1. (15 points.) Use the integral representation of $J_m(t)$,
\[ i^m J_m(t) = \int_0^{2\pi} \frac{d\alpha}{2\pi} e^{it\cos \alpha - im\alpha}, \]  
(1)

to derive the recurrence relation
\[ 2 \frac{d}{dt} J_m(t) = J_{m-1}(t) - J_{m+1}(t). \]  
(2)

2. (15 points.) The magnetic field of an infinitely long straight wire carrying a steady current $I$ is given by, (assume wire on $z$-axis,)
\[ B(r) = \hat{\phi} \frac{\mu_0 I}{2\pi \rho}, \]  
(3)

where $\rho = \sqrt{x^2 + y^2}$ is the closest distance of point $r$ from the wire. The Lorentz force on a particle of charge $q$ and mass $m$ is
\[ F = qE + qv \times B. \]  
(4)

In the absence of an electric field, qualitatively, describe the motion of a positive charge with an initial velocity in the $z$-direction. In particular, investigate if the particle will attain a speed in the $\phi$-direction. Thus answer whether the charge will go around the wire?

3. (20 points.) The Maxwell equations, in vacuum, when magnetic charges and currents are present, are given by
\[ \nabla \cdot \mathbf{E} = \frac{1}{\varepsilon_0} \rho_e, \quad -\nabla \times \mathbf{E} - \mu_0 \frac{\partial}{\partial t} \mathbf{H} = \mathbf{J}_m, \]  
(5a)
\[ \nabla \cdot \mathbf{H} = \frac{1}{\mu_0} \rho_m, \quad \nabla \times \mathbf{H} - \varepsilon_0 \frac{\partial}{\partial t} \mathbf{E} = \mathbf{J}_e. \]  
(5b)

Without introducing potentials, derive
\[ \left( -\nabla^2 + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \mathbf{E}(r,t) = -\frac{1}{\varepsilon_0} \nabla \rho_e(r,t) - \mu_0 \frac{\partial}{\partial t} \mathbf{J}_e(r,t) - \nabla \times \mathbf{J}_m(r,t). \]  
(6)
4. (20 points.) Evaluate the integral
\[ \int_{-\infty}^{\infty} dx \, g(x) \delta(a^2x^2 - b^2). \] (7)

Hint: Use the identity
\[ \delta(F(x)) = \sum_r \frac{\delta(x - a_r)}{|dF/dx|_{x=a_r}}, \] (8)
where the sum on \( r \) runs over the roots \( a_r \) of the equation \( F(x) = 0 \).

5. (15 points.) Consider the motion of a non-relativisitic particle (speed \( v \) small compared to speed of light \( c \), \( v \ll c \)) of charge \( q \) and mass \( m \). The charge moves on a circle described by
\[ \mathbf{r}(t) = \hat{i} A \cos \omega_0 t + \hat{j} A \sin \omega_0 t. \] (9)

Find the total radiated power
\[ P(t) = \frac{1}{4\pi\varepsilon_0} \frac{2q^2}{3c^3} \mathbf{a}^2(t_e), \] (10)
where \( \mathbf{a}(t_e) \) is the acceleration of the particle at the time of emission
\[ t_e = t - \frac{r}{c}. \] (11)

6. (15 points.) The spectral distribution of power radiated into a solid angle \( d\Omega = d\phi \sin \theta d\theta \) during Čerenkov radiation, when a particle of charge \( q \) moves with uniform speed \( v \) in a medium with index of refraction
\[ n = n_e n_\mu, \quad n_e = \sqrt{\varepsilon(\omega) \varepsilon_0}, \quad n_\mu = \sqrt{\mu(\omega) \mu_0}, \] (12)
is given by the expression
\[ \frac{\partial^2 P}{\partial \omega \partial \Omega} = \frac{1}{4\pi \varepsilon_0 n_e^2} \frac{q^2 \omega^2 n^2}{2\pi c} \left( \frac{v^2 n^2}{c^2} - 1 \right) \delta \left( \omega - \frac{\omega}{c} \frac{vn}{c} \cos \theta \right), \] (13)
where \( \omega \) is the frequency of light. Čerenkov light of a given frequency is emitted on a cone of half-angle \( \theta_c \). Determine the expression for \( \theta_c \). Show that for small \( \theta_c \),
\[ \theta_c \sim \sqrt{2 \left( 1 - \frac{c}{nv} \right)}. \] (14)