Chapter 5 Lecture Notes
Physics 2414 - Strauss

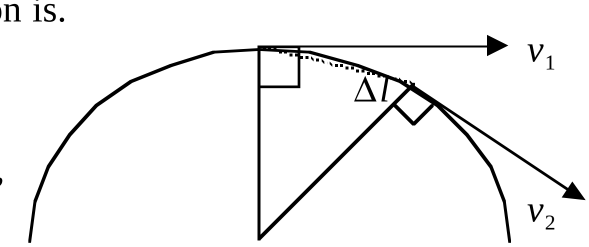
Formulas: $a_C = v^2/r$
$\mathbf{a} = \mathbf{a}_C + \mathbf{a}_T$
$F = Gm_1m_2/r^2$

Constants: $G = 6.67 \times 10^{-11}$ N-m^2/kg^2.

<u>Main Ideas:</u>
1. Uniform circular motion
2. Nonuniform circular motion
3. Universal Gravitation
4. Kepler's Laws

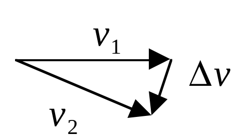
1. Uniform Circular Motion

We have talked about motion where the acceleration is either zero or is a constant magnitude in a constant direction. We will now talk about cases where the acceleration is a constant magnitude and always directed perpendicular to the direction of motion. This leads to uniform circular motion as we shall see. **Uniform circular motion is the motion of an object traveling at a constant (uniform) instantaneous speed on a circular path.** Recall that we said that velocity is a vector, so even if an object has a constant speed, but is changing direction it is accelerating. Something following a uniform circular path is always changing direction, so it is always accelerating. Let's see what the magnitude and direction of its acceleration is.

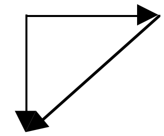


$\mathbf{a} = (\mathbf{v}_1 - \mathbf{v}_2)/\Delta t = \Delta\mathbf{v}/\Delta t$

$v_1 = v_2$ and $r = r$ and angles are the same, so triangles are the same and
$\Delta v/v = \Delta l/r$
$\Delta v = v\Delta l/r$
$\Delta v/\Delta t = (v/r)\Delta l/\Delta t = (v/r)v = v^2/r$
$a_C = v^2/r$



(The book describes one way of getting the direction. Also, the direction can be seen by looking at when $\Delta\mathbf{v}$ changes by 90°.) The direction is always toward the center of the circle.



This is called the **centripetal acceleration (center seeking)** and often given the symbol a_C**.** It is the acceleration whose direction is toward the center of the circle, and whose magnitude is equal to v^2/r. What would happen if the object did not have this acceleration? (It would go straight by Newton's first law). So everything moving at a uniform speed in a circle is accelerating by this amount. Note that the acceleration is not constant. The magnitude of the acceleration is constant, but its direction is changing and acceleration is a vector.

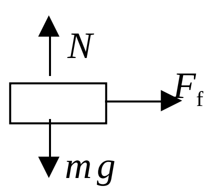
What is required according to Newton's second law in order to have an acceleration? There must be a net force in the direction of the acceleration. So,

$\Sigma F = ma_C = mv^2/r$.

It takes a net force of magnitude mv^2/r directed towards the center of the circle in order for an object to remain in a circular motion. This is called centripetal force. It is not a magic new force. It is simply the net sum of all the forces in the radial direction. These forces may be applied by a number of different methods. A string attached to a twirling ball will cause a centripetal force. Friction of tires on a road will cause a centripetal force. Friction of the car seat on your body will cause a centripetal force. There is no such thing as centrifugal force (center-fleeing). That is a psuedo-force that we say we feel because we are not going in a straight line. If there were such a force, a ball on a string would fly outward after being let go. Instead it goes in a straight line (according to Newton's first law).

Let's look at the example of circular motion for a car turning on a road.

<u>Problem:</u> A car turns a corner on a road of radius 50.0 m. (a) If the coefficient of static friction $\mu_s = 0.90$, what is the maximum speed the car can negotiate the turn without sliding? (b) What if the road is icy and the coefficient of friction is $\mu_s = 0.1$?



(center of the circle)

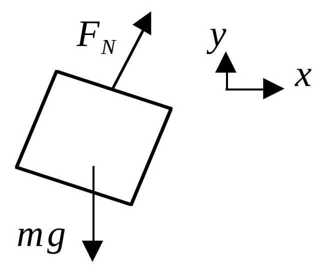
Note that we did not need the mass of the car. A designer of the road does not need to worry about the mass of the vehicle (as far as curves are concerned, not as far as wear and tear on the road), but does need to worry about how good the tires on cars are (μ_s). The designer does need to worry about where the center of gravity of a vehicle is or it might tip over, which we will talk about in chapter 9.

The centripetal acceleration always results from the sum of the components of the forces pointing toward (or away from) the center of the circle for uniform circular motion. The important forces to determine the centripetal acceleration are those pointing inward (outward). So while doing problems with circular motion, the two important axis are tangent to the circle and perpendicular to the circle.

Remember our example of a car turning a corner. There is a way to make a sharper turn with a lower coefficient of friction. We can do that by banking the corner. Why does this work? Because now, part of the normal force is used to provide the centripetal force.

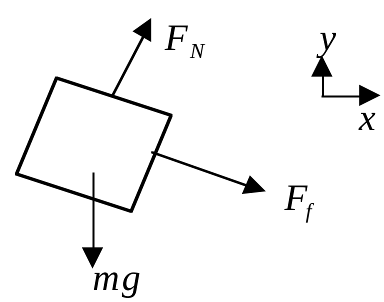
<u>Problem:</u> What is the angle we need to bank a curve so that a car doesn't need any friction to stay on the curve.

Note, here we use the normal *x* and *y* axis because the acceleration is along the horizontal direction (which is toward the center of the circle).



So, as an example, to design a road with radius 50 m for a speed of 15 m/s the road must be banked at an angle of 25°.

<u>Problem:</u> Suppose, I go back to the first case of a road with μ_s =.9, but now I add a banked turn of 15°. How fast can I safely go around the curve now?



<u>Problem:</u> What is the minimum velocity you can spin a can filled with water attached to a 1.0 m long string in a vertical direction without the water falling out?

<u>Problem:</u> What is the normal force at the bottom for such a case?

The normal is less at top or may be even zero. Think about a roller coaster. The normal (what you "feel") is less at top. The normal is strongest at the bottom. You feel the seat pressing up into you on a roller coaster.

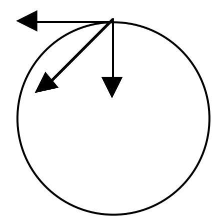
Problem: If you rotate a ball at an angle of θ. (a) What is the centripetal acceleration? (b) If the string is 0.8 meters long and θ=20°, how fast is it rotating?

2. Non-Uniform Circular Motion

Motion in a circle which is not at a constant speed is called non-uniform circular motion. For this to occur there must be a component of acceleration perpendicular to the direction of motion as well as parallel to the direction of motion. There is the centripetal acceleration, a_C, and the tangential acceleration a_T.

Since acceleration is a vector, the total acceleration is the vector sum of the two.

$\mathbf{a} = \mathbf{a}_C + \mathbf{a}_T$ so the magnitude of a is $\sqrt{(a_C^2 + a_T^2)}$.



If the speed is increasing, then it is possible to have a constant magnitude of a_T, but not of a_C.

Problem: Suppose I go from 0 to 22 m/s around a quarter circle turn with a radius of 87 m. (a) Assuming constant tangential acceleration, what is a_T, (b) What is a_C when my velocity is 15 m/s?

Note that the tangential acceleration changes as the tangential velocity changes for non-uniform circular motion.

3. Newton's Law of Universal Acceleration

Newton realized that gravity worked over a distance (the apple?), and he proposed that gravity worked over large distances, even from the earth to the moon, or between heavenly bodies.

He proposed his famous law of universal gravitation.

$F = Gm_1m_2/r^2$ The direction is toward the two objects.

What does this mean?

1. The force never dies out. Every body in the universe feels gravity from every other body in the universe.
2. The larger the mass, the stronger the force.
3. The force between two objects is the same (equal and opposite). The accelerations are different due to their different masses.
4. If I double the mass of an object, the force doubles.
5. If I double the distance of an object, the force is cut to one quarter of its value.

The value of G is $G = 6.67 \times 10^{-11}$ N-m^2/kg^2. We can measure the distance from the center of an object when the object is spherical. (This is why Newton developed calculus).

<u>Problem:</u> (a) What is the gravitational force on the moon from the earth? (b) What is the gravitational force on the earth from the moon?

This force is a vector, as are all forces, and must be added vectorially, by components.

<u>Problem:</u> Suppose I place three billiard balls, all of mass .300 kg on a table at the corners of a right triangle (.3 m , .4 m, .5 m). What is the net gravitational force on the ball at the corner from the other two balls. (Not from the earth). (Even billiard balls exert a gravitational force on each other.)

Example 5-10 in the book is just like this, but instead of 3 billiard ball, the three objects are the earth, the moon and the sun. The book shows $F_{EM} = 1.99 \times 10^{20}$ N and $F_{SM} = 4.34 \times 10^{20}$ N. Why is the force between the moon and the sun greater than the force between the moon and the earth. If so, shouldn't the moon be rotating around the sun. Is it?

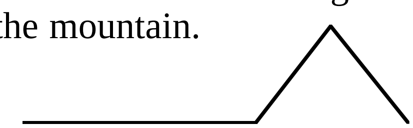
4. Gravity Near the Earth

What is force of gravity near the surface of the earth? If I am standing on the earth, we have said that the force is mg. That must come from Newton's law of universal gravitation. Let's see, how this works. I need to take the distance from the center of the earth when I am standing on the surface of the earth.

$mg = Gmm_E / r^2$
$g = Gm_E / r^2 = (6.67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2)(5.97 \times 10^{24}\text{kg})/6.38 \times 10^6 \text{ m})^2 = 9.78 \text{ m/s}^2$

<u>Problem:</u> How far above the earth must I go for the force I feel to be 1/2 of what is on the surface of the earth?

Large objects near us influence what we perceive as the acceleration of gravity. Near a mountain, there is mass pulling us toward the mountain.



Most of the mass is still underneath us, so the difference is very small, but it is there. Also if you go to higher elevations, you are farther from the center of the earth so g goes down very slightly, as well.

5. Satellites and "Weightlessness"

Suppose I am bouncing on a trampoline or bunjee jumping off a bridge. What forces are acting on me? Gravity and air resistance. Let's disregard air resistance for a moment. The only force acting on me is gravity. We say that I am in free fall. Now consider a satellite in orbit, or the moon orbiting the earth. What force is acting on it? Only gravity. **These are in free fall as well.** They are falling toward the earth. The moon and satellites are in free fall around the earth. The only force acting on them is the force of gravity. Why don't they hit the earth? Because the are moving tangent to the earth and according to Newton's first law, they continue in motion. There is no tangential force, so they are in free fall. We call this weightlessness, but it is not really. There is a weight (force=mg) pulling the object down. The object is still experiencing a force and it IS accelerating. **The acceleration is always perpendicular to the velocity so the velocity (vector) is changing, but the speed (scalar) is not.** We call it weightless, because if we were to stand on a scale in free fall, our weight would read 0. Why? Because the scale would be falling as well. The scale would exert no normal force on us and thus read 0.

Problem: What is the speed of a satellite in orbit around the earth at a distance of 12,200 km above the surface of the earth. (Note: the universal law of gravity uses distance from the center of the earth. The radius of the earth is $R = 6.38\times 10^6$ m.)

All satellites at the same radius have the same speed, regardless of their mass. Often we write the speed as a function of the period (T), where the period is the time it takes to make one orbit.

$vT = 2\pi r$ or $T = 2\pi r/v$

Problem: How long does it take the above satellite to circle the Earth?

6. Kepler's Laws

Long before Newton proposed his three laws of motion and his universal law of gravitation, Johannes Kepler had developed three laws which describe the motion of the planets. Newton showed that his law of universal gravitation **predicted** Kepler's three laws. It was an amazing prediction and helped convince people that Newton's law was correct. Kepler's laws are:

1. All planets move in elliptical orbits with the Sun at on focal point.
2. A line drawn from the Sun to any planet sweeps out equal areas in equal time intervals.

3. The square of the orbital period of any planet is proportional to the cube of the average distance from the planet to the Sun.

$T^2 \propto r^3$ or if we take the ratio of two planets.

$T_1^2 / T_2^2 = r_1^3 / r_2^3$

For this to work we must measure the period and the average radius of two orbiting objects around the same object, like earth and mars around the Sun, but not earth around the sun and moon around the earth.

<u>Problem:</u> Jupiter is 5.2 times as far from the sun as the earth is. What is the length of Jupiter's year?

<u>Problem:</u> Determine the mass of the earth from the known distance and period of the moon.

M = mass of earth, and m = mass of moon. Distance from the moon is 3.84×10^8 m and the period is 27.4 days = 2.37×10^6 s.

Look at units m^3/ s^2- N-m^2/kg^2 = m^3 kg^2/ s^2- N-m^2
a Newton is ma = kg m/s^2 so the units are
m^3 kg^2 s^2 / s^2 kg m^3 = kg