1. Compute the partition function $Z(\beta)$ for the classical one-dimensional harmonic oscillator defined by the Hamiltonian
\[ H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2. \]
Compare the result with that for the quantum harmonic oscillator discussed in class, in the high-temperature limit, $\beta \to 0$. Do they agree? What about Planck’s constant $\hbar$?

2. Consider a relativistic particle for which the Hamiltonian is
\[ H = c\sqrt{p^2 + m^2 c^2} - mc^2. \]
(a) Consider one-dimensional motion in an interval of length $L$. Find the partition function in the extreme relativistic limit, $p \gg mc$.
(b) Consider the same situation in general. Express your result in terms of the modified Bessel function,
\[ K_\nu(x) = \int_0^\infty e^{-z\cosh \theta} \cosh \nu \theta \, d\theta. \]
Look up the behavior of $K_\nu(z)$ for small $z$ to reproduce the result of part b.
(c) Consider a relativistic particle in three dimensions, where it is confined to a box of volume $V$. Find the partition function in the extreme relativistic limit.
(d) Repeat part c in general.

**Problems in Pathria:** 3.17, 3.18, 3.24, and 3.30.