Physics 3803
Homework Assignment 6
Due March 3 at 5:00 pm

Problems:

1) If $[A,B]=C$ and $\lambda << 1$, show $e^{\lambda(A+B)}=e^{\lambda A}e^{\lambda B}e^{-0.5\lambda^2 C}$

2) Consider the 2-D asymmetric box with $V(x)=0$ $(0 \leq x \leq a)$ $V(y)=0$ $(0 \leq y \leq b)$, $V(x)=V(y)=\infty$ otherwise.

a) By solving Schrodinger’s Equation: $\frac{-\hbar^2}{2m} \nabla^2 \Psi + V \Psi = E \Psi$, show the allowed energy states of the 2-D box are:
$E = \left[ \frac{n_1^2 \pi^2}{a^2} + \frac{n_2^2 \pi^2}{b^2} \right] \frac{\hbar^2}{2m}$, $n_1, n_2 = 1, 2, 3, ...$ Assume $\Psi(x,y)$ is separable, that is $\Psi(x,y) = X(x)Y(y)$.

b) If $a=b$ what values of $n$ and $r$ correspond to an energy of $50E_o$ where $E_o = \frac{\hbar^2 \pi^2}{2ma^2}$.

3) Determine the form of the position operator in momentum space. Show all work.

4) 1-D free particle.

a) A particle of mass $m$ moves in one dimension $(x)$. It is known that the momentum of the particle is $\hbar k_o$, where $k_o$ is a known constant. What is the time-independent (unnormalized) wavefunction for the particle, $\Psi_o$?

b) The particle interacts with a system. After interaction it is known that the probability of measuring the momentum of the particle is $1/5$ for $p=2\hbar k_o$ and $4/5$ for $p=4\hbar k_o$. What is the time independent wavefunction (unnormalized) in this state, $\Psi_b$?

c) What is the average momentum of the particle in state $\Psi_b$?

d) What is the particle’s average kinetic energy in the state $\Psi_b$ (Express your answer in terms of the constant $E_o=\hbar^2 k_o^2/2m$.

5) A free particle moving in one dimension is in the state

$\Psi(x) = \int_{-\infty}^{\infty} \sin(ak)e^{-\frac{(ak)^2}{2}}e^{ikx}dk$

a) What values of momentum will not be found?

b) If the momentum of the particle in this state is measured, in which momentum state is the particle most likely to be found?

c) If $a=2.1$ angstrom and the particle is an electron, what value of energy (in eV) will measurement find in the state described in part b?

6) Show that if $[\hat{A},\hat{B}]=i\hat{C}$ then $\Delta A \Delta B \geq \frac{1}{2} |\langle C | (C)|$ (See problem 5.42 in Liboff)