Physics 1215
Group Problem

A very long cylindrical shell of radius \( a \) shares the same axis with another long cylindrical shell of radius \( b, \ b > a \). The inner shell has a total charge per unit area of \( \sigma = \frac{q}{A} \), while that of the outer shell is \(-\sigma = -\frac{q}{A}\). Both charges are uniformly distributed over the entire length of the cylinders.

1) Ignoring any fringe effects at the end of the cylinders, find the electric field for (i) \( r < a \), (ii) \( a < r < b \), and (iii) \( r > b \).

2) Find the electric potential difference between (i) \( \infty \) and \( b \), (ii) \( b \) and \( a \), and (iii) \( a \) and the axis of the cylinders. (Assume \( V(\infty) = 0 \)).

3) Suppose that instead of having equal and opposite charge per unit area on each cylinder, the cylinders have equal and opposite charge per unit length, \( \lambda = \frac{q}{L} \). In this case, (i) what is the electric field for \( r > b \)? What is the electric potential between \( \infty \) and \( b \)?
1) i) \[ \oint \hat{E} \cdot d\hat{A} = \frac{Q_{enc}}{\varepsilon_0} \]
\[ E \oint dA = \frac{Q_{enc}}{\varepsilon_0} \]
\[ E 2\pi r L = 0 \]
\[ E = 0 \]

ii) \[ \oint \hat{E} \cdot d\hat{A} = \frac{Q_{enc}}{\varepsilon_0} \]
\[ E \oint dA = \frac{Q_{enc}}{\varepsilon_0} \]
\[ E 2\pi r L = \frac{2\pi Ma}{\varepsilon_0} - \frac{2\pi bL}{\varepsilon_0} \]
\[ E = \frac{\sigma (a-b)}{\varepsilon_0 r} \]

iii) \[ \oint \hat{E} \cdot d\hat{A} = \frac{Q_{enc}}{\varepsilon_0} \]
\[ E \oint dA = \frac{Q_{enc}}{\varepsilon_0} \]
\[ E 2\pi r L = \frac{2\pi Ma}{\varepsilon_0} - \frac{2\pi bL}{\varepsilon_0} \]
\[ E = \frac{\sigma (a-b)}{\varepsilon_0 r} \]

2) i) \[ \Delta V = -\oint \hat{E} \cdot ds = -\oint_0^r \frac{\sigma (a-b)}{\varepsilon_0} \cos \Theta (-dr) \]
\[ = \left[ -\frac{\sigma (a-b) dr}{\varepsilon_0} + 0 \frac{(a-b) \ln r}{\varepsilon_0} \right]_0^\infty \]
\[ = -\frac{(a-b) \ln r - (a-b) \ln r}{\varepsilon_0} \]
\[ = -\frac{\ln r}{\varepsilon_0} \]
\[ \Delta V_{b \rightarrow a} = -\oint_a^b \frac{\sigma a dr}{\varepsilon_0 r} \cos \Theta = -\frac{\sigma a \ln r}{\varepsilon_0} \]
\[ \Delta V = \oint_0^\infty \hat{E} \cdot ds = 0 \]

ii) \[ \Delta V = \oint_0^\infty \hat{E} \cdot ds = 0 \]

3) Same picture as (2ii) \[ \oint \hat{E} \cdot d\hat{A} = \frac{Q_{enc}}{\varepsilon_0} = 0 \]
\[ \oint E = 0 \]
\[ \oint \hat{E} \cdot ds = 0 \]