1) A particle of mass \( m \) and kinetic energy \( E > 0 \) approaches an abrupt potential drop \( V_0 \).

What is the probability that it will reflect back if \( E = V_0 / 8 \)?

2) a) Show that in general, the solutions to Schrodinger's Equation for different energy eigenvalues are orthonormal, i.e.

\[
\int_{-\infty}^{\infty} u_m^* u_n dx = \delta_{nm}
\]

where \( u_m \) and \( u_n \) are eigenfunctions of the Hamiltonian and \( \delta_{nm} \) is the kronecker delta function. (Assume the potential is real)

b) If \( \Psi(x) = \sum_n a_n u_n \) show \( a_n = \int_{-\infty}^{\infty} \Psi(x) u_n^* dx \)

3) A particle is in a well with \( V(x) = \infty \) for \( |x| > L \) and \( V(x) = 0 \) otherwise.

a) Find both the even and odd eigenfunctions and eigenenergies for this system.

b) The particle in the well is in the ground state. Suddenly the potential is expanded to that \( V(x) = \infty \) for \( |x| > L' \) and \( V(x) = 0 \) otherwise \( (L' > L) \).

What is the probability of the particle to have energy \( E_n \). (Write down, but do not solve the integral)

4) Suppose you wanted to describe an unstable particle that spontaneously disintegrates with a lifetime \( \tau \). In that case the total probability of finding the particle somewhere should not be constant, but should decrease at an exponential rate.

\[
P(t) = \int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = e^{-t/\tau}
\]

A crude way of achieving this results is to assign an imaginary part to the potential. \( V = V_o - i \Gamma \), where \( V_o \) is the true potential energy and \( \Gamma \) is a positive real constant.

a) Show that \( \frac{dP}{dt} = -2\Gamma P \)

b) Solve for \( P(t) \) and find the lifetime of the particle in terms of \( \Gamma \).

5) In the ground state of the harmonic oscillator, what is the probability (correct to three significant digits) of finding the particle outside the classically allowed region?