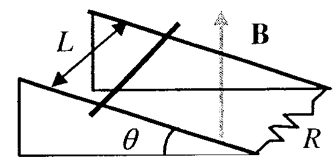
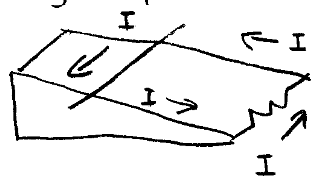
Physics 1215
Group Problem

A conducting rod of mass m and length L is free to slide without friction along two parallel rails with negligible resistance. The rails are connected to a resistor of resistance R and inclined at an angle of θ to the horizontal. The entire set-up is placed in a uniform magnetic field, **B**, pointing in the vertical direction:

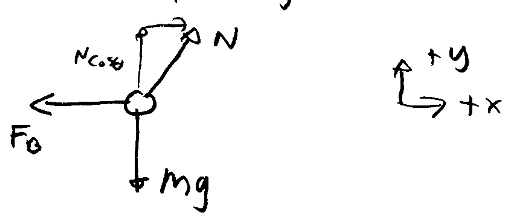


1. Indicate on the drawing which way the current will flow through the rod as the rod slides down the rails.
2. Using Faraday's law and Newton's second law, derive an expression for the velocity of the rod once it has reached a terminal velocity. (Always draw a clear free-body diagram when using Newton's second law.)
3. At the terminal velocity, what current flows through the wire?
4. Solve parts (1) and (2) again, but for the case where the magnetic field is perpendicular to the face of the inclined plane, rather than directly vertical.

1) By Lenz's Law $\Phi_{B\ \text{external}}$ is decreasing
so B_{induced} pointing up $\Rightarrow$ I is "out of page" in rod



2) We derived in class from Faraday's Law
$\mathcal{E} = vLB\sin\phi$ ① (ϕ is the angle between $\vec{B}$ and $\vec{v}$)
Draw Free body diagram of rod



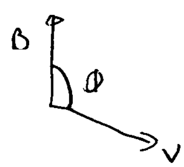
$\Sigma F_y = 0 \Rightarrow N\cos\theta = mg \Rightarrow N = \dfrac{mg}{\cos\theta}$

$\Sigma F_x \Rightarrow F_B = N\sin\theta$

$ILB = N\sin\theta = mg\tan\theta$ ②

Using Ohms Law $I = \mathcal{E}/R$.

$\frac{\mathcal{E}}{R} LB = mg\tan\theta$ ③



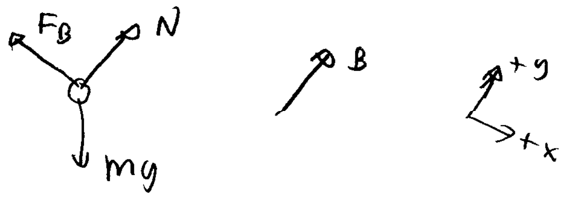
$\sin\phi = \sin(180 - \theta) = \cos\theta$

From ③ and ① $\dfrac{vL^2B^2\cos\theta}{R} = mg\tan\theta$

$$\boxed{V = \frac{Rmg\tan\theta}{L^2B^2\cos\theta}}$$

3) $I = \dfrac{\mathcal{E}}{R} = \dfrac{VLB\cos\theta}{R} = \dfrac{Rmg\tan\theta\, LB\cos\theta}{RL^2B^2\cos\theta} = \boxed{\dfrac{mg\tan\theta}{LB}}$

4)



$$\Sigma F_y = 0 \Rightarrow N = mg\cos\theta$$

$$\Sigma F_x = 0 \Rightarrow F_B = ILB = mg\sin\theta \quad (1)$$

$$I = \frac{\mathcal{E}}{R} = \frac{VLB}{R} \quad (2)$$

plug (2) in (1), $\frac{VL^2B^2}{R} = mg\sin\theta$

$$\boxed{V = \frac{Rmg\sin\theta}{L^2B^2}}$$