Group Problem

Two waves are propagating on the same very long string. A generator at one end of the string creates a wave given by

$$y = (6.0\text{ cm})\cos\frac{\pi}{2}\left[(2.0\text{ m}^{-1})x + (8.0\text{ s}^{-1})t\right]$$

and one at the other end creates the wave

$$y = (6.0\text{ cm})\cos\frac{\pi}{2}\left[(2.0\text{ m}^{-1})x - (8.0\text{ s}^{-1})t\right].$$

(a) Write an equation for the standing wave produced by these two waves. You may want to use the trigonometric identity $\cos u + \cos v = 2\cos\{(u+v)/2\}\cos\{(u-v)/2\}$.
(b) Calculate the frequency, wavelength and speed of the standing wave.
(c) Find the points on the string where there is no motion (the nodes).
(d) Find the points on the string where the motion is a maximum (the antinodes).

Group Problem Solution

a) Adding the waves up

$$y_1 + y_2 = 6.0 \cos \frac{\pi}{2}(2m^{-1}x + 8s^{-1}t) + 6.0 \cos \frac{\pi}{2}(2m^{-1}x - 8s^{-1}t)$$

$$= (12.0\,cm) \cos \frac{\pi}{2}\left(\frac{4m^{-1}x + 16s^{-1}t}{2}\right)$$

$$= 12.0\,cm \cos \frac{\pi}{2}(2m^{-1}x + 8s^{-1}t)$$

b) $\omega = 2\pi f = 4\pi$

$\Rightarrow f = 2\,Hz$

$\lambda = 2\,m$

$v = f\lambda = 4\,m/s$

c) Nodes where cosine function is zero, or equal to $n\pi + \pi/2$

$$\frac{\pi}{2}(2m^{-1}x) = n\pi + \pi/2$$

$$\pi x = n\pi + \pi/2$$

$$x = n + 1/2$$

So nodes occur at .5 m, 1.5 m, 2.5 m ...

d) Antinodes occur is between these, where $\pi x = n\pi$ or at 0 m, 1.0 m, 2.0 m ...