Additional Problem for Assignment 3
Electrodynamics II
Spring 2004

Due February 13, 2004

1. Spherical harmonics are defined by (20.23), so show that

\[
\frac{\mathbf{a} \cdot \mathbf{r}}{l+1} \sum_{m=-l}^{l} \psi_{lm} \sqrt{\frac{4\pi}{2l+1}} r^{l} Y_{lm}(\theta, \phi)
\]

\[
= \sum_{m=-l-1}^{l+1} \psi_{l+1,m} \sqrt{\frac{4\pi}{2(l+1)+1}} r^{l+1} Y_{l+1,m}(\theta, \phi),
\]

where \((\mathbf{a} \cdot \mathbf{r})\) is given by (20.20). Write out the explicit constructions this gives for the spherical harmonics of degree \(l+1\) in terms of those of degree \(l\). [As a hint, one of the three terms in the answer is

\[
\sqrt{\frac{2l+3}{2l+1}} \sqrt{(l+1+m)(l+1-m)} \cos \theta Y_{lm}(\theta, \phi)\].
\]

Begin with \(Y_{00} = 1/\sqrt{4\pi}\) and construct the three \(Y_{1m}\), and then find one or more of the five \(Y_{2m}\).

Problems in Chapter 20: 2, 3, 4, 5, 6, 7