Chapter 10 Lecture Notes
Physics 2414 - Strauss

Formulas:

$\rho = m/V$
$P_{\text{fluid}} = \rho gh$
$P_{\text{total}} = \rho gh + P_0 = P_G + P_A$
$F_2 = F_1 (A_2 / A_1)$
$B = w_{\text{fluid}}$
$Q = \Delta V/\Delta t = Av$ (Flow rate)
$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$
$P_1 + (1/2)\rho v_1^2 + \rho g y_1 = P_2 + (1/2)\rho v_2^2 + \rho g y_2$
$F = \eta Av/l$

<u>Main Ideas:</u>
1. Definition
2. Pressure
- Definitions and Units
- Properties of Pressure in a Fluid
- Pascal's Principle
- Measurements of Pressure
3. Buoyancy
4. Motion of Fluids
- Bernoulli's Equation
- Viscosity

1. Definition of Fluids

In this chapter we study fluids. A fluid is a defined as substance that can flow., that doesn't maintain a fixed shape. Gases and liquids are usually considered fluids. Any object, whether a solid, a gas, a liquid, or a plasma (a collection of ionized particles), has a **density**. The density is defined as the mass per unit volume and is given the Greek symbol rho (ρ). So the density is defined as

$\rho = m/V$. (kg/m^3)

A substance which is more dense will have more mass for a given volume. If I hold a cubic centimeter of lead, it will be heavier than a cubic centimeter of wood because lead has a higher density than wood. Table 10-1 gives the density of some materials.

Another way of specifying the density of an object is to compare its density to the density of water at 4.0°C, (which is 1.00×10^3 kg/m^3). The **specific gravity** is defined as the ratio of the density of a substance to the density of water at 4.0°C. Lead, with a specific gravity of 11.3, has a density of 11.3×10^3 kg/m^3.

2. PRESSURE

2.1 Definition and Units

Pressure is defined as the force per unit area. $P = F/A$. The force acts perpendicular to the surface area A. If a person stands on a wooden floor with spiked metal cleats, the floor will be damaged because the person's weight is not spread out over a very large area. However, if the person wears normal shoes, the weight is spread out over a larger area, so the pressure is smaller, and the floor will not be damaged. Snow shoes spread the weight out over an even larger area, and so the pressure on the snow is less, and a person does not sink into the snow. A person can lay on a bed of nails because there are so many nails that the person's weight is still spread out over a lot of area, and the pressure is not very high.

The SI units of pressure is given by N/m^2 which is given the name a Pascal (Pa), named after the French philosopher, theologian, and scientist Blaise Pascal.

1 N/m^2 = 1 Pa.

In the U.S. we also use lb/in^2 or psi (pounds per square inch).

Because the atmosphere is always pushing down on us with a certain pressure, another common unit of measurement, often used when discussing fluids, is called an atmosphere. This is defined as the standard pressure that the atmosphere presses on us at sea level and is equal to 1.013×10^5 Pa. This is given the pressure unit of **atmospheres.**

Finally, we will see that other fluids can push on us just like the air pushes on us. A commonly used measure of pressure comes from the fact that if I were immersed in a column of mercury 760 mm high, that would exert the same pressure on me as the atmosphere. We give this unit of mm/Hg (millimeters of mercury) a special name called a **torr.** So

1 atmosphere = 760 torr (mm/Hg) = 1.013×10^5 Pa

<u>Problem:</u> A water bed is 2.00 m square and 30.0 cm deep. (a) What is its weight? (b) What pressure does the bed exert on the floor, assuming the entire lower surface contacts the floor? (Volume = length × width × height, Area = length × width).

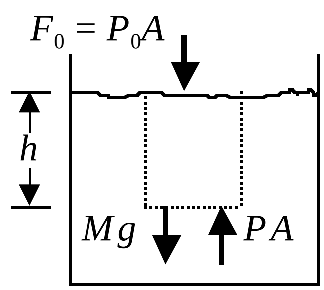
2.2 Properties of Pressure in a Fluid

There are three important observations about pressure in a fluid.

1) At any point that a fluid is in contact with a surface, the pressure is directed perpendicular to the surface.

2) At any point inside a fluid, the pressure is directed in all directions with the same magnitude. (See figure 10-1). For the block in this figure, the pressure is not exactly the same because the different faces are at different depths. But if the block is infinitesimally small, then the pressure in all directions is exactly the same.

3) The pressure at any point in a fluid depends only on the depth of the point. Suppose there is a volume of fluid with a uniform density which has a depth h and area A.



What forces act on this volume of fluid? There is the force from the atmosphere above the liquid $F_0 = P_0A$. There is the weight of the liquid Mg. There is also the force of the liquid pushing up on the column of liquid at depth h.

Since there is no acceleration of the liquid at the depth h, the forces must balance each other.

$\Sigma F_y = 0$

$PA - Mg - P_0A = 0$

$PA - \rho Vg - P_0A = 0$

$PA - \rho hAg - P_0A = 0$

$$\boxed{P = \rho gh + P_0}$$

So the pressure at any depth h in a fluid is equal to the pressure outside of the fluid (P_0) plus the fluid pressure (ρhg).

For many circumstances, the pressure outside of the fluid is the pressure of the earth's atmosphere at that point. For instance, the pressure in your tires is really the pressure of the air in your tires plus the pressure of the atmosphere on the tires. However, the pressure that is read on a pressure gauge is the pressure which is greater than the atmospheric pressure. It is called **gauge pressure.** In general, the equation above becomes

$P = P_G + P_A$

where P_G is the gauge pressure and P_A is the atmospheric pressure. We are often most concerned with the gauge pressure or fluid pressure, and not the outside pressure. For fluids, then, we often simply write the fluid pressure, or

$P_{fluid} = \rho g h$

<u>Problem:</u> A rectangular shaped dam is 70 m high and 180 m wide and water is filled to the top of the dam. (a) What is the pressure on the dam from the water at the top? (b) What is the pressure on the dam from the water at the bottom? (c) What is the average pressure on the dam from the water? (d) What is the total force on the dam? (e) What if we include the pressure from the atmosphere? (f) Suppose that the dam was holding up a thin column of water instead of a huge lake. What would be the pressure on the dam then?

2.3 Pascal's Principle

Note that the pressure is only dependent on the depth, and not on the "area" of the column of fluid. This means that the pressure at any depth of a fluid is always the same for all points at that depth. The pressure is not affected by the shape of the vessel. The pressure under 5 meters of water in a thin tube is the same as the pressure under 5 meters of water in a big swimming pool. This is why water always seeks a constant level. The pressure at any uniform height in a single liquid is the same throughout the entire liquid.

In addition, if the outside, or external pressure increases, the total pressure increases by the same amount. Pascal realized that this led to an important general principle. Pascal's principle states that **pressure applied to a confined fluid increases the pressure throughout the fluid by the same amount.** This is how a hydraulic lift works. Fluid is enclosed in a pipe with a small area at one end and a large area at the other. Pressure is applied at the end with the small area. That same pressure is transferred to the end with the large area.

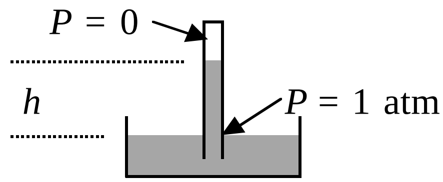
$P_1 = F_1 / A_1 = P_2 = F_2 / A_2$
$F_2 = F_1 (A_2 / A_1)$

If the area A_2 is much larger than the area A_1, then a small force F_1 can be applied to create a large force F_2 at the output end. This large force can be used to jack up a car or lift heavy objects.

Problem: In the hydraulic press used in a trash compactor, the radii of the input piston and the output plunger are 6.4×10^{-3} m and 5.1×10^{-2} m, respectively. What force is applied to the trash when the input force is 330 N?

2.4 Measurements of Pressure

Pressure is measured using the two principles discussed above, that $P = \rho gh$, and Pascal's Principle. If a fluid, like mercury or water is put in a container which is open to the atmosphere at one end and closed at the other, with the closed end having zero pressure (it is a vacuum), then the following situation occurs.



The pressure at any height is equal, so the pressure of the atmosphere, just equals the pressure of the liquid or ρgh.

For different liquids with different densities, the height of the column at sea level will be different. For mercury it is 760 mm. For water, it is 10.3 m.

What happens when you suck on a straw? You take the air out of the straw and the atmospheric pressure pushes the liquid up the straw. You do not suck the liquid up. Instead you create a vacuum and the atmosphere pushes the liquid up. Similarly, a water pump pumping at the top of a well can only pump water up 10.3 m, since it works the same way, and atmospheric pressure is 10.3 m of water. To pump water up larger distances, you must use a series of pumps, or place the pump at the bottom of the well, and then it pushes the water up, rather than using atmospheric pressure to push it up.

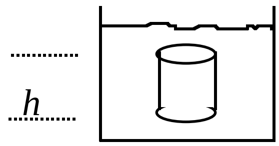
3. BUOYANCY

Any object which is partially or totally submerged in a liquid has a buoyant force acting on it which pushes the object up. That is why a rock appears to weigh less when it is submerged in liquid, or why it is very difficult to push a beach ball underwater. The famous Greek mathematician, Archimedes' developed a principle which describes this around 250 B.C. Archimedes' principle can be stated as **any body completely or partially submerged in a fluid is buoyed up by a force equal to the weight of the fluid displaced by the body.**

$B = w_{\text{fluid}}$

where B is the magnitude of the buoyant force and W_{fluid} is the weight of the displaced fluid. The reason for this is that the pressure of the fluid is dependent on the depth of the fluid. So the pressure at the top of an object is less than the pressure at the bottom of the object which creates a net force.

$$
\begin{aligned}
B &= P_2A - P_1A \\
&= (P_2 - P_1)A = \rho ghA \\
&= \rho gV = mg = w
\end{aligned}
$$



Problem: (a) What is the buoyant force on a balloon filled with 1.0 m^3 helium at sea level. (b) What is the gravitational force on the same balloon?

So the upward (buoyant) force is much greater than the downward force (weight) and the helium balloon rises.

Problem: Every fluid exerts a buoyant force. Even the air exerts a buoyant force on your body. Estimate how strong that force is.

Problem: Archimedes supposedly determined whether or not the king's crown was made of gold by using this law. Suppose a crown is weighed on a scale and it weighs 9.8 N. The crown is then weighed in water and it weighs 9.00 N. Is the crown pure gold?

Problem: A raft is made of wood having a density of 600 kg/m^3. Its surface area is 5.7 m^2, and its volume is 0.60 m^3. How much of it is below water level?

Problem: A hollow log is used as a canoe. It has a length of 3.00 m and a radius of 0.350 m. The canoe weighs 1.00×10^3 N. What is the maximum weight it can carry without sinking?

As the weight in the canoe is increased, it sinks lower in the water. The maximum weight it can carry is when the water level is at the top of the canoe's side. The canoe has an upward buoyant force F_B, and two downward forces, the weight of the canoe w_C, and the weight of the cargo, w.

4. MOTION OF FLUIDS

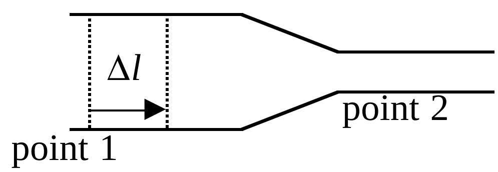
Up until this point, we have discussed fluids which are static. That is, they are not in motion. We now turn our attention to fluids in motion, or hydrodynamics. There are many categories of fluids in motion, categorized by whether the fluid flow is steady, or not steady, compressible or incompressible, viscous or nonviscous. In steady flow, the velocity of the fluid particles at any point is constant as time goes by. Different parts of the fluid may be flowing at different rates, but the fluid in one location is always flowing at the same rate. An incompressible flow is the flow of a fluid which cannot be compressed. Most liquids are nearly incompressible. A viscous fluid is one which does not flow easily, like honey, while a nonviscous fluid is one which flows more easily, like water. We will mostly be concerned with the steady flow of incompressible, nonviscous fluids.

If the flow is steady, then the velocity of the fluid particles at any point is a constant with time. The various layers of the fluid slide smoothly past each other. This is called **streamline** or **laminar flow.** Above some certain velocity, the flow is not smooth and becomes **turbulent.** Illustrations of turbulent and laminar flow are shown in Figure 10-14.

We first consider the steady flow of a fluid through an enclosed pipe. We want to determine how the speed of the fluid changes when the size of the pipe changes.

At some point along the pipe, we look at how much fluid flows past us (Δm) in a short period of time (Δt).

$\Delta m/\Delta t = \rho\Delta V/\Delta t = \rho A\Delta l/\Delta t = \rho Av$



This is the flow rate for any point along the pipe. Because no fluid flows in or out of the sides, the mass flowing past any point during a short period of time must be the same as the mass flowing past any other point, so

$$\boxed{\rho_1 A_1 v_1 = \rho_2 A_2 v_2}$$

where the subscript 1 and 2 refer to two different points along the pipe. This equation is called the **equation of continuity**. If the fluid is incompressible, then the density is the same at all points along the pipe and this equation becomes

$$\boxed{A_1 v_1 = A_2 v_2}.$$

We see that if the cross sectional area is decreased, then the flow rate increases. This is demonstrated when you hold your finger over part of the outlet of a garden hose. Because you decrease the cross sectional area, the water velocity increases.

The **flow rate** or **flux** is defined as the rate with which the fluid flows and is given by the variable Q.

$$\boxed{Q = \Delta V/\Delta t = Av}$$

Problem: A water hose with a radius of 1.00 cm is used to fill a 20.0 liter bucket. If it takes 1.00 min to fill the bucket, what is the speed, v, at which the water leaves the hose? (1.00 liters = 10^3 cm^3)

Problem: The approximate inside diameter of the aorta is 1.0 cm (radius = 5.0×10^{-3} m) and that of a capillary is 10 μm = 10×10^{-6} m. The approximate blood flow speed is .3 m/s in the aorta and 0.5 mm/s in the capillaries. Approximately how many capillaries get blood from the aorta?

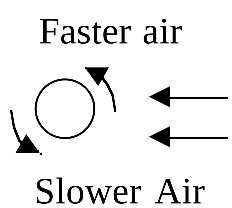
4.1 Bernoulli's Equation

In the 18th century, the Swiss physicists Daniel Bernoulli derived a relationship between the velocity of a fluid and the pressure it exerts. Qualitatively, Bernoulli's principle states that **swiftly moving fluids exert less pressure than slowly moving fluids.**

Bernoulli's principle is extremely important in our everyday life. It is the primary principle which leads to lift on an airplane wing and allows the plane to fly. It is the primary reason a sailboat can sail into the wind. It is the primary reason a baseball can curve. It is an important reason that smoke is drawn up a chimney.

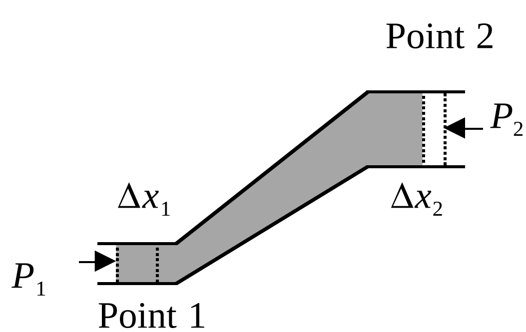
Airplane wing

Curve Ball



Bernoulli's equation is really a consequence of a fundamental principle of physics: the conservation of energy. It can be derived using energy principles.

Consider a fluid moving through a pipe. The pipe's cross sectional area changes, and the pipe changes elevation. At one point the pipe has a cross sectional area of A_1, a height of y_1, a pressure of P_1, a velocity of v_1 and moves a distance of Δx_1 in a time of Δt. At another point along the pipe these quantities are given by A_2, y_2, P_2, v_2, and Δx_2.



We are going to push a certain amount of fluid up the pipe from point 1 to point 2. P_1 is opposite in direction from P_2 because the rest of the fluid pushes to the left of fluid at point 2 and to the right of the fluid at point 1.

Recall, the fundamental equation that

$W_{NC} = \Delta K + \Delta U$

We want to look at each of these terms individually to derive Bernoulli's equation.

$\Delta K = (1/2)mv_2^2 - (1/2)mv_1^2 = (1/2)\rho V_2 v_2^2 - (1/2)\rho V_1 v_1^2$

$\Delta U = mgy_2 - mgy_1 = \rho V_2 g y_2 - \rho V_1 g y_1$

$W_1 = F\Delta x_1 = P_1 A_1 \Delta x_1$
$W_2 = F\Delta x_2 = -P_2 A_2 \Delta x_2$

$W_{NC} = W_1 + W_2 = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 = P_1 V_1 - P_2 V_2$

$W_{NC} = \Delta K + \Delta U$
$P_1 V_1 - P_2 V_2 = (1/2)\rho V_2 v_2^2 - (1/2)\rho V_1 v_1^2 + \rho V_2 g y_2 - \rho V_1 g y_1$

We know that since this is an incompressible fluid, $V_1 = V_2$

$P_1 - P_2 = (1/2)\rho v_2^2 - (1/2)\rho v_1^2 + \rho g y_2 - \rho g y_1$

$$\boxed{P_1 + (1/2)\rho v_1^2 + \rho g y_1 = P_2 + (1/2)\rho v_2^2 + \rho g y_2}$$

which is Bernoulli's equation.

Remember, that Bernoulli's principle applies when a fluid is moving. The pressure is not the same if the velocities are different. Pascal's principle applies when the fluid is stationary. A pressure applied at one point is transferred to every point of the fluid. Pascal's principle can be used for something moving slowly or something that moves a little then stops like a hydraulic jack.

As shown in the book, for a large storage tank of height *h* which has a small pipe open at the bottom of it, the pressure at the top and at the small opening is atmospheric pressure. This is actually an important point for solving problems. *The pressure of any part of a container open to the atmosphere is the same pressure as the atmosphere for the entire section of container that is at the same* *height as the opening and has the same cross sectional area as the opening.* Also in this case, the velocity at the top of the storage tank is basically zero because it has such a huge cross sectional area, so Bernoulli's equation gives the velocity at the bottom of the tank to be:

$P_1 + (1/2)\rho v_1^2 + \rho g y_1 = P_2 + (1/2)\rho v_2^2 + \rho g y_2$
$P_1 = P_2 = 1$ atm, $v_1 = 0$, $y_1 - y_2 = h$, so
$v_2 = \sqrt{2gh}$

Problem: A water pipe is inclined 30° below the horizontal. The radius of the pipe at the upper end is 2.00 cm. If the gauge pressure at a point at the upper end is 0.100 atm, what is the gauge pressure at a point 3.00 m downstream where the pipe has narrowed to 1.00 cm radius? The flow rate is 20.0π cm^3/s.

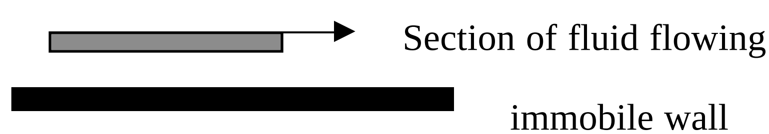
Problem: A hypodermic syringe contains a fluid with the density of water. The barrel of the syringe has a cross sectional area of 2.50×10^{-5} m^2 and the cross sectional area of the needle is 2.50×10^{-8} m^2. Before the plunger is pushed, the pressure everywhere is at 1.00 atmospheres. A force, F, of 2.00 N is exerted on the plunger. Assume that the gauge pressure in the needle remains at 1.00 atm. If the syringe is horizontal, what is the speed of the liquid as it flows through the needle and into the arm?

Don't confuse this situation with one where there is a piston pushing on the fluid, but the fluid doesn't move. If you are considering the situation before the fluid starts to move Pascal's principle applies. After the fluid starts to move Bernoulli's principle applies.

4.2 Viscosity

Viscosity is the amount of internal friction in a fluid. As stated before, a fluid like honey is very viscous, and a fluid like water has a low viscosity. The viscosity of a fluid is quantitatively given by its coefficient of viscosity η (the Greek letter, eta). It takes a constant force to move a fluid, because there is a frictional viscous force opposing the motion. (Remember from Newton's 2nd law that if there was no frictional force, it would not take any external force to keep a body moving at constant velocity). The force it takes to keep a liquid moving in a tube is found to follow the equation

$F = \eta A v / l$



where A is the area of the fluid in contact with the walls, v is the velocity of fluid, and l is the perpendicular distance from the fluid to the immobile surface.

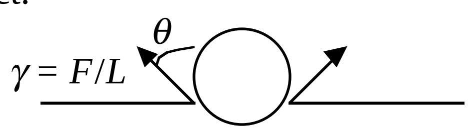
So a fluid has more drag near the surface of a stationary plate, and as l approaches zero it takes an infinite force to move the fluid. In other words, the fluid touching the wall does not actually move at all. Table 10-3 gives coefficients of viscosity for some fluids.

5. SURFACE TENSION AND CAPILLARITY

We now deal with what happens at the surface of a fluid. The surface of a fluid does not act like the rest of the fluid, but instead acts something like a stretched membrane. This allows insects to walk on water, and water drops to hang from a faucet. This phenomena is due to attractive forces between the fluid molecules called **surface tension.** The surface tension is defined as the force per unit length that acts along any line of a surface. Surface tension is defined by

$\gamma = F/L$

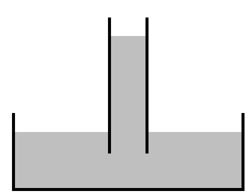
where γ is the Greek letter gamma, and is defined as the surface tension. Table 10-4 lists surface tensions for various fluids. When a small object is placed in a pool of water it depresses the water and this surface tension force tends to hold up the object. Look at the forces on the object.



The vertical forces are given by
$\Sigma F = 0$
$w - 2\gamma L\cos\theta = 0$
If the object is totally submerged, then L for each side is half of the circumference, or $L = \pi r$.
Then,
$w = 2\gamma\pi r\cos\theta$.

So for a spherical object to float on a liquid, its weight must be approximately less than $2\gamma\pi r$, since the maximum value of $\cos\theta$ is 1. If the weight is more than that, then surface tension will not play a real role in the problem, and we just use Archimedes' principle and buoyancy to analyze the situation.

Surface tension also explains why water rises up slightly in a glass, or can even climb up a very thin tube. This phenomena is called capillarity. Suppose we put a thin tube vertically into a pool of liquid.



Using Newton's second law, we look at the forces on the water

$\gamma L - mg = 0$
$\gamma(2\pi r) - \rho(\pi r^2 h)g = 0$

$h = 2\gamma/(\rho g r)$

If r gets very large, then h is very small. So the fluid will only rise any appreciable height if the radius of the tube is very small.

5.1 Drag

When an object moves through a fluid it experiences a drag. When we move through the ocean of air that we live in, we experience a drag that we call air resistance. This drag, analogous to air resistance, is experienced by any object moving through any fluid. The drag that an object experiences is characterized by something we call the Reynolds number. The Reynold's number is defined as

$$R = vL\rho/\eta$$

where v is the velocity of the object relative to the fluid, ρ is the density, η is the coefficient of viscosity, and L is a characteristic length of the object which depends on its shape. As long as the Reynold's number is less than about one, then the fluid flow around the object is laminar and the force of drag is proportional to the velocity

$$F_D \propto v = kv.$$

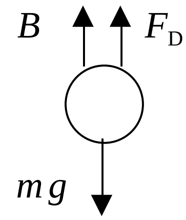
That means that as long as the Reynold's number is below one, when your speed doubles, the force of drag acting against your direction of motion also doubles. When the Reynold's number is greater than one, the flow of air is no longer laminar. Instead it is turbulent, and the drag is proportional to the square of the velocity.

$$F_D \propto v^2$$

This means when you velocity doubles, your drag is increased by a factor of four. The more aerodynamic an object is the lower the Reynold's number. We try to make objects aerodynamic not only to decrease wind resistance, but because if the Reynold's number is below about 1, then the drag of an object will increase linearly with velocity rather than quadratically. On a bicycle, the drag increases approximately with the square of the velocity, so when your speed doubles, the drag from wind resistance is increased by a factor of four.

Let us consider the case of Reynold's number below one with the fluid flow laminar. Then $F_D = kv$, where k depends on the size and shape of the object and on the viscosity of the fluid. For a spherical object, $k = 6\pi r\eta$, so the force of drag becomes

$\boxed{F_D = 6\pi r\eta v}$ which is called Stoke's Equation.



Consider an object falling in a fluid. There are three forces acting on it, gravity, buoyancy, and drag. By Newton's second law,

$$mg - B - F_D = ma$$

So when the flow is laminar,

$$mg - B - kv = ma$$
$$\rho_O Vg - \rho_F Vg - kv = ma$$

where ρ_O is the density of the object, and ρ_F is the density of the fluid.
When the sum of the forces equal, and there is no further acceleration, then the object reaches its terminal velocity. (Remember this is all for the case of laminar flow)

$$\rho_O Vg - \rho_F Vg - kv_T = 0$$

$$\boxed{v_T = (\rho_O - \rho_F)Vg/k}$$