

### Group Problem

The Doppler effect can be observed for electromagnetic radiation. But because there is no absolute frame of reference where something is at rest (like the air in the Doppler equations we derived in class), the equation for the electromagnetic Doppler effect is slightly different.

$$f_R = f_S \sqrt{\frac{c-v}{c+v}}$$

where  $f_R$  is the frequency of the receiver,  $f_S$  is the frequency of the source,  $c$  is the speed of light, and  $v$  is the velocity of the source in the frame of reference where the receiver is at rest. If the source and the receiver are moving directly toward each other then the sign of  $v$  is negative. If the source and the receiver are moving directly away from each other then the sign of  $v$  is positive.

- a) Show that the above equation can be written as  $f_R = f_S \left(1 - \frac{v}{c}\right)^{1/2} \left(1 + \frac{v}{c}\right)^{1/2}$
- b) Use the binomial theorem to show that if  $v \ll c$ , then this is approximately equal to

$$f_R = f_S \left(1 - \frac{v}{c}\right)$$

- c) Microwaves with a wavelength of 0.100 m are reflected off a distant airplane approaching the wave source. The reflected wave interferes with the wave source to produce beats with a frequency of 990 Hz. How fast is the airplane approaching?

## Group Problem

$$a) f_R = f_S \sqrt{\frac{c-v}{c+v}} = f_S \sqrt{\frac{1-v/c}{1+v/c}} = f_S (1-v/c)^{1/2} (1+v/c)^{-1/2}$$

$$b) \text{ Using } (1+x)^n = 1+nx + \frac{1}{2}n(n-1)x^2$$

Then, keeping only terms linear in  $\frac{v}{c}$  since  $\frac{v}{c} \ll 1$

$$\begin{aligned} f_R &= f_S \left(1 - \frac{1}{2}\frac{v}{c} + \dots\right) \left(1 + \frac{1}{2}\frac{v}{c} + \dots\right) \\ &= f_S \left(1 - \frac{1}{2}\frac{v}{c} - \frac{1}{2}\frac{v}{c} + \dots\right) \\ &= f_S \left(1 - \frac{v}{c}\right) \end{aligned}$$

$$c) f_S = \frac{v}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{.100 \text{ m}} = 3 \times 10^9 \text{ Hz}$$

for this problem the plane acts first as a receiver.

then as a source when it reflects the wave

With  $f_1 = 3 \times 10^9 \text{ Hz}$ , the plane receives a signal  $f_2$

$f_2 = f_1 (1 - v/c)$ , then transmits that signal, so the

source receives a final signal  $f_3$

$$f_3 = f_2 (1 - v/c) = f_1 (1 - v/c)^2 = f_1 (1 - 2v/c + v^2/c^2) \sim f_1 (1 - 2v/c)$$

$$f_{\text{beat}} = f_3 - f_1 = f_1 - 2f_1 \frac{v}{c} - f_1$$

$$f_{\text{beat}} = -2f_1 \frac{v}{c}$$

( $v < 0$  since approaching)

$$v = -\frac{c}{2} \frac{f_{\text{beat}}}{f_1} = \frac{3.0 \times 10^8 \text{ m/s}}{2} \frac{990 \text{ Hz}}{3 \times 10^9 \text{ Hz}} = \boxed{49.5 \text{ m/s}}$$