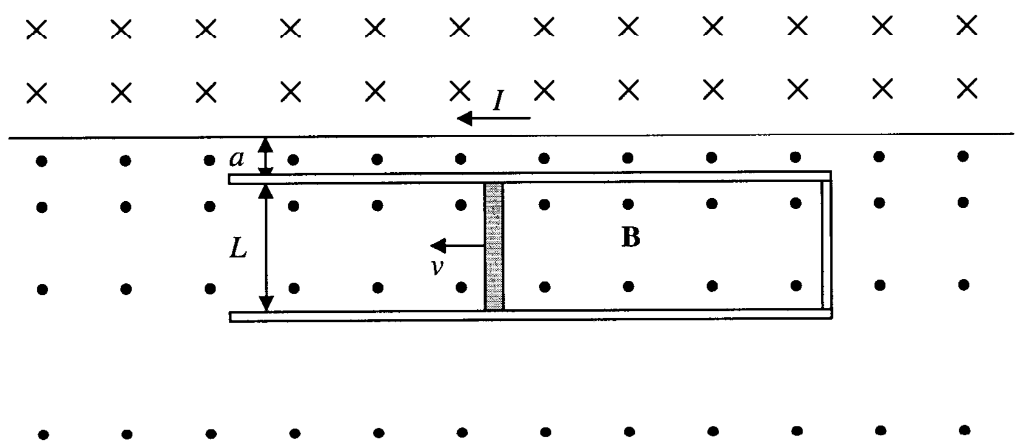
Physics 1215
Group Problem

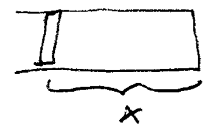
A rod of length L is moved at a constant speed v along horizontal conducting rails as shown in the figure. In this case, the magnetic field in which the rod moves is not uniform, but is provided by a current I in a long wire parallel to the rails. Assume that $v = 5.00$ m/s, $a = 10$ mm, $L = 10$ cm, and $I = 100$ A.

a) Calculate the emf in the rod by using Faraday's law.
b) What is the current in the rod if the rod has a resistance of 0.400 Ω and the rails and connecting strip have no resistance?
c) What is the rate that thermal energy is being produced in the rod?
d) Calculate the external force that must be applied to the rod to maintain its motion by calculating the magnetic force on the rod and using Newton's 2nd law.
e) At what rate does the agent exerting the external force do work? Compare your answer to your answer in part c.



Group Problem

a) First calculate Φ_B inside the rails



From the wire $B = \frac{\mu_0 I}{2\pi r}$

so

$$\Phi_B = \int_a^{a+L} B \cdot dA = \int_a^{a+L} \frac{\mu_0 I}{2\pi r} x \, dr$$

$$= \frac{\mu_0 I x}{2\pi} \int_a^{a+L} \frac{dr}{r} = \frac{\mu_0 I x}{2\pi} \ln\left(\frac{a+L}{a}\right)$$

then by Faraday's Law

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{\mu_0 I}{2\pi} \ln\left(\frac{a+L}{a}\right) \frac{dx}{dt} = \frac{\mu_0 I v}{2\pi} \ln\left(\frac{a+L}{L}\right)$$

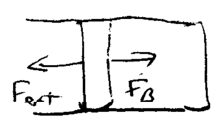
$$\mathcal{E} = \frac{(4\pi \times 10^{-7}\ T\cdot m/A)}{2\pi} (100\ A)(5.0\ m/s) \ln\left(\frac{1+10\ cm}{1\ cm}\right)$$

$$= \boxed{2.40 \times 10^{-4}\ V}$$

b) $I_i = \mathcal{E}/R = \frac{2.40 \times 10^{-4}\ V}{.400\ \Omega} = \boxed{6.00 \times 10^{-4}\ A \text{ clockwise}}$

c) $P = I_i V = (2.40 \times 10^{-4})(6.00 \times 10^{-4}) = \boxed{1.44 \times 10^{-7}\ W}$

d)



By Newton's 2nd Law $F_{ext} = F_B$

On a small section of the rod $dF_B = I_i B\, dr$

$$\text{So} \quad F_B = \int_a^{a+L} I_i B\, dr = \int_a^{a+L} I_i \frac{\mu_0 I}{2\pi r} dr = \frac{I_i \mu_0 I}{2\pi} \ln r \Big|_a^{a+L}$$

$$= \frac{\mu_0 I_i I}{2\pi} \ln\left(\frac{a+L}{L}\right)$$

$$= \frac{(4\pi \times 10^{-7}\ T\cdot m/A)}{2\pi} (6.00 \times 10^{-4}\ A)(100\ A) \ln\left(\frac{11\ cm}{1\ cm}\right)$$

$$= \boxed{2.87 \times 10^{-8}\ N}$$

e) $W = FV = (2.87 \times 10^{-8}\ N)(5.00\ m/s) = \boxed{1.44 \times 10^{-7}\ W}$ – Same as (c)