Physics 3803
Homework Assignment 7
Due March 10 at 5:00 pm

Problems:

1) Prove the following theorem: If \( V(x) \) is an even function [i.e. \( V(-x) = V(x) \)] then \( \Psi(x) \) can always be taken to be either even or odd. Hint. If \( \Psi(x) \) satisfies the time-independent Schrodinger equation for a given \( E \), so too does \( \Psi(-x) \), and hence also the even and odd linear combinations \( \Psi(x) \pm \Psi(-x) \)

2) Consider the double delta-function potential
\[
V(x) = -\alpha[\delta(x + a) + \delta(x - a)]
\]
where \( \alpha \) and \( a \) are positive constants.

a) Sketch this potential

b) How many bound states does it possess? Find the allowed energies, for \( \alpha = \hbar^2/ma \) and for \( \alpha = \hbar^2/4ma \) and sketch the wave functions.

3) Find the transmission coefficient for the potential in problem 2.

4) Calculate the momentum representation of the energy eigenstates of the infinitely deep square well potential. For simplicity use the asymmetric well defined as: \( V(x) = 0 \) \((0 \leq x \leq L)\); \( V(x) = \infty \) (otherwise). Discuss the meaning of the eigenstates in the momentum representation; do they make physical sense; how do they compare to the classical version of this problem.

5) The position representation of the momentum eigenstates is
\[
\Psi(x) = \frac{1}{\sqrt{2\pi\hbar}}e^{-ipx/\hbar}
\]
Calculate the momentum representation of the momentum eigenstates. Discuss the physical meaning of the eigenstates in this representation, do they make sense when compared to the position representation.