Chapter 7 Lecture Notes
Physics 2414 - Strauss

Formulas:
$$\mathbf{p} = m\mathbf{v}$$
$$\Sigma \mathbf{F} = \Delta \mathbf{p}/\Delta t$$
$$\mathbf{F}\Delta t = \Delta \mathbf{p}$$
$$\Sigma \mathbf{p}_i = \Sigma \mathbf{p}_f$$
$$x_{CM} = (\Sigma mx)/\Sigma m, \quad y_{CM} = (\Sigma my)/\Sigma m$$

Main Ideas:
1. Momentum and Impulse
2. Conservation of Momentum.
3. Elastic Collisions
4. Inelastic Collisions
5. Center of Mass and motion.

1. Momentum and Impulse.

Last chapter we talked about the fundamental law of the conservation of energy. Energy is always conserved in any process. This chapter will introduce another conservation law (which is not applicable in all circumstances, but in many important ones). That is the conservation of linear momentum. First we have to define what linear momentum is.

$$\boxed{\mathbf{p} = m\mathbf{v}}$$

Linear momentum ($\mathbf{p}$) is defined as the mass of an object times the velocity of that object. It is a vector, having the same direction as the velocity of the object. We use momentum in this way in everyday language. A 300 lb lineman has more momentum than a 180 lb flanker when they are both moving at the same velocity because the lineman has more mass. A car going 60 mph has more momentum than one going 10 mph because it has a higher velocity.

Now consider Newton's second law:

$$\Sigma \mathbf{F} = m\mathbf{a} = m\Delta\mathbf{v}/\Delta t = m(v_2 - v_1)/\Delta t = (mv_2 - mv_1)/\Delta t = (\mathbf{p}_2 - \mathbf{p}_1)/\Delta t$$

$$\boxed{\Sigma \mathbf{F} = \Delta \mathbf{p}/\Delta t}$$

So Newton's second law can be restated as **the rate of change of momentum of a body is proportional to the net force applied to it.** In fact, this way of stating Newton's law as the change in momentum is how Newton originally formulated his second law.

I can rewrite this equation as $\Sigma\mathbf{F}\Delta t = \Delta\mathbf{p}$.

In physics you should understand that the equations mean something. This equation says that the net force acting on an object over a period of time (Δt) produces a change in momentum ($\Delta\mathbf{p}$). Often when the momentum of an object is changed, it is because there was a very large force acting on the object for a very brief time or the force of impact is in a different direction from other forces acting on an object. This happens when you hit a ball, or bounce a ball, or hammer a nail, or in many collisions. In such cases, we neglect all forces except the major force changing the momentum and write (dropping the "summation"),

$$\boxed{\mathbf{F}\Delta t = \Delta\mathbf{p}}$$

where the quantity $\mathbf{F}$ is the major force changing the momentum, and $\mathbf{F}\Delta t$ is called the **impulse.** It is the quantity which gives the impetus necessary to change the momentum.

Problem: A 50-g golf ball is struck with a club. The ball is deformed by about 2.0 cm during the time of collision and the ball leaves the club face with a velocity of 44 m/s (a) What is the impulse during collision? (b) How long is the collision? (c) What is the average force during the collision?

Problem: A 100-g ball is dropped from 2.00 m above the ground. It rebounds to a height of 1.50 m. What was the average force exerted by the floor if the ball was in contact with the floor for 1.00×10^{-2} s.

2. Conservation of Momentum

One of the most important concepts in understanding conservation of momentum is to understand what we mean by a **system.** A system will be defined by the person trying to solve the problem or look at the situation.

Suppose we look at a collection of objects that we want to define as a system. If we have two billiard balls, that could be the system. A ball bouncing off the floor could consist of the ball and the floor as the "system," or I could define just the ball itself as the system. Let's define two classes of forces. The first are forces **internal** to the system. These are forces between only objects that are in our defined system. For example, if our system consists only of two balls, the only forces internal to the system would be the equal and opposite force between the two balls when they collide or the force of gravity *between* the two balls. Other forces, like the force of gravity from the earth, or the normal force of the table on the balls are **external** to the system. That is they require agents which

are not a part of the system. They require the earth to make gravity, or the table to impart the forces to the system.

If there are no external forces, we say that the system is **isolated.** In many cases where a collision occurs, the internal forces are much greater than the external forces. If I hit a baseball or two cars collide, the internal forces between the ball and the bat, or between the two cars, are much greater over the very brief period of time that the collision took place, than the external forces like gravity. In such a case the system **acts as if it is isolated** during the collision. So we approximate collisions as isolated systems, and treat them as isolated systems.

A major question, then is what happens in an isolated (or nearly isolated) collision?

Look at two balls colliding

<div style="text-align:center">Before collision: After collision</div>

$$m_1 v_{1i} \qquad\qquad m_2 v_{2i} \qquad\qquad\qquad m_1 v_{1f} \qquad\qquad m_2 v_{2f}$$

Looking at ball 1   and at ball 2

$$F_1 \Delta t = \Delta p_1 = m_1 v_{1f} - m_1 v_{1i} \qquad\qquad F_2 \Delta t = \Delta p_2 = m_2 v_{2f} - m_2 v_{2i}$$

where F_1 and F_2 are the force of ball 2 on ball 1 and the force of ball 1 on ball 2, respectively, during the collision.

From Newton's third law we know that the force of ball 2 on ball 1 (F_1) is equal to the force of ball 1 on ball 2 ($-F_2$), and the time of the collision is the same for each ball, so we set the two equations above equal to each other with a minus sign ($F_1 \Delta t = -F_2 \Delta t$).

This leads to two important results. First, when two objects collide, the magnitude of the *change* in momentum for each object is exactly the same:
$$\Delta p_1 = -\Delta p_2$$

Second, when we set the two equations equal to each other we get

$$m_1 v_{1f} - m_1 v_{1i} = -(m_2 v_{2f} - m_2 v_{2i}) \quad \text{or} \quad m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

which generalizes for any number of objects to be

$$\boxed{\Sigma \mathbf{p}_i = \Sigma \mathbf{p}_f}$$

which is the law of the conservation of momentum. It states:

The total momentum of an isolated system of bodies remains constant.

Problem: A 90-kg fullback attempts to dive over the goal line with a velocity of 6.00 m/s. He is met at the goal line by a 110-kg linebacker moving at 4.00 m/s in the opposite direction. The linebacker holds on to the fullback. Does the fullback cross the goal line?

Since $\mathbf{p}_i = \mathbf{p}_f$ is a vector equation, if we have a collision in two dimensions, we must conserve momentum in each direction. So the conservation of momentum can be written as two equations $p_{xi} = p_{xf}$ and $p_{yi} = p_{yf}$. We must make the momentum initially and finally equal in both the x direction, and the y direction.

Problem: A firecracker weighing 100 g, initially at rest, explodes into 3 parts. One part with a mass of 25 g moves along the x axis at 75 m/s. One part with mass of 34 g moves along the y axis at 52 m/s. What is the velocity of the third part?

3. Elastic Collisions

In a certain class of collisions, both the momentum and the kinetic energy are conserved. That is, the **total** momentum **and** the **total** kinetic energy are the same before and after the collision. **The only collisions that conserve kinetic energy are elastic collisions. All collisions that can be considered isolated conserve momentum.** Remember that momentum is a vector, so the total momentum must be summed like a vector. Kinetic energy is a scalar, so the total energy is summed like regular numbers. An example of a nearly elastic collision is a super ball.

Problem: A ball with a mass of 1.2 kg moving to the right at 2.0 m/s collides with a ball of mass 1.8 kg moving at 1.5 m/s to the left. If the collision is an elastic collision, what are the velocities of the balls after the collision?

4. Inelastic Collisions

An **inelastic collision** is one in which kinetic energy is not conserved. A collision is called a **completely inelastic collision** if the objects stick together after colliding. In an inelastic collision, kinetic energy is not conserved. However, linear momentum is conserved even in inelastic collisions.

Demonstration/Problem: The ballistic pendulum is used to determine the velocity of a projectile. Suppose we have a projectile with mass m and velocity v_1, and a pendulum with mass M. The projectile hits the pendulum and sticks to it, so that the projectile and pendulum have a mass $(m + M)$ and final velocity v. The

pendulum then rises a distance h from its original position. From conservation of momentum during the collision we know that

$$mv_1 = (m+M)v$$

After the collision, then mechanical energy is conserved and we have
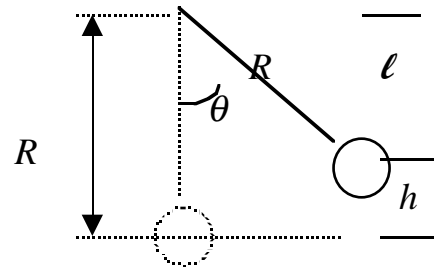$K_i = U_f$
$(1/2)(m+M) \, v^2 = (m+M)gh$
$(1/2) \, m^2 v_1^2/(m+M) = (m+M)gh$
$v_1 = \sqrt{2gh} \, (m+M)/m$

Now h is not easy to measure but it is given by $h+\ell = R$
$\ell/R = \cos\theta \Rightarrow \ell = R\cos\theta$
$h = R - R\cos\theta$

$v_1 = \sqrt{2gR \, (1-\cos\theta)} \, (m+M)/m$

If the gun is horizontal, then from kinematic equations we know
$y = v_{yi}t - (1/2)gt^2$
$y = -(1/2)gt^2$
$t = \sqrt{2y/g}$
$x = v_1 t$
$\quad = 2\sqrt{yR \, (1-\cos\theta)} \, (m+M)/m$

5. Center of Mass

We have talked about the motion of an object a lot in this class. We have usually used simple objects and described their movement. But sometimes the movement of an object is not so simple, (like a spinning, wobbling tossed frisbee). How do we describe the motion of that object. The motion of most of the points on the object is quite complicated. However, there is one point of the object which behaves in the same way as a single particle would move subject to the same forces. That point of the body is called the center of mass. Even with rotating, and spinning, the center of mass moves in the same way that a single particle would move. That is, the same way as all of

We find the center of mass of an object along a certain axis by using the equation
$x_{CM} = (\Sigma mx)/\Sigma m$ or $y_{CM} = (\Sigma my)/\Sigma m$.

<u>Problem:</u>   If the mass distribution of a person sitting down with his legs outstretched can be approximated by the two rectangles, where is the person's center of mass?

10 cm

65 kg

25 kg

45 cm

6 cm

20 cm 40 cm
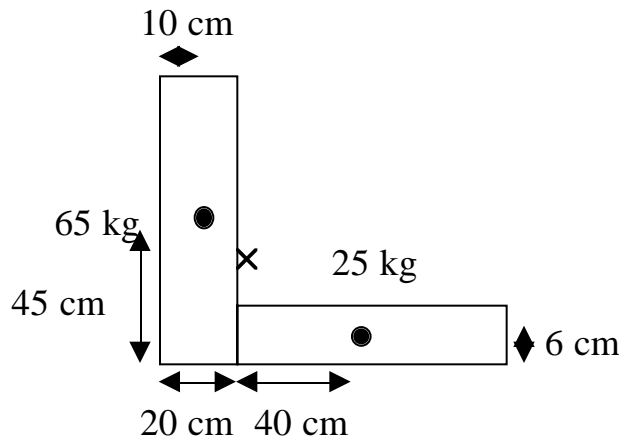
Note a few things about this example.
1) The CM may be outside the body.
2) The CM depends on the shape of the body.
If this person stands up, the center of mass will change location relative to his body. In fact, it will then be somewhere in his torso.

The high jumper described in the book can actually clear the bar without the center of mass clearing the bar.   A basketball player or gymnast in flight has their center of mass follow the same path that a single particle would follow (parabolic near the earth if we neglect air resistance) although their extremities may be changing direction, and they may be twisting and tumbling.

This means that even if there are internal force, like muscles pulling, the motion of the center of mass of the system is governed by the external forces only. Suppose, I shoot fireworks up in the air and they explode. What is the motion of the center of mass of all the particles? What is the motion of each particle? Look at the rocket example in the book.

Case where $m_{II} = m_I$

m_I

m_{II}

D

D

D
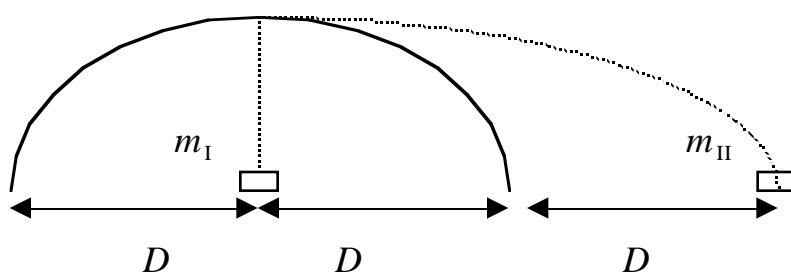
<u>Problem:</u> Suppose that $m_{II} = 3m_I$. Where would m_I land?

m_I would still land straight down, and now the CM must be at $2D$, so

Note that every one of the individual objects, as well as the center of mass of all the objects follow a parabolic path. (A straight line down is also a type of parabola).