Physics 5583. Electrodynamics II.
Final Examination
Spring 2003

May 7, 2003

Instructions: This examination consists of three problems. If you get stuck on one part, assume a result and proceed onward. Do not hesitate to ask questions. GOOD LUCK! and have a great summer!

1. Consider a static magnetic field $\mathbf{B}$ in a region $V$ with magnetic permeability $\mu = 1$ and with no electric field present. Let $V$ be bounded by a closed superconducting wall $S$. Let the only electric currents be present on or outside the surface $S$. The only property of a superconductor needed is that, within the superconducting material, the magnetic field $\mathbf{B}$ is zero.

(a) Show that $\mathbf{B}$ may be derived from a magnetic scalar potential,

$$\mathbf{B} = -\nabla \phi,$$

at all points in $V$ interior to $S$.

(b) What differential equation does $\phi$ satisfy?

(c) Show that $\phi$ satisfies the boundary condition on the surface $S$

$$\mathbf{n} \cdot \nabla \phi \bigg|_S = \frac{\partial}{\partial n} \phi \bigg|_S = 0,$$

where $\mathbf{n}$ is the unit normal to $S$.

(d) Then show that $\mathbf{B} = \mathbf{0}$ in $V$. [Hint: This is a T mode.]
This is the basis of ideas of ways to create regions of very low magnetic fields in the laboratory.

2. Consider a nonrelativistic particle of charge $e$ moving in a circular orbit of radius $R$ and with angular velocity $\omega_0$.

(a) Calculate the dipole moment $d(t)$ of this charge distribution.

(b) Calculate the instantaneous power radiated at an angle $\phi$ above the plane of the orbit using the nonrelativistic Larmor formula,

$$\frac{dP}{d\Omega} = \frac{1}{4\pi c^3} |\mathbf{n} \times \mathbf{d}(t)|^2.$$  

(c) What is the above result when averaged over one cycle?

(d) Integrate this result over all angles $\phi$ and show that the result coincides with that calculated from

$$P = \frac{2}{3} \frac{1}{c^3} (\dot{d})^2.$$  

(e) Calculate the magnetic dipole moment for this current and show that there is no magnetic dipole radiation.

3. It is easy to see where the relativistic factor $(E/mc^2)^4$ comes from in the formula for the total power emitted in synchrotron radiation, that is, for the radiation produced by a particle of charge $e$ moving in a circular orbit of radius $R$ with speed $v = \omega_0 R$. Here $E$ is the total energy of the particle, and $m$ is its rest mass.

(a) Argue that because power $P = dE/dt$ is a relativistic invariant [why is this?] the generalization of the Larmor formula must be

$$P = \frac{2}{3} \frac{e^2}{c^3} \left( \frac{d^2 x^\lambda}{d\tau^2} \frac{d^2 x^\lambda}{d\tau^2} \right) = \frac{2}{3} \frac{e^2}{m^2 c^3} \left( \frac{dp^\lambda}{d\tau} \frac{dp^\lambda}{d\tau} \right),$$

where the four-vector describing the space-time position of the particle is

$$x^\lambda = (ct, \mathbf{x}), \quad x^\lambda = (-ct, \mathbf{x}), \quad \lambda \in \{0, 1, 2, 3\},$$

2
\( p^\lambda = (E/c, p) \) is the 4-vector momentum of the particle, and the proper time interval is

\[
d\tau = \sqrt{dt^2 - dx^2/c^2}.
\]

Show that if this is written in terms of the velocity \( v = dx/dt \) when \( |v| = v = \beta c \) is constant,

\[
P = \frac{2 e^2}{3 c^3} \frac{1}{(1 - \beta^2)^2} \left( \frac{dv}{dt} \right)^2.
\]

(b) Show that given the relation between \( \frac{dv}{dt} \) and \( v \) and \( \omega_0 \), the angular velocity of the particle in its circular orbit, we obtain

\[
P = \frac{2 e^2}{3} \frac{\omega_0 \beta^3}{R} \left( \frac{E}{mc^2} \right)^4.
\]