Physics 1215
Group Problem

1. A cylindrical telephone cable has a charge per unit length of λ. The cable stretches along the x axis from $-a$ to a.
   a) Find the electric field anywhere along the y axis. In doing this, there will be three variables that can change for a specific y value. What are those three variables? You will have to write your integral in terms of only one of those three variables.
   b) If you were to do this problem again, but you were to find the electric field at a location not on the y axis, would you get a different answer? Why or why not?

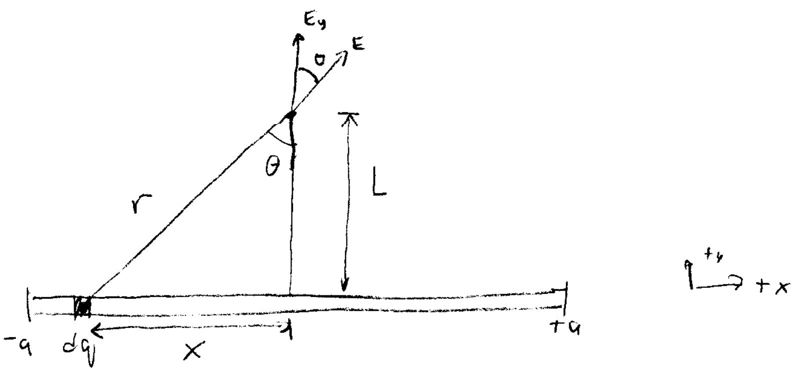
2. Now let the length of the wire be infinite by letting a go to ∞.
   a) What is the electric field for an infinite length wire?
   b) If you were to do this problem again, but you were to find the electric field at a location not on the y axis, would you get a different answer? Why or why not?

3. Consider the same infinitely long cable given in part (2).
   a) Solve (2a) above, but this time use Gauss's law
   b) Compare your answer to that found in (2). Which is easier to do, integrate over the whole line, or use Gauss's law?
   c) Could you use Gauss's law in (1)? Why or why not?
   d) Would it be easy to use Gauss's law in (1)? Why or why not?

4. Now use Gauss's law to find the electric field outside of a spherical charge distribution with total charge Q. Compare your answer with Coulomb's law.

5. An infinitely long nonconducting cylinder has a radius of R and a charge distribution given by $\rho(r) = ar$ where a is a constant.
   a) Find the electric field inside this cylinder.
   b) Find the electric field outside of this cylinder.
   c) Find the total charge per unit length, λ, for the cylinder and write the answer for part (b) in terms of λ. Then compare your answer in 5b with the answer you found in parts 2 and 3 above.

1)



a) $dq = \lambda dx$

by symmetry, the field will only be in the "y" direction

$$E_y = \frac{k\,dq}{r^2}\cos\theta = \frac{k\lambda\cos\theta\;dx}{r^2}$$

There are 3 variables that change, r, x, and θ. I will write all in terms of θ

$$\frac{x}{L} = \tan\theta \Rightarrow x = L\tan\theta \Rightarrow dx = \frac{L}{\cos^2\theta}\,d\theta$$

$$\frac{L}{r} = \cos\theta \Rightarrow r = \frac{L}{\cos\theta}$$

$$\therefore\; E_y = k\lambda\cos\theta\;\frac{\cos^2\theta}{L^2}\;\frac{L}{\cos^2\theta}\;d\theta$$

$$= \frac{k\lambda\cos\theta\;d\theta}{L}$$

Let $\theta_0 = \tan^{-1} a/L$, $r_0 = r$ when $\theta = \theta_0$, $r = \sqrt{a^2+L^2}$

$$E = 2\int_0^{\theta_0} \frac{k\lambda\cos\theta\;d\theta}{L} = \frac{2k\lambda}{L}\sin\theta\Big|_0^{\theta_0}$$

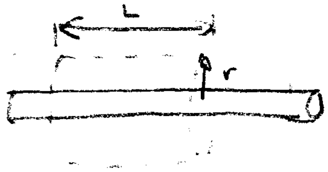
$$= \frac{2k\lambda}{L}\sin\theta_0 = \frac{2k\lambda}{L}\frac{a}{r_0} = \boxed{\frac{2k\lambda}{L}\frac{a}{\sqrt{a^2+L^2}}}$$

b) Sure, there would be a different answer. The x components do not cancel out.

2 a) As a gets Large, $\sqrt{a^2+L^2} \Rightarrow a$

so $E = \frac{2k\lambda \cancel{a}}{L\cancel{a}} = \boxed{\frac{2k\lambda}{L}}$

b) I would get the same answer. Because the line charge is infinite, the x components will always cancel out

3a) 

$$\oint \vec{E}\cdot d\vec{A} = q_{encl}/\varepsilon_0$$

$$\int_{CYLINDER} \vec{E}\cdot d\vec{A} + 2\int_{ENDS} \vec{E}\cdot d\vec{A} = q_{encl}/\varepsilon_0$$

$$\int_{CYLINDER} \vec{E}\cdot d\vec{A} = q_{encl}/\varepsilon_0$$

$$\vec{E}\cdot \int_{CYLINDER} d\vec{A} = q_{encl}/\varepsilon_0$$

$$\vec{E}\cdot\vec{A} = q_{encl}/\varepsilon_0$$

$$E\,2\pi r L = \lambda L/\varepsilon_0$$

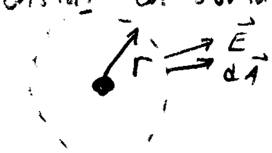
$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2k\lambda}{r} \qquad \text{since } k = \frac{1}{4\pi\varepsilon_0}$$

b) This is exactly the same as (2a) since $L = r$

c) Yes you could, because Gauss's Law can be used in any circumstances

d) It would be hard because the electric field would vary in direction and magnitude along a cylindrical Gaussian surface, so the integral would be very hard

4) Choose a spherical Gaussian surface so E is constant on surface



$$\oint \vec{E}\cdot d\vec{A} = q_{encl}/\varepsilon_0$$

$$\vec{E}\cdot \oint d\vec{A} = q_{encl}/\varepsilon_0$$

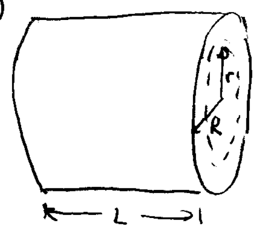
$$\vec{E}\cdot\vec{A} = q_{encl}/\varepsilon_0$$

$$EA = q/\varepsilon_0$$

$$E\,4\pi r^2 = q/\varepsilon_0$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2} = \frac{kq}{r^2} \quad \text{which is Coulomb's law}$$

5) a)



$$\oint \vec{E} \cdot d\vec{A} = q_{encl}/\varepsilon_0$$

$$\int_{sides} \vec{E} \cdot d\vec{A} + \int_{ends} \vec{E} \cdot d\vec{A} = q_{encl}/\varepsilon_0$$

$$\int_{sides} \vec{E} \cdot d\vec{A} = q_{encl}/\varepsilon_0$$

(I) $\vec{E} \cdot \int d\vec{A} = \frac{1}{\varepsilon_0} \int_0^r \rho(r)\, dV$ $\qquad V = \pi r^2 L$

$\vec{E} \cdot \vec{A} = \frac{1}{\varepsilon_0} \int_0^r a r L\, 2\pi r\, dr$ $\qquad dV = L\, 2\pi r\, dr$

$$E\, 2\pi r L = \frac{2\pi a L}{\varepsilon_0} \int_0^r r^2 dr = \frac{2\pi a L}{\varepsilon_0} \frac{r^3}{3}$$

$$\boxed{E = \frac{a r^2}{3\varepsilon_0} \text{ in the radial direction (INSIDE)}}$$

b) Like (I)

$$\vec{E} \cdot \int d\vec{A} = \frac{1}{\varepsilon_0} \int_0^R \rho(r)\, dV = \frac{1}{\varepsilon_0} \int_0^R a r L 2\pi r\, dr$$

$$E\, 2\pi r L = \frac{2\pi a L}{\varepsilon_0} R^3/3$$

$$\boxed{E = \frac{a R^3}{3\varepsilon_0 r} \text{ in the radial direction (outside)}}$$

c) The total charge per length inside the cylinder is

$$\lambda = \frac{Q_{TOT}}{L} = \frac{1}{L} \int \rho(r)\, dV = \frac{1}{L} \int_0^R a L 2\pi r^2 dr$$

$$= \frac{2\pi a L}{L} \frac{R^3}{3} = \frac{2\pi a R^3}{3}$$

so $E = \frac{a R^3}{3\varepsilon_0 r} = \frac{2\pi a R^3}{3} \frac{1}{2\pi\varepsilon_0 r} = \boxed{\frac{\lambda}{2\pi\varepsilon_0 r} \text{ which is the Same as } 3_4 + 2_9}$