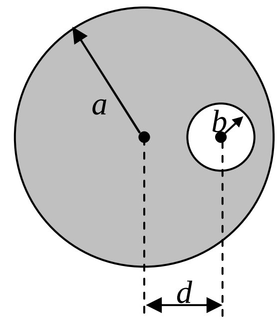
Physics 1215
Group Problem

Solve one of the two problems below.

1. The figure to the right shows a cross section of a long cylindrical conductor of radius a containing a long cylindrical hole of radius b. The axes of the cylinder and hole are parallel and are a distance d apart. A current I is uniformly distributed over the tinted area.



   (a) Use superposition to show that the magnetic field at the center of the hole is

$$B = \frac{\mu_0 I d}{2\pi\left(a^2 - b^2\right)}.$$

   (Hint: Regard the cylindrical hole as filled with two equal currents moving in opposite directions, thus canceling each other. Assume that each of these currents has the same current density as that in the actual conductor. Then superimpose the fields due to two complete cylinders of current, of radii a and b, each cylinder having the same current density.)

   (b) Discuss the two special cases, b=0 and d=0.

2. (a) In the figure to the right, use the Biot-Savart law to calculate the magnetic field at the point P.

   (b) If the length of the wire L is infinite in only one direction, (negative x) in the figure, what is the magnetic field at P?

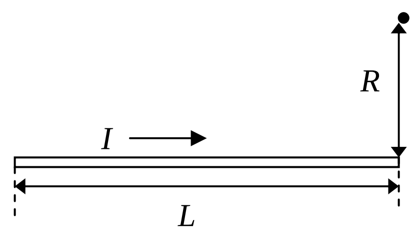


Figure for parts (a) and (b)

   (c) Using class notes, the book, or your own calculation, what is the magnetic field at the center of a circular arc with radius R and angle ϕ.

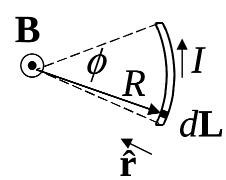


Figure for part (c)

   (d) Finally, in the figure below, two semi-infinite straight sections carrying a current I are connected to a semicircular arc of angle θ with all sections lying in the same plane. What must be the angle θ in order for B to be zero at the center of the arc?

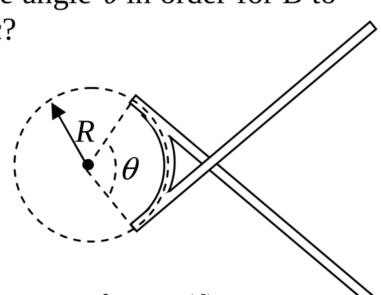
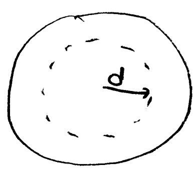


Figure for part (d)

1) a) The current density in the cylinder is

$$J = \frac{I}{A} = \frac{I}{\pi a^2 - \pi b^2} = \frac{I}{\pi (a^2-b^2)}$$

Now consider the shaded cylinder, but without the hole
What is the magnetic field from this cylinder at point d?
Use Ampere's Law



$$\oint \vec{B}\cdot d\vec{L} = \mu_0 I_{enc}$$

$$\oint B\,dL\cos\theta = \mu_0 \int_0^d J\,dA$$

$$B \oint dL = \mu_0 \int_0^d J\,2\pi r\,dr = \mu_0 \int_0^d \frac{I}{\pi(a^2-b^2)}\,2\pi r\,dr$$

$$B\,2\pi d = \mu_0 \frac{2I}{(a^2-b^2)} \int_0^d r\,dr$$

$$B\,2\pi d = \frac{I\,d^2\,\mu_0}{(a^2-b^2)}$$

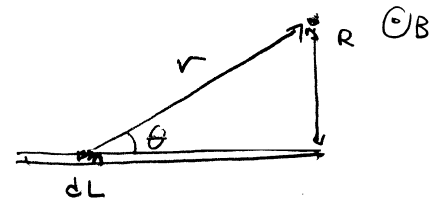
$$\therefore \quad B = \frac{\mu_0 I d}{2\pi(a^2-b^2)}$$

Now consider a negative current density <u>also</u> flowing through the cut out hole. At the very center of this cut out hole $B = 0$,
So the total B at d is $\boxed{B = \frac{\mu_0 I d}{2\pi(a^2-b^2)}}$

b) When $b=0$, there is no hole and we get what we would expect inside a solid cylinder with constant charge density $B = \frac{\mu_0 I d}{2\pi a^2}$

when $d=0$, we get $B=0$, which is correct for the axis of a cylindrical current.

2) a) $B = \frac{\mu_0}{4\pi} \int_{-\infty}^{0} \frac{I\, d\vec{L} \times \hat{r}}{r^2}$



$= \frac{\mu_0}{4\pi} \int_{-\infty}^{0} \frac{I\, dL}{r^2} \sin\theta$

Now $dL = dx$, $r^2 = x^2 + R^2$, $\sin\theta = R/\sqrt{x^2+R^2}$

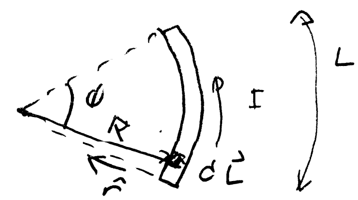
So $B = \frac{R\mu_0 I}{4\pi} \int_{-\infty}^{0} \frac{dx}{(x^2+R^2)^{3/2}} = \frac{R\mu_0 I}{4\pi R^2} \frac{x}{(x^2+R^2)^{1/2}} \Big|_{-\infty}^{0}$

$B = \frac{\mu_0 I}{4\pi R} \frac{L}{(L^2+R^2)^{1/2}}$

b) when $L \Rightarrow \infty$ $\frac{L}{(L^2)^{1/2}} \Rightarrow 1$

so $B = \frac{\mu_0 I}{4\pi R}$

c) $\vec{B} = \int_0^L \frac{\mu_0}{4\pi} I \frac{d\vec{L} \times \hat{r}}{r^2}$

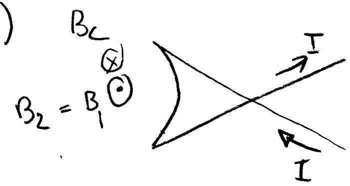


$= \frac{\mu_0}{4\pi} \frac{I}{R^2} \int_0^{R\phi} dL$

$= \frac{\mu_0}{4\pi} \frac{I}{R^2} R\phi$

$= \frac{\mu_0}{4\pi} \frac{I\phi}{R}$

d)



From the straight wires the magnetic fields will both point out of the page & be equal $B_1 = B_2 = \frac{\mu_0 I}{4\pi R}$

So $B_{STRAIGHT} = B_1 + B_2 = \frac{\mu_0 I}{2\pi R}$

The curved section has B in the opposite direction

$B_C - B_{STRAIGHT} = 0$

$B_C = B_{STRAIGHT}$

$\frac{\mu_0 I \theta}{4\pi R} = \frac{\mu_0 I}{2\pi R}$

$\therefore$ $\boxed{\theta = 2 \text{ (in radians)}}$