1) A delta-function potential is placed in the center of the infinite square well of width $a$ centered on the origin.

$$H' = \alpha \delta(x - a/2)$$

where $\alpha$ is a constant.

a) First the first order correction to the allowed energies.
b) Find the second order correction to the energies.

2) Calculate the second order correction to the ground state energy for the harmonic oscillator where

a) $k \rightarrow (1 + c)k$, (i.e. spring cools so is less flexible)
b) $H' = -qEx$ (i.e. charged particle in an electric field)

3) An electron is in a 1D simple harmonic oscillator potential of frequency $\omega$. For this problem, use atomic units ($\hbar = m_e = e = 1$).

At first, the electron is in a state that is an equal superposition of the ground state and the second excited state. Thus, the wavefunction is given by:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} |\Psi_0\rangle + \frac{1}{\sqrt{2}} |\Psi_2\rangle$$

a) You make a measurement and find the electron in the ground state. You then adiabatically apply a perturbation to the system given by $H' = ax^4$. To first order, what must $\alpha$ be such that the ground state energy in the new system (simple harmonic oscillator plus perturbation) is equal to the energy of the second excited state in the unperturbed system?
b) You turn off the perturbation instantly. Using perturbed wavefunctions corrected to first order, what is the probability that the electron is now in the second excited state of the original simple harmonic oscillator potential.