N.B. Problems 3 and 4 have relativistic decays and collisions.

1). A mass $m$ moving horizontally with velocity $v_0$ strikes a pendulum of mass $m$.
   
   (a) If the masses scatter elastically along the line of the initial motion, find the resulting maximum height of the pendulum.
   
   (b) If two masses stick together, find the maximum height reached by the pendulum.

2). A mass $3m$ moves with velocity $2v_0 \hat{z}$ and overtakes a mass $m$ moving with velocity $v_0 \hat{z}$. The masses collide with a coefficient of restitution $e = 1/2$ and in the CM frame both leave the collision in the $\hat{x}$ direction. Make a Galilean transformation to determine the velocities in the CM frame before and after the collision. Find the final velocity of $3m$ in the original coordinate system.

3). Particle $A$ at rest, decays into particles $B$ and $C$ ($A \rightarrow B + C$).
   
   (a) Find the energy of the outgoing particles ($B$ and $C$) in terms of the various masses ($m_A, m_B,$ and $m_C$).
   
   (b) Find the magnitudes of the outgoing momenta ($|\vec{p}_B|$ and $|\vec{p}_C|$).

4). Let us consider a two-body scattering event, $A + B \rightarrow C + D$ with $B$ initially at rest. It is convenient to introduce the Mandelstam variables

$$s \equiv (p_A + p_B)^2,$$
$$t \equiv (p_B - p_C)^2,$$
$$u \equiv (p_A - p_C)^2.$$ 

(a) Show that $s + t + u = (m_A^2 + m_B^2 + m_C^2 + m_D^2)c^2$.

(b) Find the Lab energy of $A$ ($E_A^{\text{Lab}}$).

(c) Find the CM energy of $A$ ($E_A$).

(d) Find the total CM energy $E = E_A + E_B = E_C + E_D$.

In (b), (c) and (d), express $E_A^{\text{Lab}}, E_A,$ and $E$ in terms of $s, t, u$ and the masses.