Problems in Whittaker and Watson: Problems 1, 2, and 3, Chapter III, page 59.

1. For what range of positive values of \(x\) is
\[
\sum_{n=0}^{\infty} \frac{1}{1 + x^n}
\]
(a) Convergent?
(b) Uniformly convergent?

2. In numerical analysis it is often convenient to replace derivatives by finite differences. For example, we might represent the second derivative of a function as follows:
\[
\frac{d^2}{dx^2} \psi(x) \approx \frac{1}{h^2} [\psi(x + h) - 2\psi(x) + \psi(x - h)].
\]
Regarding \(h\) as a small parameter, find the error in this approximation.

3. Compute \(e\) from the Taylor series of \(e^x\) about 0 to 16 significant figures.

4. Given that
\[
\int_{0}^{1} \frac{dx}{1 + x^2} = \tan^{-1} x \bigg|_{0}^{1} = \frac{\pi}{4},
\]
expand the integrand into a series and integrate term by term, obtaining
\[
\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \cdots + (-1)^n \frac{1}{2n+1} + \cdots,
\]
which is Leibnitz’s formula for $\pi$ (actually discovered by Gregory in 1671). This formula converges so slowly that it is quite useless for numerical work: compute the first 100 terms and see for yourself!