Eleusis
The game of predictions

1 Equipment

- A deck of playing cards,
- Scoring sheets,
- a keen intellect.

2 Introduction

Part of studying science is learning induction, the art of reasoning from specific observations to general laws. Induction can be a complicated business, and even modern logicians argue about it. In this lab you will try a simple investigation of inductive reasoning by playing a card game called Eleusis.*

In most card games you know the rules which determine what cards can and cannot be played. In the game of Eleusis you are trying to determine the rules themselves!

3 The Meta-Rules

Eleusis is played by 4–7 people, with a deck of standard cards. (If you have more than four people you may need a second deck.) At the beginning of each hand, one person is designated the Rulemaker, and everyone else is referred to simply as a “player”. The role of Rulemaker rotates from hand to hand. A game consists of a set of hands where everyone has played the role of Rulemaker once.

The Rulemaker decides upon a rule for the hand and secretly records it on a piece of paper. The rule must decide whether a card played by a player is right or wrong based on any of the following properties:

1. the card’s suit (or color),
2. the card’s value,
3. the card’s position in the sequence (in relation to previously laid correct cards’ suit or value).

The Jack, Queen and King are counted as 11, 12 and 13, respectively. Also, a rule must cover all possibilities: any card laid down must be considered either right or wrong. A few examples of rules:

- Only even cards (counting the queen as even) are correct.
- The number of the card played must be one more or one less than the last correct card.
- The colors must alternate. (If the 2♣ was first laid down, the next card would be correct if it were a diamond or a heart, and incorrect if it were another club or spade.)

*This is a simplified version of the game. See section 6 below for references to the full rules.
The rule cannot depend on who laid the card, how he laid it, which deck the card came from, whether the card is aesthetically pleasing at the moment or not, etc. Only the suit, value, and position in the sequence determine its rightness or wrongness.

It is very easy to come up with more complicated rules. For example:

- If the last correct card is red, play an even number. If the last correct card is black, play an odd number.
- The cards must be the digits of $\pi$, starting after the decimal (ace four ace five . . .).
- The correct card equals the square root of the sum of the previous cards evaluated modulo 13.

These rules, while legal, will result in a low score for the Rulemaker (as will be explained below), as well as frustrated and annoyed players. Thus it is in the Rulemaker’s best interest to choose simple rules. It is strongly recommended that you stick to simpler rules in the beginning.

The Rulemaker places a single starter card down that obeys the rule. He or she then deals out twelve cards to each of the other players. (You may need more than one deck). Play proceeds clockwise from the Rulemaker. The player lays down a card which s/he thinks is correct. (This card is like a small “experiment” to test the rule.) After the card is laid down, the Rulemaker says whether the card was right or wrong. If the card is right, it is played in a horizontal line with the starter card, and the player wins a point. If the card is wrong, the card is placed vertically below the last correctly played card. The rulemaker does not play any cards after placing the starter; he or she simply states if the other cards played are correct or incorrect. The game ends after 10 such rounds.

Below is a sample of a game. The Rulemaker used the $A\heartsuit$ as a starter. The first player tried the $K\clubsuit$, which was correct. The next player tried the $4\diamondsuit$, which was incorrect, and placed below the line of correct cards. The third tried the $8\diamondsuit$, which was also incorrect. The fourth tried the $3\diamondsuit$ which was correct, and placed in line with the correct cards.

Starter

\[
A\heartsuit \quad K\clubsuit \quad 3\heartsuit \quad 5\spadesuit \quad 9\spadesuit \quad \Leftarrow \text{Correct Cards}
\]

Incorrect Card ⇒ \[
4\diamondsuit \quad 4\spadesuit
\]

\[
8\diamondsuit
\]
Can you guess the rule that generated the above sequence of cards? It is given at the bottom of the page. (Try before looking)†

It may arise that the player thinks that they have no card in their hand which they can play. In this case they declare this as their “play”. The Rulemaker then looks at their hand. If the player is correct, they score a single point, discard their entire hand and place it below the last correct card, and draw a new hand with one fewer cards. If the players is incorrect, the Rulemaker selects from their hand a playable card, and adds it to the string of correct cards. The player has two points subtracted from their score as a penalty.

You may have realized that one of the goals of the game is for the players to figure out the rule. Once someone thinks they have done that, they may declare discovery. You can declare discovery at any time, regardless of whose turn it is. You do not have to state your rule when declaring discovery. The discoverer is given the honorary title of the Landau.‡ A marker (such as a coin) is put on the card where the discovery was declared, and the Landau puts down his cards; they will not be used again this hand. Only one player can be Landau at a time.

The game continues as before, except that the Landau says whether a card is right or not, and the Rulemaker says whether the Landau is right or not. So long as the Landau is right, the game continues; but when the Landau is wrong, he or she is demoted to the status of Lab Rat. The card on which the player fell from grace is marked, and the discredited player waits until everyone else is done with the hand. As soon as one Landau is deposed, another player may declare discovery. If all the players are turned into Lab Rats, the hand is over.

Play continues until the players are out of cards. The Rulemaker then reveals the rule. The scores are computed from the scoresheet, with the added rules below.

It is best to keep a running score on the tally sheets provided. As each player places a card, the player (and the Landau, if there is one) is given the appropriate score.

Normal players score one point for every correctly played card or declared “no play”.

The Landau scores one point for every correct card before announcing discovery, and one point for every correct ruling after their discovery.

Lab Rats score as The Landau up until their discovery is proved fraudulent. A penalty of 5 points is subtracted from this total due to the besmirching of their reputation.

The Rulemaker scores twice the difference between the highest and lowest scores of the other players. Thus it the Rulemaker does well when at least one player determines the rule and at least one other player does not.

Below is an example of a section of a scoresheet from a game played by Abigail, Berthold, Carruthers, Dillsworth, and Elspeth, or A, B, C, D and E. A is the rulemaker for this hand. She writes down the rule, and places the A♥ as the starter card.

†The rule is: “Alternating red and black cards, all odd”.
‡The title is named after Lev Davidovich Landau, one of the most brilliant physicists of the 20th century. Landau was famous for his annoying habit of making brief, elegant arguments that were always correct.
On the first turn B guesses the 3♦, which is wrong. C guesses the Q♣ which is correct. D guesses the 4♦, and E guesses the 8♦ which are both wrong.

On the second turn poor B guesses the 4♠ which is wrong. C guesses 3♥ and is correct. D guesses the 6♠ and is also correct. E guesses the 4♣ and is wrong.

On the third turn, B plays the 9♥ and is correct. Boldly he states that he has discovered the rule. He now sets down his cards, and does not play them for the rest of the hand. C plays the 8♠ and B claims this is correct. A agrees, so both B and C get a point. Next, D plays a 2♣, which B claims is incorrect. A agrees and B gets one point. Finally, E plays 9♣, which B claims is incorrect. A says this is wrong, and B is demoted to the status of "lab rat". E gets the point for her correct play.
4 Procedure

Split up into teams of 4-6 players. Play enough hands so that every player has been rulemaker at least once.

In the last half-hour of class, the laboratory TA will act as the rulemaker, and each team will cooperate as a single player. Play as many hands as time allows, rotating which team guesses first. The highest scoring team by the end of class will not have to turn in written answers to the questions below. Everyone else in the lab will have to write up answers and turn them in by the start of the next lab.

5 Questions

Answer the following essay questions in complete sentences. It will take at least four sentences for each question. Your answers must be neat and legible. Incomprehensible, messy or fragmentary answers will receive little or no credit.

1. Did you ever discover a rule simpler than the Rulemaker’s, and was very often right? Does this ever happen in scientific research? Give an example.

2. Why are the time of day, the gender of the player, the room temperature or the mood of the Rulemaker not good quantities to be considered in writing rules? Write at length and in detail on how this relates to tests of physical laws.

3. Give at least three ways in which the game of Eleusis is a model that resembles scientific research. Give at least three ways in which it fails to model research.

4. One of the fundamental assumptions in Eleusis is: there is a rule that determines if a card is correct or not. What is the corresponding assumption in scientific research? How justifiable is this assumption?

6 Supplemental Information

Eleusis can be found in the June 1959 issue of Scientific American in the “Mathematical Games” department by Martin Gardner. An updated version appeared in the October 1977 issue in the same department. You can also find it in a chapter of Abbott’s New Card Games by Robert Abbott, and in a book titled New Rules for Old Games by the same author.

Other versions of the game call for players to pick up a card when they are incorrect; the game continues until one player uses up all of their own cards. In this version you can also play a run of several cards at once, with a large penalty if you are incorrect. These versions tend to take much longer to play. However, you are encouraged to experiment with the meta-rules. After all - experimenting with rules is the whole point of this particular lab!