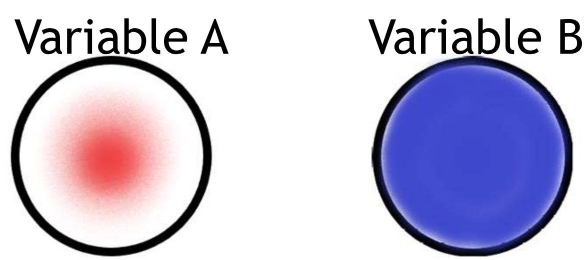
# OU Physics REU Final Talk: Quantum Metrology in an Interferometer

By Samuel Bayliff

# Background: Quantum Metrology

- Study of quantum measurements, especially the uncertainty
- Traditional uncertainty: $\Delta C^2 = \Delta A^2 + \Delta B^2$
- Doesn't work for variables that aren't independent
  - Instead, we have covariance terms
    - $\Delta C^2 = \Delta A^2 + \Delta B^2 + Cov(A, B) + Cov(B, A)$
    - We can use this to get a better result




- The inter-parameter dependencies come from

the Hamiltonian and the input state.

# Background: The Quantum Fisher Information Matrix (QFIM)

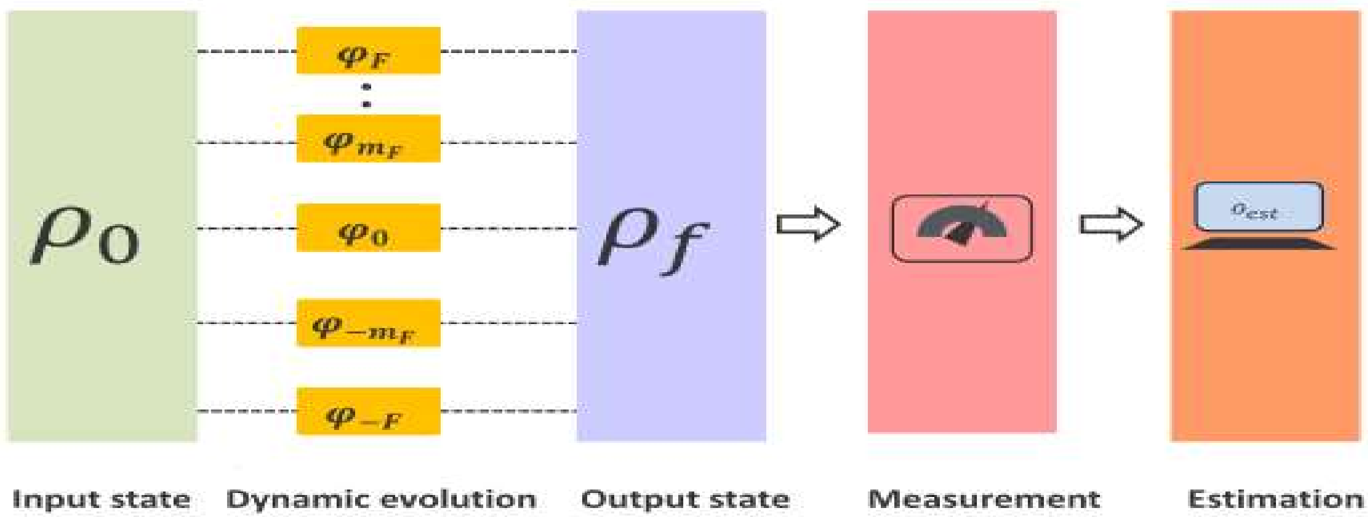
- A measure of the information a random variable carries about some unknown parameter.
- For example, the Hamiltonian $\mathcal{H} = \theta\hat{A} + \Phi\hat{B}$, where $\hat{A}$ and $\hat{B}$ are operators, yields the QFIM below for measuring $\theta$ and $\Phi$

$$\begin{pmatrix} (\Delta A)^2 & Cov(A,B) + Cov(B,A) \\ Cov(A,B) + Cov(B,A) & (\Delta B)^2 \end{pmatrix}$$

- Square *n x n* matrix, for *n* parameters.
- The Cramér-Rao bound
  - Variance is greater than the corresponding diagonal element of the inverse of the QFIM
- The system Hamiltonian determines what values are present in the QFIM, while the input wavefunction is used to evaluate them.

# Input States

- The input state is used to evaluate the QFIM
  - In a Spinor-Bose-Einstein condensate interferometer, the input state is the state of the BEC before the Hamiltonian is applied to imprint the phase.




PHYSICAL REVIEW A98, 033603 (2018)

- States are a multiparticle state, such as the Fock state $|N_{-1}, N_0, N_1\rangle$.
- My research has used the Vacuum, Pure Fock, and coherent spin states as input states.

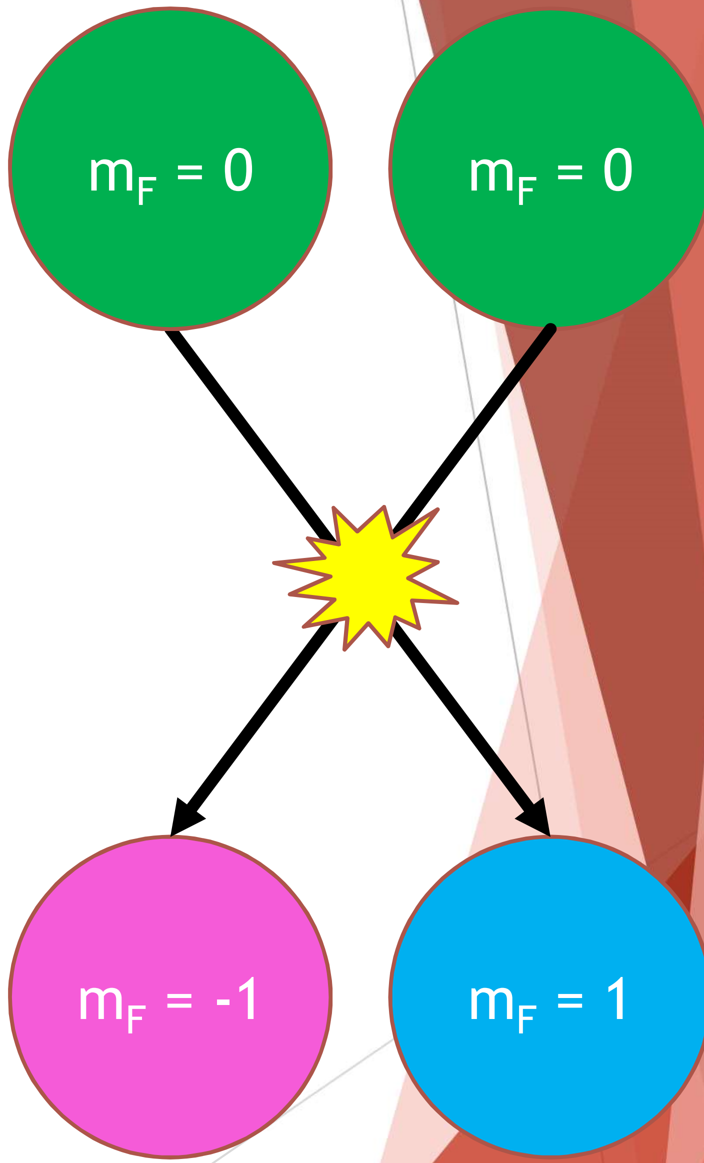
# The System Hamiltonian

- The Hamiltonian determines what parameters and operators can appear in the QFIM, and how they are related.

- The Hamiltonian in my research comes from spin-mixing dynamics under the undepleted pump approximation.  The Hamiltonian contains terms which describe what changes in angular momentum can occur.

- The parameter we wish to estimate is imprinted as a phase.

- The split modes are recombined in such a way to yield a particle number difference dependent on the phase.



# Results for Input State into Phase Imprinting.

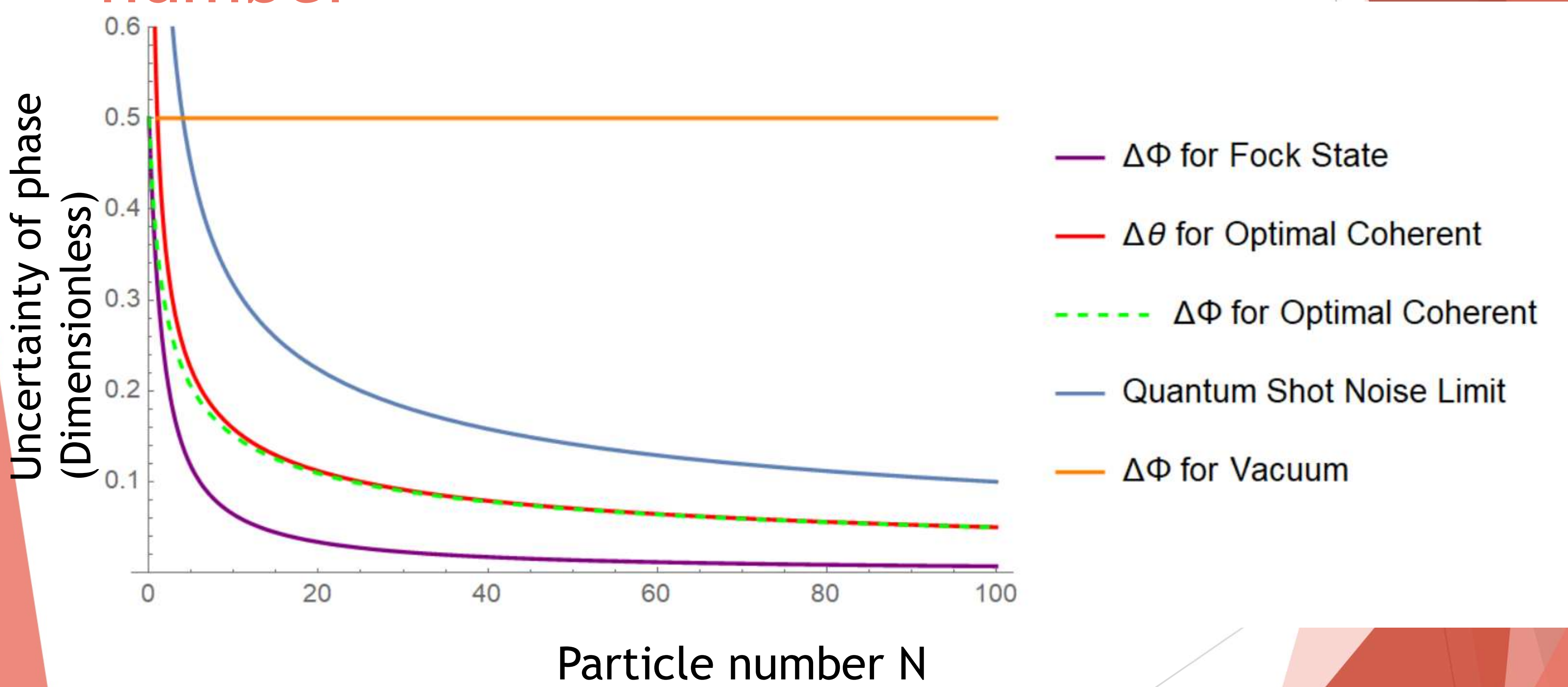
- For our system, where $\tilde{b} = \hat{a}_1^\dagger \hat{a}_{-1}^\dagger + \hat{a}_1 \hat{a}_{-1}$ and $\widehat{N_s} = \widehat{N_1} + \widehat{N_{-1}}$ we have a Hamiltonian $\mathcal{H} = \theta \hat{N}_s + \Phi \tilde{b}$ and a general QFIM as below

$$\begin{pmatrix} 4(\Delta \hat{N}_s)^2 & Cov(\hat{N}_s, \tilde{b}) + Cov(\tilde{b}, \hat{N}_s) \\ Cov(\hat{N}_s, \tilde{b}) + Cov(\tilde{b}, \hat{N}_s) & 4(\Delta \tilde{b})^2 \end{pmatrix}$$

- For the vacuum state, it reduces to a single value of 4 corresponding to the upper bound for precision in $\Phi$.
- Likewise, the pure Fock state also reduces to a single value for $\Phi$, $4(\widehat{N}_s + \widehat{N}_{-1}\widehat{N}_1 + 1)$.
- The coherent state yields a QFIM of

$$4\begin{pmatrix} \langle \hat{N}_s \rangle & \langle \tilde{b} \rangle \\ \langle \tilde{b} \rangle & \langle \hat{N}_s + 1 \rangle \end{pmatrix}$$

# Graphs of uncertainty vs particle number



# Future Work

- The initial state will be allowed to evolve in our Hamiltonian before being input to the phase imprinting
- This allows a more informative input state to be used with the ease of setting up a simpler initial state.
- As our system is not ideal, it also accounts for the non-negligible time between when the Hamiltonian is applied and when the phase is imprinted.
- Likewise, evolution after phase imprinting will be taken into account.

# Acknowledgements

- I’ve been working under Dr. Blume with Post-Doctoral student Jianwen Jie.

# Questions!