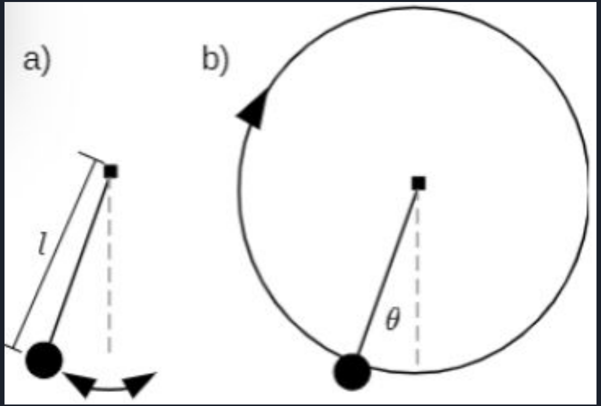
δ = Zeeman shift: energy shift between spin states due to an applied magnetic field

c = Interaction strength ($c \equiv 1$)

Pendulum like dynamics within the BEC



Contour map of equal energy lines, it shows the different motions of the BEC similar to the classical pendulum

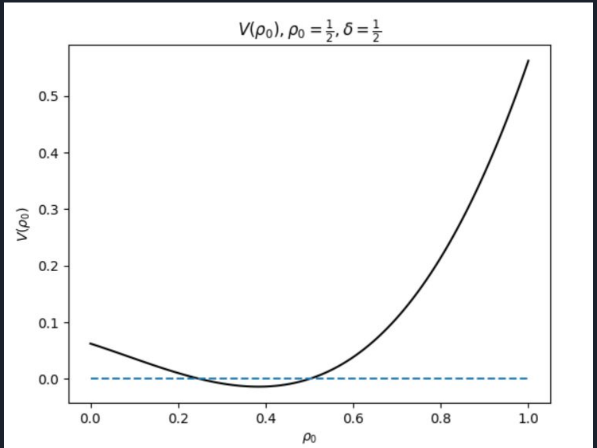


Understanding motion by mapping to a particle in a potential

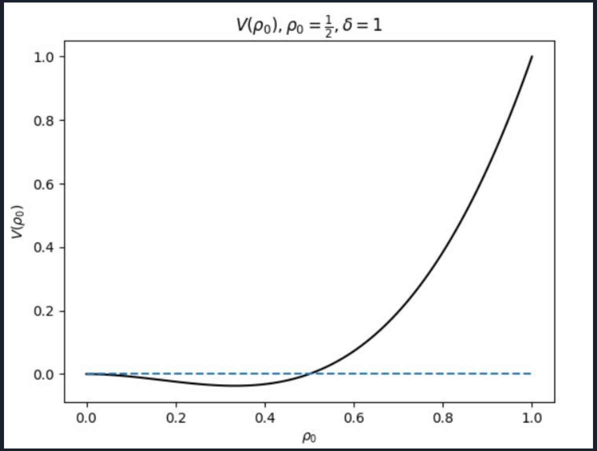
- Energy is conserved in the system, which means we can reduce the Hamiltonian dimensionally, and eliminate either ρ_0 or θ

$$(\dot{\rho}_0)^2 + V(\rho_0) = 0$$

- By eliminating θ we get the LHS
- This equation looks like a classical particle in a 1D potential well



- The particle is stuck between two turning points, leading to oscillation similar to the pendulum’s libration

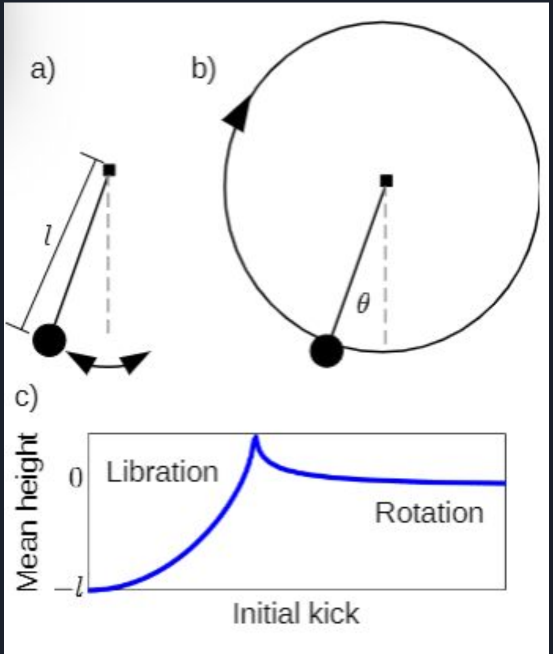


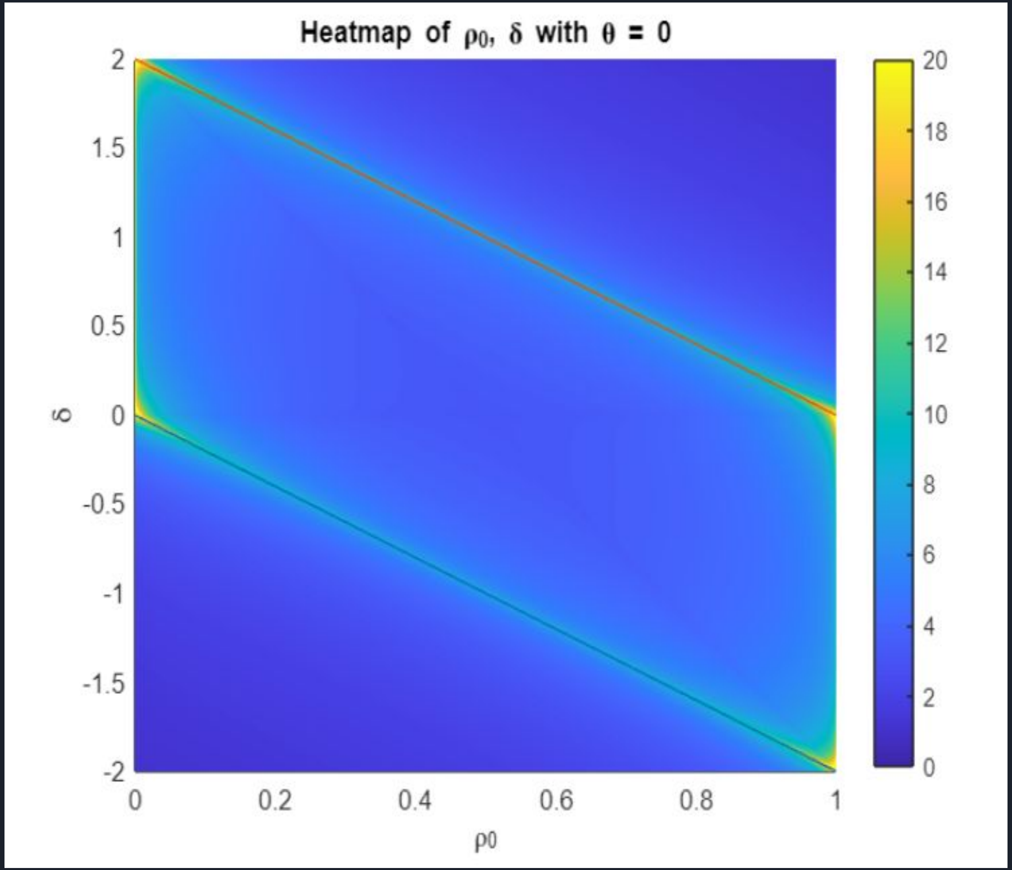
- The particle travels to the LHS, where it stops as the graph levels out, leading to a period of infinity

Revisiting Dynamical Phases

$$(\dot{\rho}_0)^2 + V(\rho_0) = 0$$

- Once we have the potential, we can find its roots, which will give us information on turning points
- We find when turning points coincide and solve for delta to find a period of infinity
- We can then use these delta values to find Dynamical Phase Transitions (DPTs)





$$T = \frac{\sqrt{2}\hbar}{\sqrt{\delta c}} \frac{K\left(\sqrt{\frac{x_3 - x_2}{x_3 - x_1}}\right)}{\sqrt{x_3 - x_1}}$$

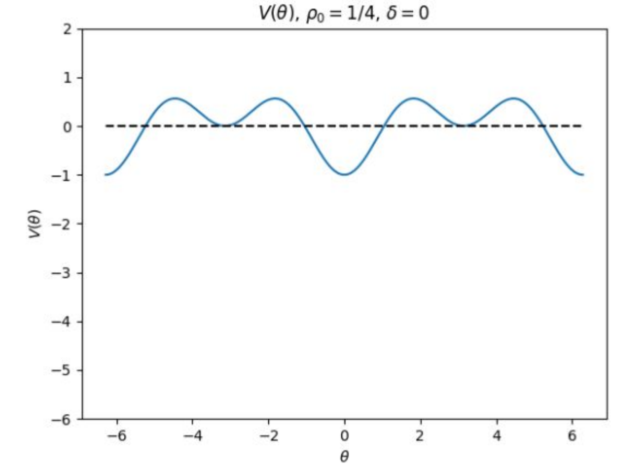
- A graph showing the period of the motion of ρ_0 as a function of time
- Shows the different dynamical phases of the BEC, separated by a sharp transition (two diagonal lines)

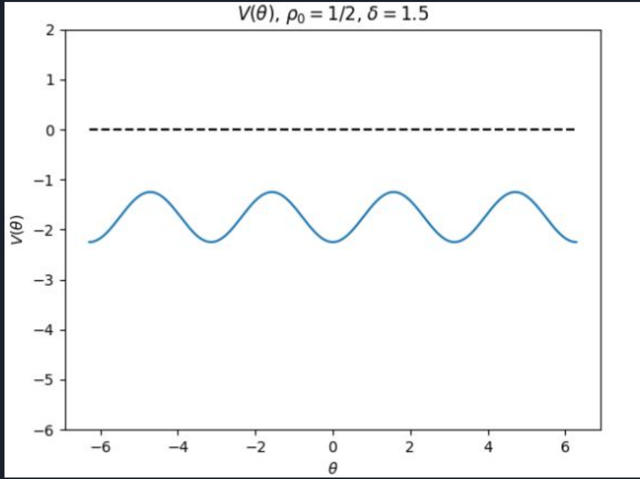
Difficulties in analysis

- The lines on the previous graph indicated DPTs, however, there is not enough information about what phase each different section of the graph is in

- To find this information, we need to apply the same steps for the earlier potential, but with respect to θ

Potential as a function of θ



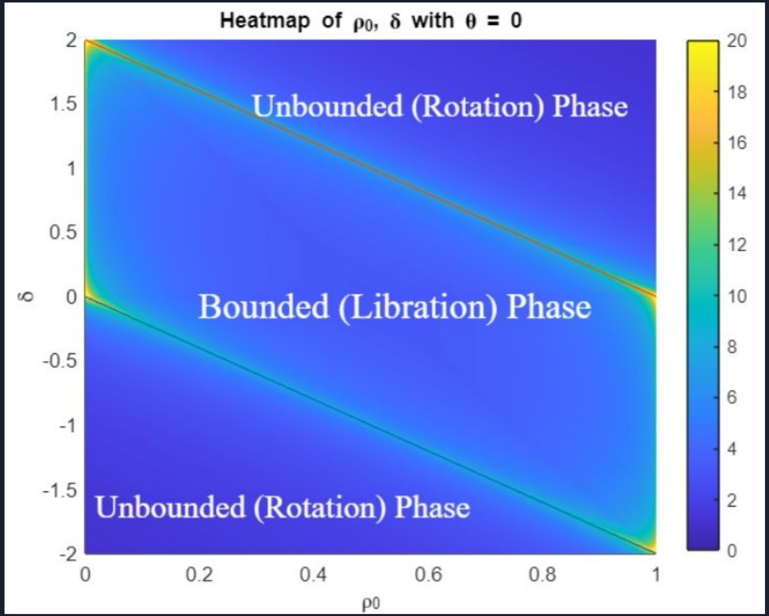


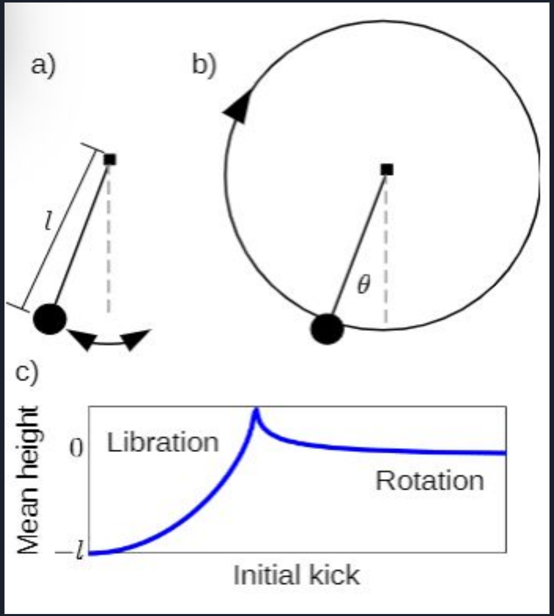
The particle is bounded

The particle is unbounded

*note: $\theta_i = 0$, but the particle is not at rest initially

Identifying Dynamical Phases





Further Research

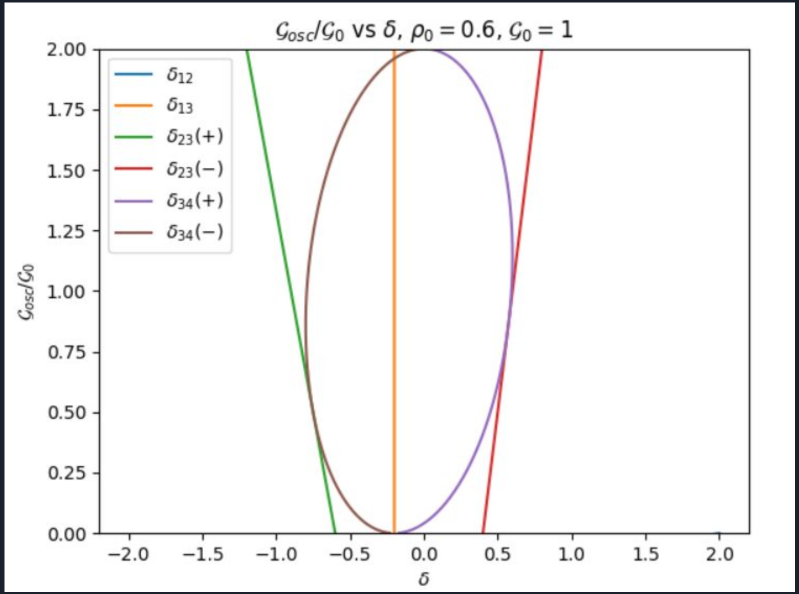
- The next step in researching DPTs within a spin 1 BEC is altering the Hamiltonian to account for different interaction strength for different collision processes, and trying to find an as yet unknown analytical solution to the dynamics of this new system

$$H = \mathcal{G}_0\rho_0(1-\rho_0) + \frac{\mathcal{G}_{osc}}{2}\rho_0\sqrt{(1-\rho_0)^2 - M^2}cos(2\theta) + \delta(1-\rho_0)$$

$\mathcal{G}_0$: Elastic

$\mathcal{G}_{osc}$: Inelastic

DPTs of new Hamiltonian



- A graph of critical delta values found from the new Hamiltonian, further research is needed to differentiate actual DPTs from unimportant delta values

Summary

- We have characterized the dynamics of a spin 1 BEC, and found it was similar to a classical non-rigid pendulum

- Characterized the dynamical phase diagram: Found phase transitions and identified distinct phases

- We are now analyzing a more general and more sophisticated model where the interaction strength is tuneable. We have found DPTs, and continued research will be done to characterize the dynamical phases, as well as interpret how the tunability of interaction strength changes the physics