1. Derive the result
\[
\frac{\Gamma\left(\frac{3N}{2}\right)}{\Gamma\left(\frac{3N-1}{2}\right)} \approx \left(\frac{3N}{2}\right)^{1/2}, \quad N \to \infty
\]
using the Stirling approximation for the Gamma function. What order in \(N\) is the correction to this leading asymptotic approximation?

2. Using the microcanonical ensemble, derive, for an ideal gas, the distribution function
\[P(p_1, p_2, p_3)\]
for the three Cartesian momentum components of a single molecule.

3. Starting from the expression for the volume of an \(n\)-dimensional sphere of radius \(r\),
\[V_n(r) = \frac{\pi^{n/2}}{\Gamma(n/2 + 1)} r^n,\]
calculate the difference between the volumes of two spheres of slightly different radii,
\[V_n(r) - V_n(r - t),\]
in the limit
\[n \gg 1, \quad \frac{t}{r} \ll \left(\frac{2}{n}\right)^{1/2} \ll 1.\]
Show that if \(n \sim 10^{22}\), \(t/r \sim 10^{-22}\), the shell so defined contains all but about 1/3 of the volume, and just a bit more would include it all!
4. Assume

\[ \rho(q,p) = \text{const.} \times \psi_E(H), \]

where \( \psi_E(H) \) is the characteristic function,

\[ \psi_E(H) = \begin{cases} 
1, & \text{if } E > H, \\
0, & \text{if } E < H,
\end{cases} \]

for the \( N \)-particle gas. According to Problem 3, nearly all the volume of the energy sphere lies very, very close to the surface. Show that the single momentum distribution \( \mathcal{P}(p_1) \) is still the Maxwell distribution.