Problem #7: A 1200-kg car traveling south at 24-m/s collides with and attaches itself to a 2000-kg truck traveling east at 16-m/s. Calculate the velocity (magnitude and direction) of the two vehicles when locked together after the collision. (Similar to Fishbane, Gasiorowicz and Thornton 1993, example 8-5)

**FOCUS the PROBLEM**

**Picture and Given Information**

![Diagram of collision with velocities and masses](image)

**Question(s)**

What is the velocity (magnitude and direction) of the two vehicles when locked together after the collision?

**Approach**

Use conservation of momentum. System: car and truck

**Time:** Initial time is the instant just prior to the collision.

**Final time** is the instant just after the collision.

Initial momentum due to car and truck separately.

Final momentum due to car/truck locked together.

Assume no momentum transfer between system and environment.

**DESCRIBE the PHYSICS**

**Diagram(s) and Define Quantities**

![Momentum diagram with states and variables](image)

**Target Quantity(ies)** $V_f, \theta$

**Quantitative Relationships**

\[ P_{fx} - P_{ix} = J_x = 0 \]

\[ P_{fy} - P_{iy} = J_y = 0 \]

\[ P_{ix} = m_t v_{txi} \quad \therefore p_{fx} - m_t v_{txi} = 0 \]

\[ P_{iy} = m_c v_{cyi} \quad \therefore p_{fy} - m_c v_{cyi} = 0 \]

\[ P_f = M_v_f \]

\[ \begin{align*}
M &= m_c + m_t = 3200 \text{ kg} \\
V_f &= ? \\
\theta &= ?
\end{align*} \]

**Components of** $P_f$

\[ P_{fx} = P_f \cos \theta \]

\[ P_{fy} = P_f \sin \theta \]

\[ \begin{align*}
&\text{Also: } P_f^2 = P_{fx}^2 + P_{fy}^2
\end{align*} \]
PLAN the SOLUTION
Construct Specific Equations

\[ v_f = Mv_f \quad (1) \]

\[ p_f = \sqrt{p_{fx}^2 + p_{fy}^2} \quad (2) \]

\[ p_{fx} - mTv_{txi} = 0 \quad (3) \]

\[ p_{fy} - m_cv_{cyi} = 0 \quad (4) \]

\[ \frac{p_f}{p^2} = \frac{mTv_{txi}}{Mv_f} \]

\[ V_f = \frac{\sqrt{\left(mTv_{txi}\right)^2 + \left(m_cv_{cyi}\right)^2}}{M} \]

Find \( \theta \)

\[ p_{fy} = p_f \sin \theta \quad (5) \]

\[ \sin \theta = \frac{p_{fy}}{p_f} \]

\[ \theta = \arcsin \left( \frac{p_{fy}}{p_f} \right) \]

\[ \theta = \arcsin \left( \frac{m_cv_{cyi}}{Mv_f} \right) \quad \text{result (A)} \]

\[ \theta = \arcsin \left( \frac{[kg][m/s]}{[kg][m/s]} \right) \quad \text{equation (1)} \]

Check Units

\[ V_f: \left[ \frac{[kg][m/s]}{[kg]} \right]^2 + \left[ \frac{[kg][m/s]}{[kg]} \right]^2 \]

\[ \left[ \frac{[kg]}{[kg]} \right] = \left[ \frac{[kg][m/s]}{[kg]} \right] = [m/s] \quad \text{OK} \]

EXECUTE the PLAN
Calculate Target Quantity(ies)

\[ V_f = \sqrt{\left(2000 \text{ kg}\right)^2 \left(6 \text{ m/s}\right)^2 + \left(1200 \text{ kg}\right)^2 \left(24 \text{ m/s}\right)^2} \]

\[ V_f = 13.5 \text{ m/s} \]

\[ \theta = \arcsin \left( \frac{1200 \text{ kg} \left(24 \text{ m/s}\right)}{3200 \text{ kg} \left(13.5 \text{ m/s}\right)} \right) = \arcsin (0.6) \]

\[ \theta = 42^\circ \text{ South of East} \]

EVALUATE the ANSWER
Is Answer Properly Stated?

Yes. As expected \( v_f \) has units of velocity and \( \theta \) is an angle.

Is Answer Unreasonable?

No. \( v_f \) is in the right ballpark and \( \theta \) is nearly 45\(^\circ\) as expected since the two initial perpendicular momenta are similar in size.

Is Answer Complete?

Yes. The magnitude and direction of the final velocity have been found which answers the question.

(extra space if needed)

\[ \theta = \arcsin \left( \frac{[kg][m/s]}{[kg][m/s]} \right) = \arcsin ([1]) \quad \text{OK} \]

continued

5-23
Problem #8: A billiard ball at rest is hit head-on by a second billiard ball moving 1.5 m/s toward the east. If the collision is elastic and we ignore rotational motion, calculate the final speed of each ball. (Based on Fishbane, Gasiorowicz and Thornton 1993, example 8-8)

FOCUS the PROBLEM
Picture and Given Information

\[ V_{i1} = 1.5 \text{ m/s} \quad V_{2i} = 0 \]
\[ \begin{array}{c}
\Rightarrow \quad \text{Ball 1} \\
\end{array} \quad \begin{array}{c}
\Rightarrow \quad \text{Ball 2} \\
\end{array} \]
\[ m_1 \quad m_2 \]

\[ V_{f1} = ? \quad V_{2f} = ? \]
\[ \begin{array}{c}
\Rightarrow \quad \text{Ball 1} \\
\end{array} \quad \begin{array}{c}
\Rightarrow \quad \text{Ball 2} \\
\end{array} \]
\[ m_1 \quad m_2 \]

Question(s) What is the final speed of each ball?

Approach Use conservation of momentum and conservation of energy.

System: ball 1 and ball 2

Time: Initial time is the instant before the collision.
Final time is the instant after the collision.

Initial mom. is due to ball 1. Final mom. is due to balls 1 and 2.
No net momentum transfer between system and environment.
Initial energy is kinetic. Final energy is kinetic.
No energy input or output. "Elastic" collision \( \Rightarrow \) no kinetic energy lost.
Assume the balls have the same mass.

DESCRIPT the PHYSICS
Diagram(s) and Define Quantities

\[
\begin{align*}
\text{momentum diagram} & \quad \text{energy diagram} \\
\begin{array}{c}
\text{Initial state} \\
\to \quad \text{Final state} \\
\end{array} & \quad \begin{array}{c}
\text{Initial state} \\
\to \quad \text{Final state} \\
\end{array} \\
\Rightarrow & \quad \Rightarrow \\
\begin{array}{c}
P_{i} \\
\to \quad P_{f} \\
\end{array} & \quad \begin{array}{c}
V_{i} \quad \Rightarrow \quad V_{f} \\
\to \quad \Rightarrow \\
\end{array} \\
+x & \quad +x \\
\end{align*}
\]

Relative sizes of final momenta unknown at this point.

Target Quantity(ies) \( V_{f1}, V_{2f} \)

Quantitative Relationships

\[
\begin{align*}
P_{f} - P_{i} & = J = 0 \\
P_{f} & = m_{1}V_{1f} + m_{2}V_{2f} \\
P_{i} & = m_{1}V_{1i} \\
\therefore \quad m_{1}V_{1f} + m_{2}V_{2f} - m_{1}V_{1i} & = 0 \\
5-24 & \Rightarrow V_{1f} + V_{2f} - V_{1i} = 0 \\
E_{f} - E_{i} & = 0 \\
E_{f} & = \frac{1}{2}m_{1}V_{1f}^{2} + \frac{1}{2}m_{2}V_{2f}^{2} \\
E_{i} & = \frac{1}{2}m_{1}V_{1i}^{2} \\
\therefore \quad \frac{1}{2}m_{1}V_{1f}^{2} + \frac{1}{2}m_{2}V_{2f}^{2} - \frac{1}{2}m_{1}V_{1i}^{2} & = 0 \\
\Rightarrow V_{1f}^{2} + V_{2f}^{2} - V_{1i}^{2} & = 0
\end{align*}
\]
PLAN the SOLUTION
Construct Specific Equations

Find $v_{1f}$

$\begin{align*}
    v_{1f} + v_{2f} - v_{i1} &= 0 \quad \text{(1)} \\
    v_{1f} &= v_{i1} - v_{2f} \quad \text{(A)}
\end{align*}$

Find $v_{2f}$

$\begin{align*}
    v_{1f}^2 + v_{2f}^2 - v_{i1}^2 &= 0 \quad \text{(2)} \\
    (v_{i1} - v_{2f})^2 + v_{2f}^2 - v_{i1}^2 &= 0 \\
    (v_{i1} - v_{2f})^2 &= v_{i1}^2 - v_{2f}^2 \\
    v_{2f} &= v_{i1} \quad \text{used result (A)}
\end{align*}$

$\begin{align*}
    \sqrt{v_{2f}^2} &= \sqrt{v_{i1}^2} \\
    v_{2f} &= v_{i1} \quad \text{assumed $v_{2f} \neq 0$}
\end{align*}$

$v_{1f} = v_{i1} - v_{i1}$

$\begin{align*}
    v_{1f} &= 0
\end{align*}$

EXECUTE the PLAN
Calculate Target Quantity(ies)

$\begin{align*}
    v_{2f} &= 1.5 \text{ m/s} \\
    v_{1f} &= 0
\end{align*}$

EVALUATE the ANSWER
Is Answer Properly Stated?
Yes. As expected $v_{2f}$ has units of velocity.

Is Answer Unreasonable?
No. All the momentum and kinetic energy get transferred from ball 1 to ball 2 in the collision.

Is Answer Complete?
Yes. $v_{1f}$ and $v_{2f}$ are the final speeds of the two balls. This answers the question.

Check Units

$[\frac{\text{m}}{\text{s}}]$ OK
Problem #9: An 80-g arrow moving at 80 m/s hits embeds itself in a 10-kg block resting on ice. How far does the block slide on the ice following the collision if it is opposed by a 9.2-N force? (Similar to Fishbane, Gasiorowicz and Thornton 1993, example 8-7)

**FOCUS the PROBLEM**

Picture and Given Information

![Diagram of the problem with initial and final states and the energy diagram](attachment:image.png)

**Question(s)** How far does the block slide before coming to rest?

**Approach** Use conservation of momentum to determine the recoil velocity of the block/arrow just after the collision. System: block & arrow. Then use conservation of energy to determine how far the block/arrow slides following the collision. Same system as above.

**Time:**
- Initial time is the instant just before arrow strikes block.
- Middle time is the instant just after the collision.
- Final time is the instant block/arrow comes to rest.

Just before collision momentum due to arrow. Assume arrow motion is strictly horizontal when it strikes the block.

Just after collision momentum due to block/arrow.

No net momentum transfer between system and environment.

Energy just after collision is kinetic. Final energy is zero. Input energy is zero. Output energy due to frictional force.

**DESCRIBE the PHYSICS**

Diagram(s) and Define Quantities

<table>
<thead>
<tr>
<th>(Initial state)</th>
<th>(Momentum Transfer)</th>
<th>(Final state)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_i )</td>
<td>( +x )</td>
<td>( p_f )</td>
</tr>
</tbody>
</table>

\( V_{0a} = 80 \text{ m/s} \)

\( m_a = 80 \text{ g} = 0.080 \text{ kg} \)

\( V_{0b} = 0 \)

\( m_b = 10 \text{ kg} \)

\( M = m_a + m_b = 10.08 \text{ kg} \)

\( x_1, x_2 \)

\( x_i = 0 \)

\( x_f = 9.2 \text{ N} \)

Target Quantity(ies) \( x_2 \)

Quantitative Relationships

\[ p_f - p_i = J = 0 \]

\[ p_f = M_v_i \]

\[ p_i = m_a V_{0a} \]

\[ M_v_i - m_a V_{0a} = 0 \]

\[ \frac{1}{2} M_v_i^2 = \int_{x_i}^{x_2} F \cdot dx \]

\[ \frac{1}{2} M_v_i^2 = \int_{x_i}^{x_2} F^2 \cdot dx \]

\[ E_f - E_i = W_{nc} \]

\[ E_i = \frac{1}{2} M v_i^2 \]

\[ W_{nc} = \int_{x_i}^{x_2} F \cdot dx \]
PLAN the SOLUTION
Construct Specific Equations

\[
\begin{align*}
\text{Find } x_2 & \\
\frac{1}{2} M v_1^2 &= \int_{x_1}^{x_2} F_f \cdot dx \\
\frac{1}{2} M v_i^2 &= \left[ F_f \cdot x \right]_{x_1}^{x_2} \\
\frac{1}{2} M v_i^2 &= F_f (x_2 - x_1) \rightarrow 0 \\
\text{Find } V_i & \\
M v_i - m_a v_{oa} &= 0 \quad (2) \\
M v_i &= m_a v_{oa} \\
V_i &= \frac{m_a v_{oa}}{M} \\
\frac{1}{2} M \left( \frac{m_a v_{oa}}{M} \right)^2 &= F_f x_2 \\
\frac{m_a^2 v_{oa}^2}{2M} &= F_f x_2 \\
\underbrace{\hfill x_2 = \frac{m_a^2 v_{oa}^2}{2M F_f}}_{\text{Answer}} \\
\end{align*}
\]

EXECUTE the PLAN
Calculate Target Quantity(ies)

\[
\begin{align*}
x_2 &= \frac{(0.080 \text{ kg})^2 (80 \text{ m/s})^2}{2(10.08 \text{ kg})(9.2 N)} \\
x_2 &= 0.22 \text{ m}
\end{align*}
\]

EVALUATE the ANSWER
Is Answer Properly Stated?
Yes. As expected \( x_2 \) has units of length.

Is Answer Unreasonable?
No. The block is quite massive so it doesn't slide back very far.

Is Answer Complete?
Yes. \( x_2 \) is the distance the block slides which answers the question.

Check Units
\[
\begin{align*}
\left( \frac{[\text{kg}] [\text{m/s}]^2}{[\text{kg}] [\text{N}]} \right)^2 &= \frac{[\text{kg}]^2 [\text{m/s}]^2}{[\text{kg}] [\text{kg} m/s^2]^2} \\
&= [\text{m}] \quad \text{OK}
\end{align*}
\]

(Extra space if needed)