The law of gravitation and the gravitational constant I, acceleration method

Literature: P o h l, Mechanics, §§ 29, 30, 32

Principle: Observation of mutual attraction with the gravitation torsion balance and determination of the gravitational constant from the accelerated motion on changing the position of the spheres.

All bodies are attracted towards the earth. This fact is well known to everybody and appears self-evident. However, the fact that all bodies exercise mutual attraction lies outside the range of everyday experience; for the forces of attraction are so small that they cannot be perceived without special experimental aids. E. g. for two lead spheres of masses 1·5 and 0·015 kg distant 4·5 cm from each other — the experiment described here will be carried out using these dimensions — the force is less than 10^{-4}mg wt.

Hence, to prove the existence of the mutual attraction, a very sensitive dynamometer is used, preferably the torsion balance constructed by C o u l o m b in 1784, which was first used by C a v e n d i s h in 1798 for the measurement of the gravitational force.

With a balance, and also with a torsion balance, nearly always **measurements of forces in equilibrium** are made. Such a procedure for the determination of the gravitational constant is described in DC 531.51; b. In the experiment described here, a different, dynamic method of measurement is used. It is comparatively easily understood and requires a time of observation of only one minute.

The gravitation torsion balance of Prof. S c h ü r h o l z consists of a ⊥-shaped system suspended on a torsion fibre and carrying a small lead sphere at each end. On account of its large moment of inertia, this system has a long period of oscillation (about 10 minutes!). Two large outer spheres, lying approximately in the positions shown dotted (see Fig. 1) attract the two inner spheres. Equilibrium is established between this force and the restoring force due to the twisting of the torsion fibre. When the equilibrium position is attained and the null point is checked by the position of the light beam serving as a pointer, the experiment is begun. The outher large spheres are swung on their rotatable holders so that their positions are changed in the manner shown in the accompanying diagram. The equilibrium is thereby disturbed, for the torsion fibre, on account of the long period of oscillation, is still twisted in the earlier direction, and the large spheres attract the small spheres in the opposite direction. With this arrangement, the force F_0 acting between each pair of spheres at the beginning of the motion is twice as great as the mutual attraction alone, because the whole twist of the fibre, which still remains at first, corresponds to a force of equal magnitude:

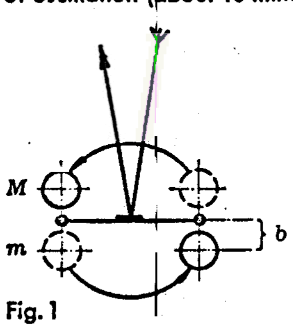


Fig. 1

$$F_0 = 2\,G\,\frac{m\,M}{b^2}$$

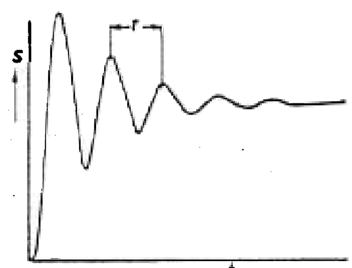


Fig. 2

Under the action of this force F_0, the two small spheres begin to be accelerated towards the large spheres lying opposite them. The result is that the torsion fibre is more and more relaxed at first and then twisted in the opposite direction. Thus, the system of the balance executes an oscillatory motion (Fig. 2), of which only the initial part will be considered here. For this, the change of the twist in the fibre can be neglected, and the motion of the spheres is (up to about $^1/_{10}$ of a period of oscillation, with an accuracy of about 5 %) still uniformly accelerated with the acceleration a_0. Hence, for this part of the motion,

$$m\,a_0 = 2\,G\,\frac{m\,M}{b^2}\,.$$

In this relation, all the quantities are determinable, so that the gravitational constant $G = \frac{a_0}{2}\,\frac{b^2}{M}$ is found. The acceleration a_0 is ascertained from the displacement S of the light pointer in the first minute after the interchanging of the spheres. As shown in the accompanying Fig. 3, let

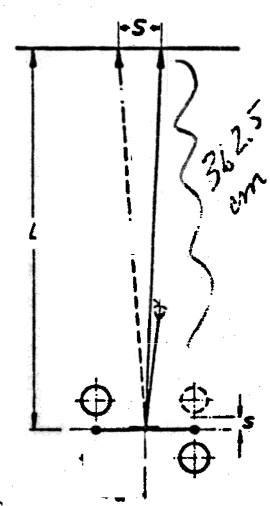


Fig. 3

s = the path traversed by the small sphere on the balance,
d = the separation of the small sphere from the axis,
S = the distance traversed by the spot of light on the screen,
L = the separation of the screen from the mirror of the balance.

Then, on account of the doubling of the angle on reflection, $\frac{s}{d} = \frac{S}{2L}$. The deflection of the spot of light is thus $\frac{2L}{d}$ times the distance moved through by the smaller sphere on the balance. The acceleration a_0 can therefore be determined from the deflection of the light spot in the first minute by means of the equation

$$a_0 = \frac{2\,s}{t^2} = \frac{S\,d}{t^2\,L}\,.$$

d = 6.5 cm
not used.

M = 1501 gm

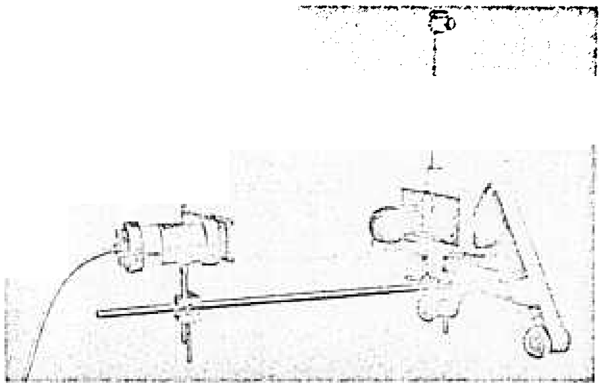
Case = 2.8 cm

The mass M of the large sphere is found by means of a balance, the separation b from the radius of the large sphere and half the thickness of the case of the torsion balance. An example of measurement and a simple method of error correction is given overleaf.

Gravitation Torsion Balance - Reprint from "Physics Leaflets" Cat. No. 59921 - First Collection - Sheet Nos. 1/17 and 1/18.

Apparatus required for the experiment:
(in order of assembly)

	Cat. No.
1 Stand base, 28 cm	300 01
1 Pair of adjusting screws	300 06
1 Stand rod, 60 cm long, bent	300 52
2 Leybold bosses	301 01
1 Gravitation torsion balance	332 10
1 Lamp-house	450 60 a
1 Lamp, 6 V, 30 W	450 51
1 Single-lens condenser with diaphragm	460 17
1 Transformer, 6 V, 30 W	562 76
1 Stopclock	313 05
or	
1 Large electric stopclock	313 04/08



Useful hints:

1. For the erection and adjustment of the gravitation torsion balance, see directions for use 332 10.
2. Before the commencement of the experiment, null point observations must be made for several consecutive minutes.
3. The changing around of the spheres must be carefully performed. Knocking of the sphere against the glass plate of the apparatus should be avoided.
4. After the changing round of the spheres, the path of the light pointer must be read every 15 sec. a_0 is found by graphical or computational averaging of the first four values.
5. The mean value is subject to the following systematic error: the small sphere is attracted with a very small force also by the more distant large sphere (see Fig. 4).

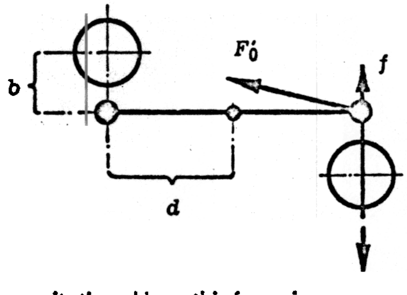


Fig. 4

From the gravitational law, this force is

$$F'_0 = G\,\frac{m\,M}{b^2 + 4\,d^2}$$

and has a component f opposite to the force F_0 being measured:

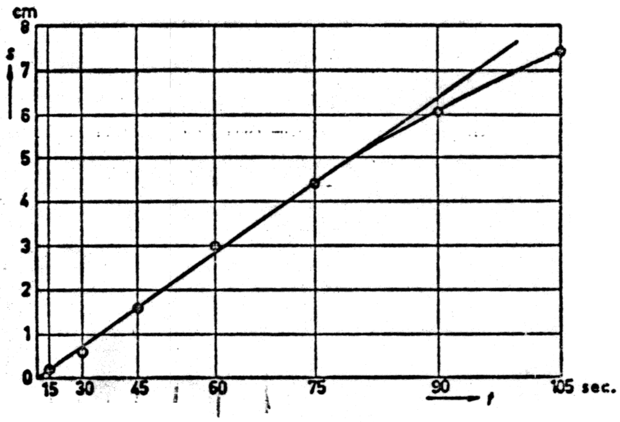
$$f = G\,\frac{m\,M}{b^2 + 4\,d^2}\,\frac{b}{\sqrt{b^2 + 4\,d^2}} = \beta\,F_0$$

if β denotes the fraction by which the observed force F_0 is too small.

$$\beta = \frac{b^3}{\left(b^2 + 4\,d^2\right)\sqrt{b^2 + 4\,d^2}}$$

With $d = 0{\cdot}05$ m and $b = 0{\cdot}045$ m, β becomes 0·069. The value of G calculated without this correction must therefore be increased by 6·9 %.

6. Example of measurements: Fig. 5



$$M = 1{\cdot}46 \text{ kg}$$
$$b = 0{\cdot}045 \text{ m}$$
$$L = 5{\cdot}0 \text{ m}$$
$$d = 0{\cdot}05 \text{ m}$$

$$a_0 = \frac{S\,d}{t^2 L} = 8 \times 10^{-6} \times \frac{1}{100} = 8 \times 10^{-8}\,\frac{\text{m}}{\text{sec}^2}$$

$$G = \frac{a_0\,b^2}{2M} = 5{\cdot}55 \times 10^{-11}\,\frac{\text{m}^3}{\text{kg sec}^2}$$

With the correction for $\beta = 0{\cdot}069$, this value for G must be increased by 6·9 %. It becomes, therefore,

$$G = 5{\cdot}93 \times 10^{-11}\,\frac{\text{m}^3}{\text{kg sec}^2}$$

instead of the exact value

$$G = 6{\cdot}67 \times 10^{-11}\,\frac{\text{m}^3}{\text{kg sec}^2}.$$

The law of gravitation and the gravitational constant II, final deflection method

Literature: P o h l, Mechanics, §§ 29, 30, 32

Principle: Determination of the gravitational constant from two successive positions of equilibrium of the torsion balance.

In DC 531.51; a, a description is given of the observation of the mutual attraction between two lead spheres and the determination of the gravitational constant by means of a dynamic method of measurement using a gravitation torsion balance. This procedure has the advantages of ease of observation and a short time of measurement; it does not provide, however, very accurate results. Here the torsion balance will be used for a more exact determination of the gravitational constant by the usual procedure of measuring the final deflections. To do this, some further information is necessary, especially about the torsional oscillations. The couple per unit angle of twist for the torsion fibre cannot be determined directly, but only from the equation for oscillation.

For the determination of the gravitational constant, observations are made of the initial and final equilibrium positions of the measuring system used with the torsion balance, as well as of the damped oscillatory motion between these positions (time of observation about 45 minutes).

The experiment is started by carefully checking the position of rest of the torsion balance. Then the larger spheres are placed in the diagonally opposite positions. The measuring system consequently proceeds after a few swings from a final position relative to that denoted as the position of rest to a new final position. Let the angle between these two final positions be α. The angle can be obtained from the dimensions of the arrangement and the deflections of the light spot, as shown in Fig. 1. The turning moment N acting on the measuring arrangement in a final position due to the attraction between the masses is given by $N = 2 \times F \times d$, if F denotes the force of attraction between each pair of spheres and d is the axial distance from the suspension fibre of the small spheres on the balance arm. This turning moment will attain an equilibrium by twisting the torsion fibre through an angle of $\frac{\alpha}{2}$. If the couple per unit angle of twist of the fibre is denoted by c, then this turning moment $N = c\,\frac{\alpha}{2}$.

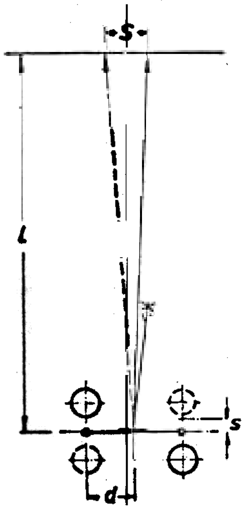


Fig. 1

Referring to Fig. 1, let:

- s = the deflection of the small spheres on the arm of the torsion balance,
- d = the separation of the small spheres from the axis of the suspension,
- S = the deflection of the light spot at the screen,
- L = the distance of the screen from the mirror of the balance.

On account of the doubling of the angle on reflection

$$\frac{s}{d} = \frac{S}{2L} = \tan \alpha.$$

The couple per unit angle of twist, c, can only be derived from the period of torsional oscillation, T, of the measuring system:

$$T^2 = 4\pi^2 \cdot \frac{I}{c}$$

or

$$c = \frac{4\pi^2 I}{T^2}.$$

The moment of inertia I involved here can be put equal to the moment of inertia of the two small spheres, i. e $I = 2\,m\,d^2$ because the moment of inertia of the suspension with the mirror is negligible, therefore,

$$c = \frac{8\pi^2 m\, d^2}{T^2}$$

Putting in the equation $N = c\,\frac{\alpha}{2}$, the quantity

$N = 2\,F\,d = 2\,G\,\frac{M\,m}{b^2}\,d$, where G is the gravitational constant, and the values for c and α obtained above, it follows that, since $2\,N = c\,\alpha$

$$4\,G\,\frac{M\,m}{b^2} = \frac{8\pi^2 m\, d}{T^2}\;\frac{S}{2L}$$

provided that α is small so that $\tan \alpha = \frac{S}{2L} = \alpha$.

From this is obtained

$$G = \frac{\pi^2 b^2 d\, S}{M\, T^2 L}.$$

This formula for the gravitational constant contains only measurable quantities. The mass of the small sphere does not appear in the result, so a knowledge of it is unnecessary.

In connection with the example given overleaf, the unavoidable systematic error with this arrangement is discussed together with its correction.

Apparatus required for the experiment:
(in order of assembly)

		Cat. No.
1	Stand base, 20 cm	300 01
1	Pair of adjusting screws	300 06
1	Stand rod, 60 cm, bent	300 52
2	Leybold bosses	301 01
1	Gravitation torsion balance	332 10
1	Lamp-house	450 60 a
1	Lamp, 6 V, 30 W	450 31
1	Single-lens condenser	460 17
1	Transformer, 6 V, 30 W	562 76
1	Stopclock	313 05
	or	
1	Large electric stopclock	313 04/08

Useful hints:

1. For the erection and adjustment of the gravitation torsion balance, see directions for use 332 10.
2. Before the commencement of a measurement, the null point must be observed for several minutes.
3. The placing of the spheres must be done carefully. The spheres should be prevented from knocking against the glass window of the apparatus.
4. After placing the spheres in position, the position of the light spot must be read first, as in the acceleration method, every 15 sec and, after 1 or 2 minutes, only every half or whole minute. A curve is obtained which is important in other connections: that of a damped oscillation. The period of oscillation can be determined on plotting the graph. Since the square of this period appears in the result, the average of several values must be taken.
5. Before the beginning as well as after the end of the experiment, the constancy of the final deflection is observed for some minutes. The correct motion of the torsion balance under consideration can be confirmed in this way.
6. Example:

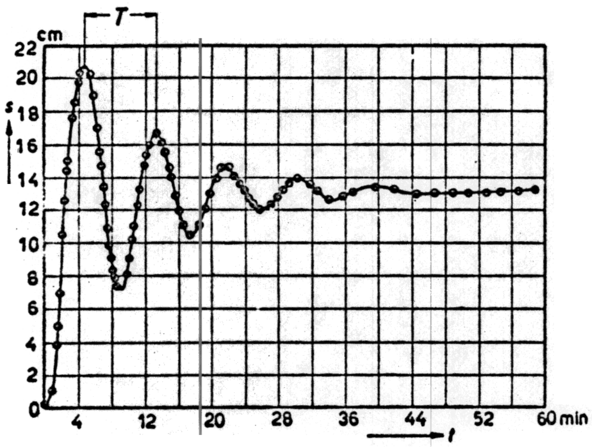


Fig. 2

Separation of spheres

Numerical values:

$b = 0{\cdot}045$ m $d = 0{\cdot}05$ m $S = 0{\cdot}13$ m
$M = 1{\cdot}46$ kg $T = 525$ sec $L = 1{\cdot}05$ m
1.501 kg 3.645 m

Result:

$$G = \frac{\pi^2\, b^2\, d\, S}{M\, T^2\, L} = 6{\cdot}39 \times 10^{-11}\ \frac{\mathrm{m}^3}{\mathrm{kg\ sec}^2}.$$

7. The value determined is subject to the following systematic error: The small sphere will also attract the distant large sphere with a very much smaller force (see Fig. 3).

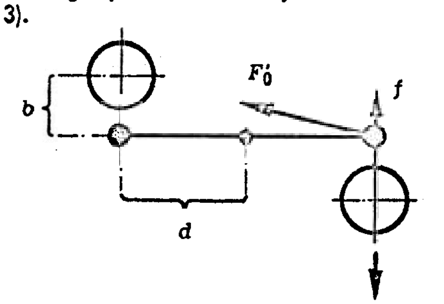


Fig. 3

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This force is given by the gravitational law

$$F_0' = G\,\frac{m\,M}{b^2 + 4\,d^2}$$

and has a component f opposite to that of the force F_0.

$$f = G\,\frac{m\,M}{b^2 + 4\,d^2}\,\frac{b}{\sqrt{b^2 + 4\,d^2}} = \beta\,F_0$$

if β is the fraction by which the measured force F_0 is too smal.

$$\beta = \frac{b^3}{(b^2 + 4\,d^2)\,\sqrt{b^2 + 4\,d^2}}.$$

With $d = 0{\cdot}05$ m and $b = 0{\cdot}045$ m, β becomes 0·069. The value of G found with this correction must therefore be increased by 6·9 %.

The value obtained is then $G = 6{\cdot}83 \times 10^{-11}\ \dfrac{\mathrm{m}^3}{\mathrm{kg\ sec}^2}$

instead of the exact value $G = 6{\cdot}67 \times 10^{-11}\ \dfrac{\mathrm{m}^3}{\mathrm{kg\ sec}^2}$.