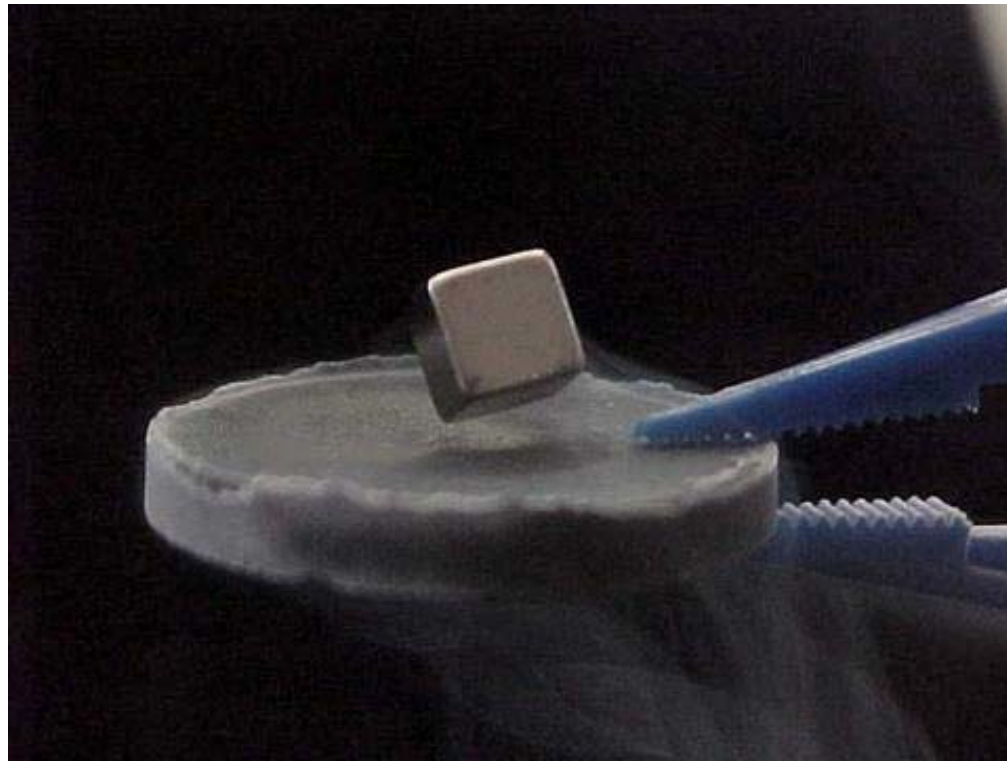
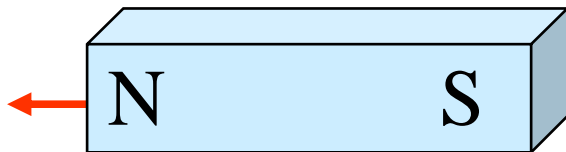
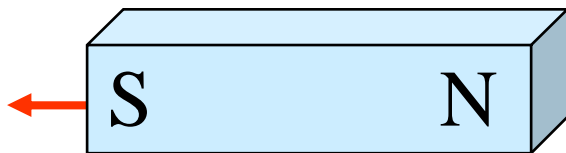
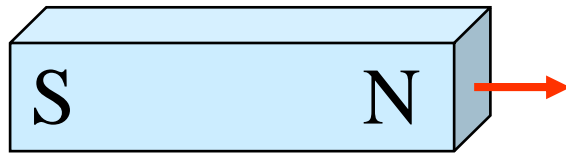
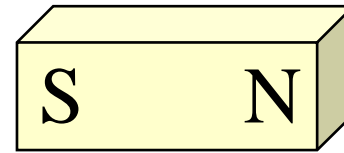
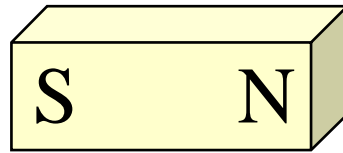
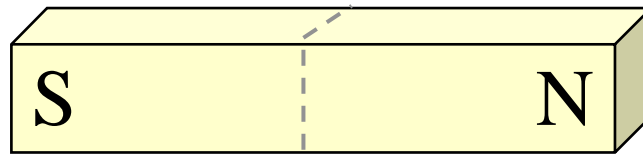
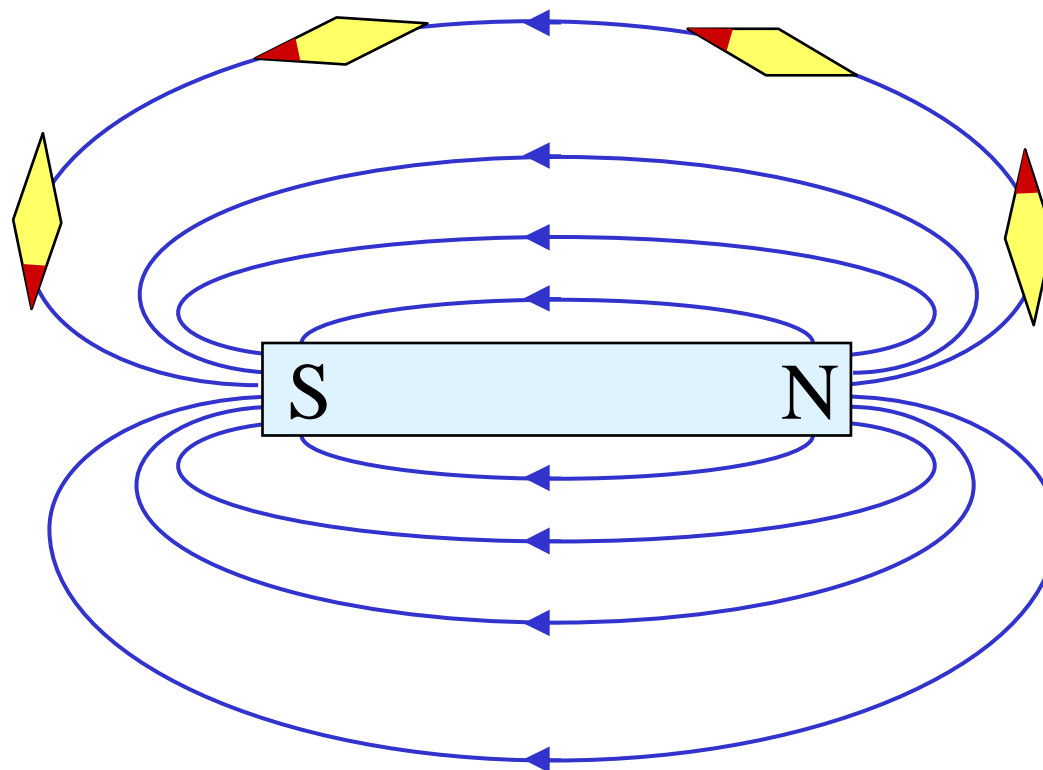


Chapter 20

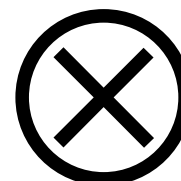
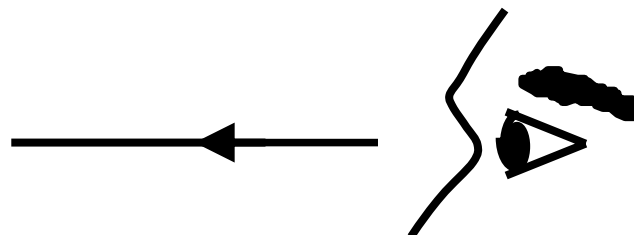
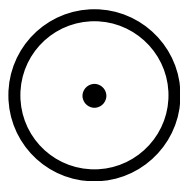
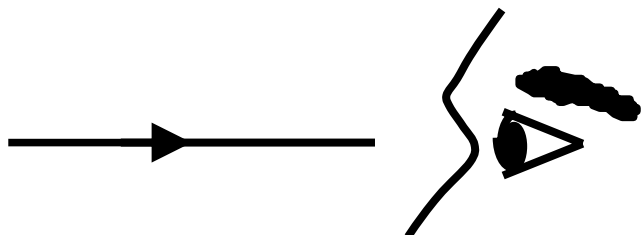
Magnetic Field Forces and the Magnetic Field







Looking at Vectors



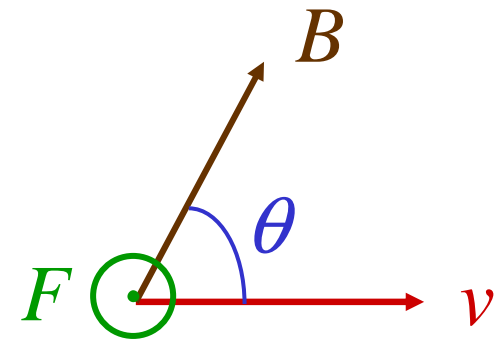
Magnetic Forces on Objects

Magnetic fields will produce a force on an object, if the object satisfies three criteria:

1. The object must have an electric charge.
2. The object must be moving.
3. The velocity of the object must have a component that is **perpendicular** to the direction of the magnetic field.

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$
$$F = Bqv \sin \theta$$

This shows the direction of the force for a positively charged particle:

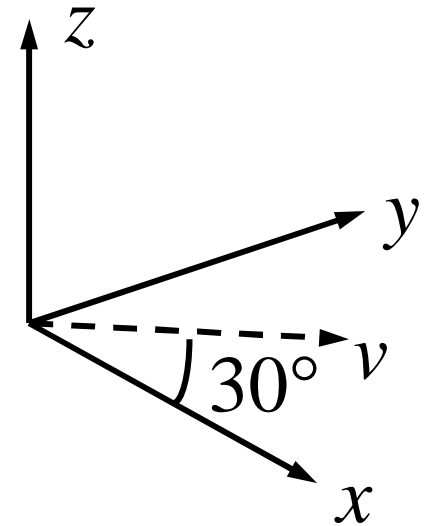


SI units of B: Tesla (T)=1 N·s/m·C=1 N/A·m, 1G=10⁻⁴ T

$$\begin{aligned}
 \mathbf{A} \times \mathbf{B} &= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \\
 &= (A_y B_z - A_z B_y) \hat{\mathbf{i}} + (A_z B_x - A_x B_z) \hat{\mathbf{j}} + (A_x B_y - A_y B_x) \hat{\mathbf{k}}
 \end{aligned}$$

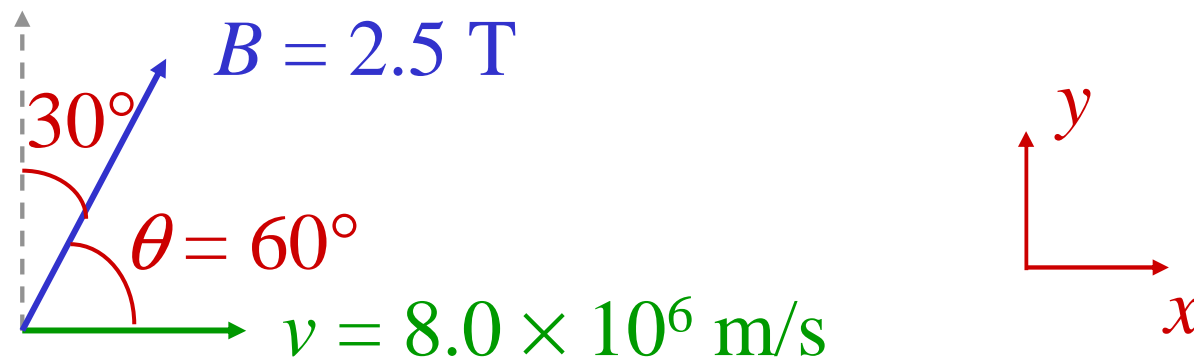
Interactive Question

An object with a positive charge travels in the xy plane at an angle of 30° above the x axis. A magnetic field points along the y axis. What direction is the force that the object feels?



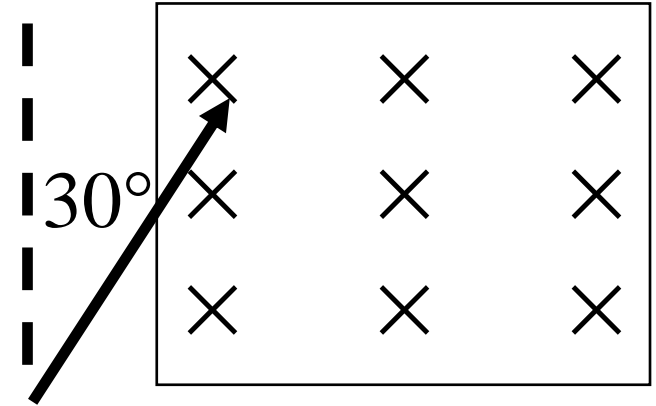
- A) 60° below the positive x axis
- B) 120° above the positive x axis
- C) Along the negative z axis
- D) Along the positive z axis
- E) There is no force on the object

Problem: A proton moves at 8.0×10^6 m/s along the x axis. It enters a region where there is a magnetic field of magnitude 2.5 T directed at an angle of 30° from the positive y axis and lying in the xy plane. What is the initial force the proton feels and what is its acceleration?



Interactive Question

A proton enters a region that contains a uniform magnetic field directed into the paper as shown. The velocity vector of the proton makes an angle of 30° with the positive y axis as shown. What is



the direction of the magnetic force on the proton when it enters the field?

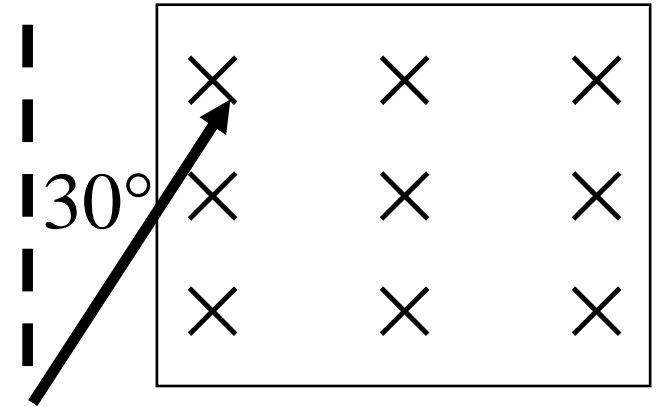
- A) up out of the paper
- B) at an angle of 30° below the negative x axis
- C) at an angle of 30° above the negative x axis
- D) at an angle of 60° below the negative x axis
- E) at an angle of 60° above the negative x axis

Interactive Question

An electron enters a region that contains a uniform magnetic field directed into the paper as shown.

The velocity vector of the electron makes an angle of 30° with the positive y axis as shown. What is

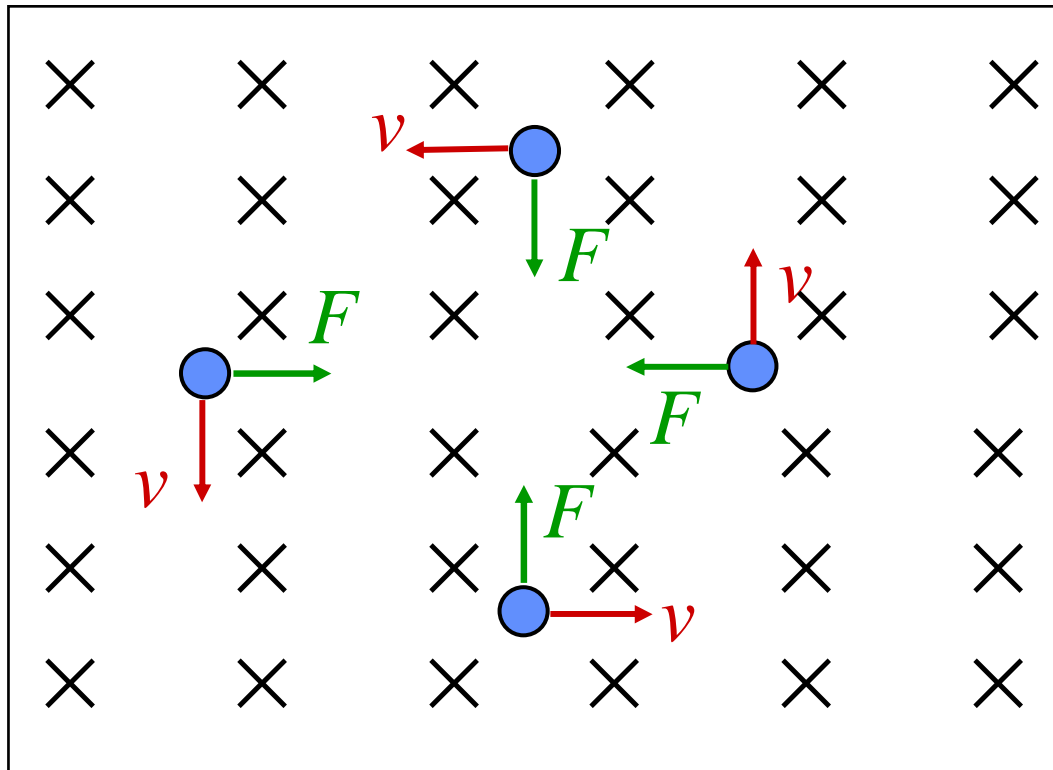
the direction of the magnetic force on the electron when it enters the field?



- A) up out of the paper
- B) at an angle of 30° below the positive x axis
- C) at an angle of 30° above the positive x axis
- D) at an angle of 60° below the positive x axis
- E) at an angle of 60° above the positive x axis

Motion of a Charged Particle in a Magnetic Field

Positive Particle in Magnetic Field



$$F = ma$$

$$Bqv \sin \theta = mv^2/r$$

$$Bqv = mv^2/r$$

$$mv/r = Bq$$

$$r = mv/Bq$$

$$T = d/v = 2\pi r/v$$
$$= 2\pi rm/rBq$$

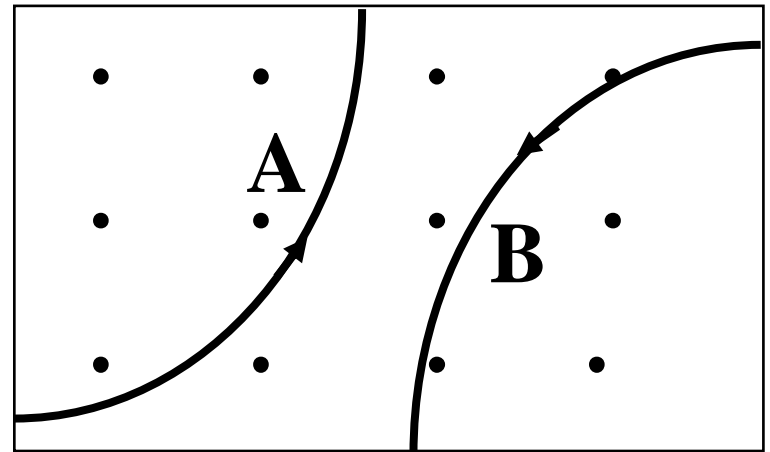
$$T = 2\pi m/Bq$$

$$f = Bq/2\pi m$$

Interactive Question

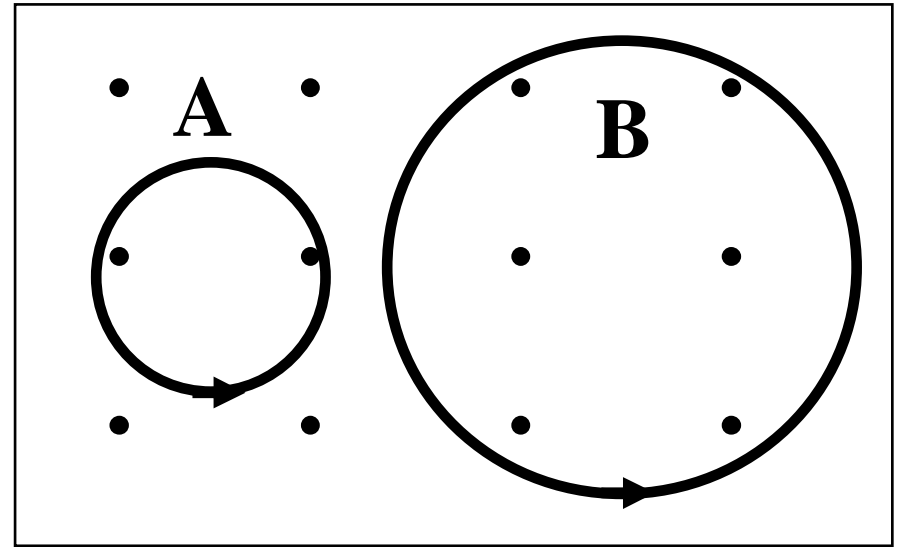
Two particles move through a magnetic field that is directed out of the page. The figure shows the paths taken by two particles as they move through the field. Which statement concerning these particles is true?

- A) The particles may both be neutral.
- B) **A** is positively charged:
B is negative
- C) **A** is positively charged:
B is positive
- D) **A** is negatively charged: **B** is negative
- E) **A** is negatively charged: **B** is positive



Interactive Question

Two particles of equal mass and equal charge move in circular orbits in a uniform magnetic field as shown. Which is the correct entry in the table?



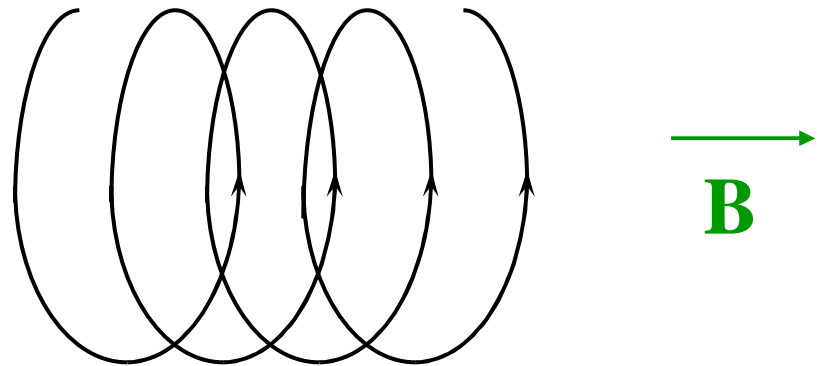
	<u>Charge of A</u>	<u>Charge of B</u>	<u>Greater Velocity</u>
A)	+	+	B
B)	+	+	A
C)	-	-	B
D)	-	-	A
E)	-	+	A

If a particle has a component of velocity perpendicular to the magnetic field, and *also* parallel to the magnetic field, the parallel component will not be affected.

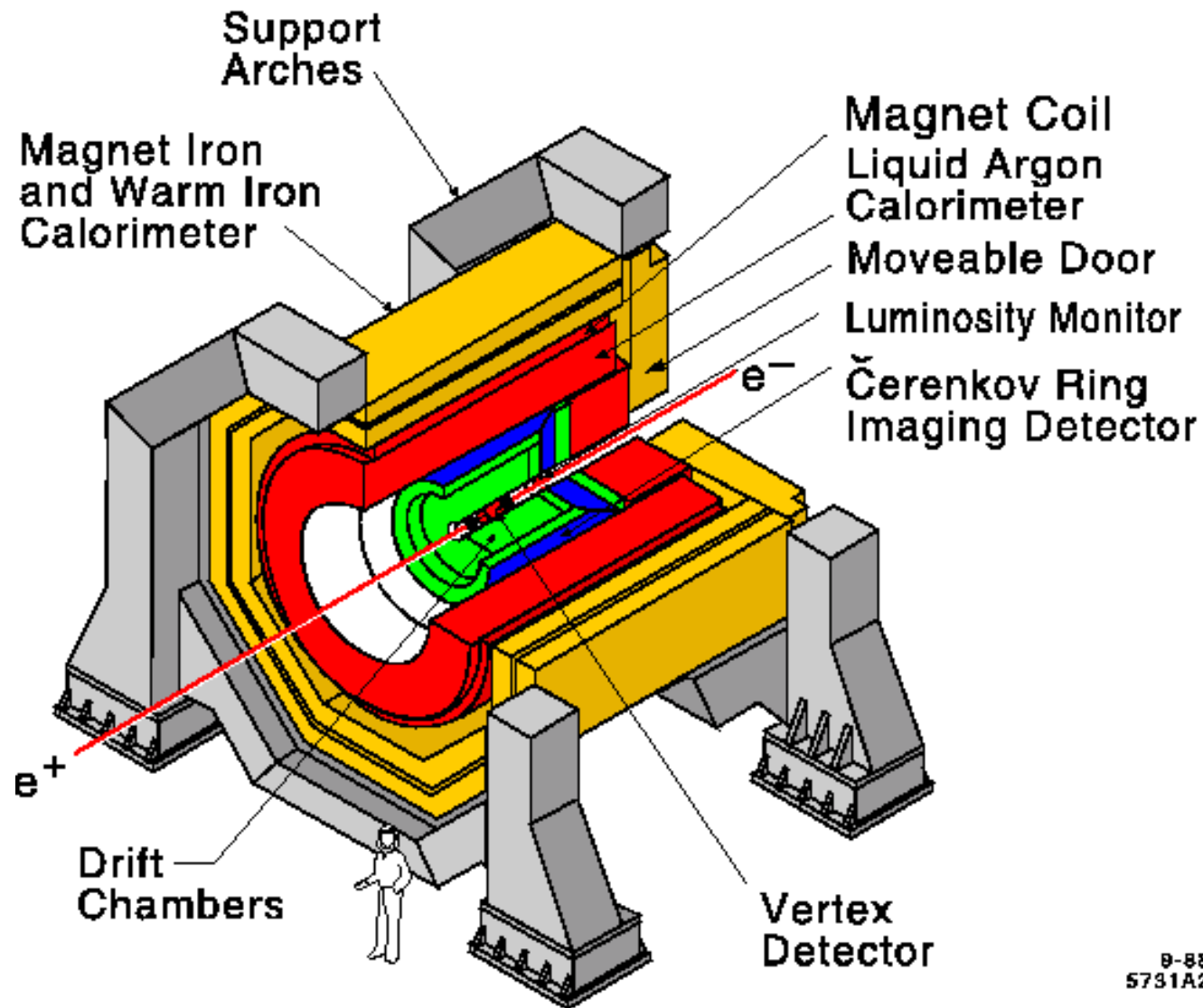
$$\mathbf{v} = \mathbf{v}_{||} + \mathbf{v}_{\perp}$$

↑ ↖
circle straight line

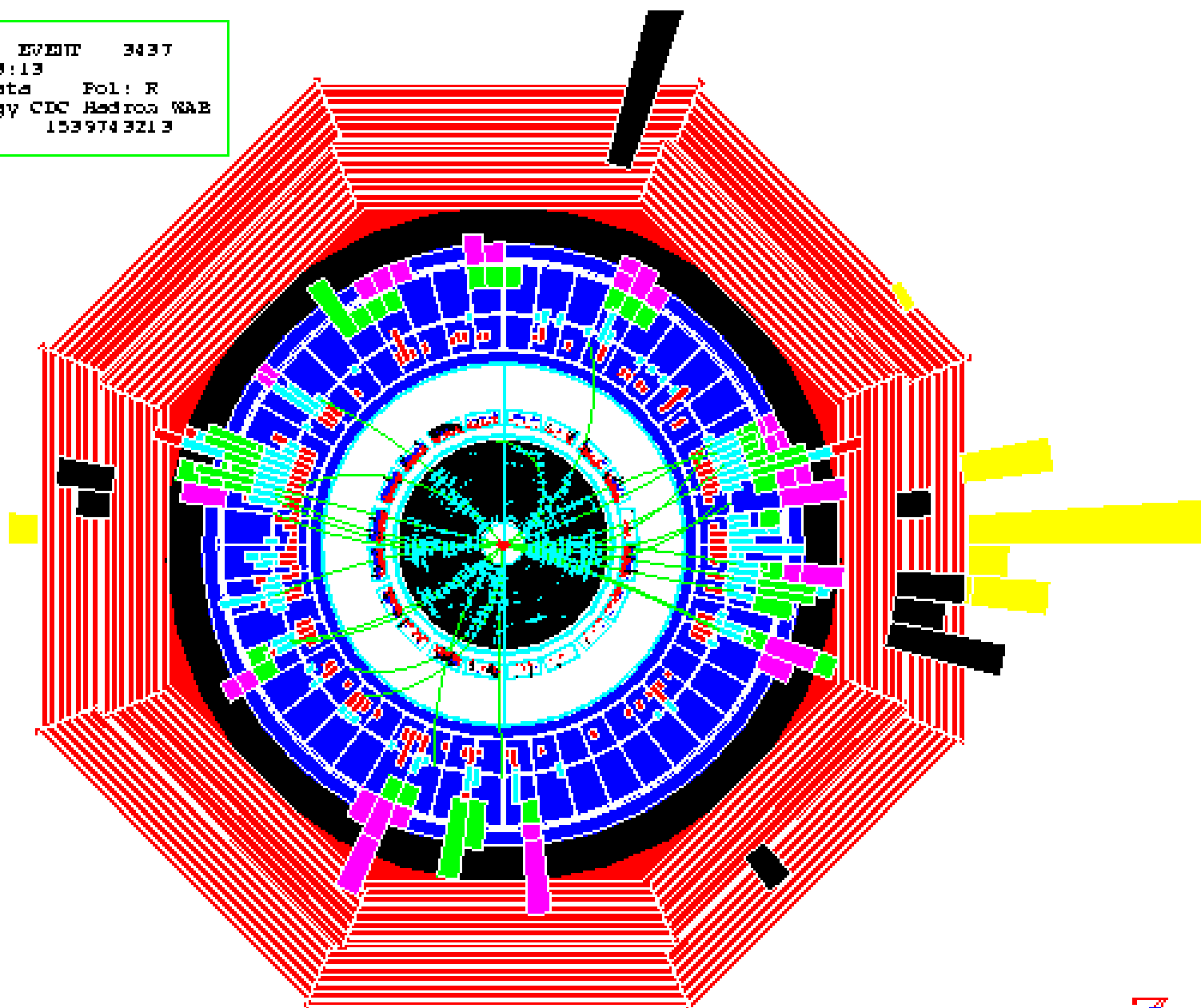
which gives a
helix in a uniform
magnetic field



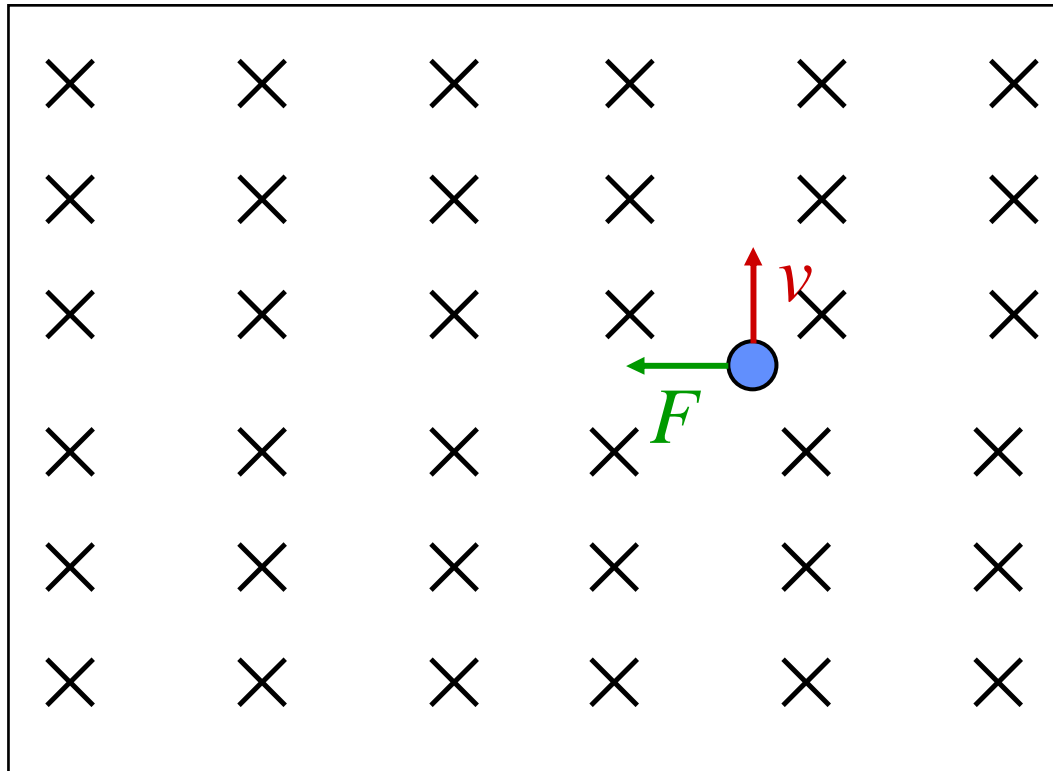
SLD



Run 34356, EVENT 3437
28-MAY-1990 13:13
Source: Run Data Pol: R
Trigger: Energy CDC Hadron WAB
Beam Crossing 1539743213

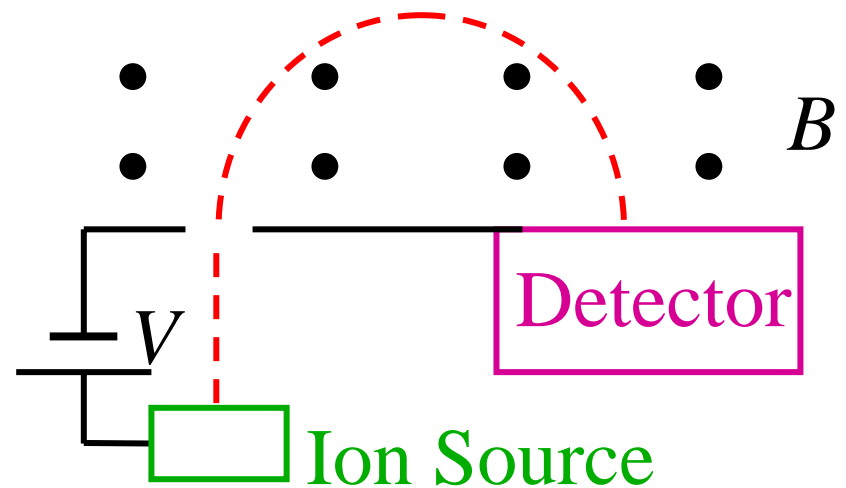


Work Done by a Magnetic Field



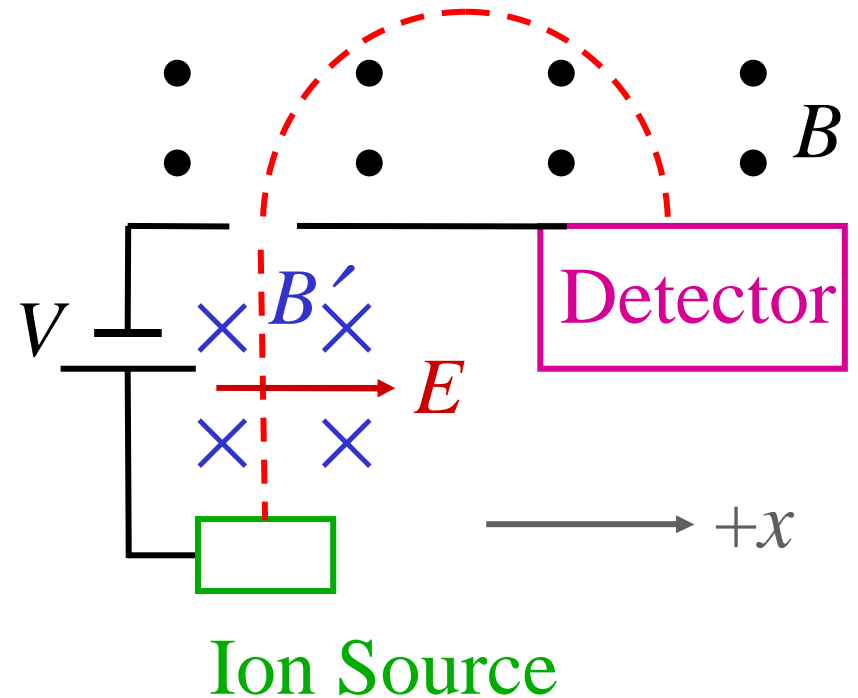
$$W = Fd \cos \theta = 0$$

Problem: An ion in a mass spectrometer is accelerated across a potential difference of V and enters a region with a magnetic field B .



- (a) What is the speed of the ion when it enters the region with the magnetic field?
- (b) What is the work done by the magnetic field?
- (c) What is the mass of the ion?
- (d) What force does the ion feel when it enters the field?
- (e) How do things change if B is in direction of motion of the ion?

Sometimes “crossed” electric and magnetic fields are placed in the same region to select ions with a particular velocity.



For the ion to go straight,

$$\Sigma \mathbf{F} = m\mathbf{a} = 0$$

$$q\mathbf{E} - q\mathbf{v} \times \mathbf{B}' = 0$$

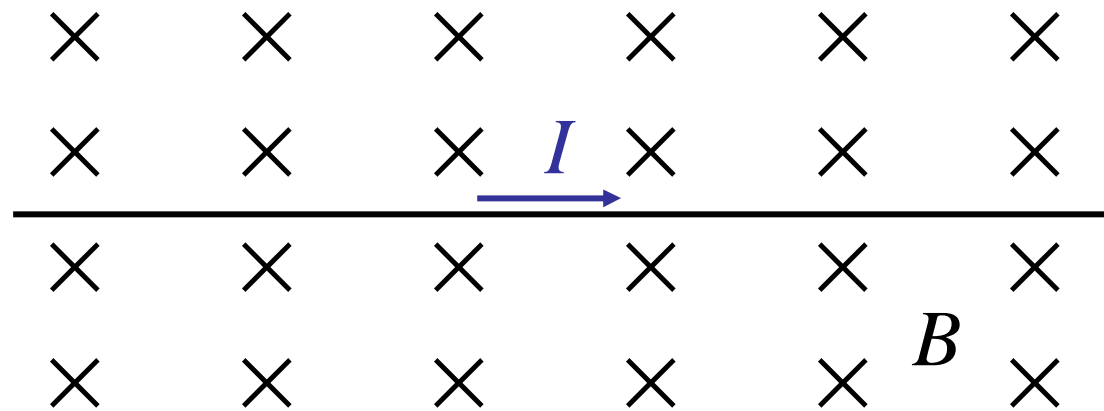
If \mathbf{E} , \mathbf{v} and \mathbf{B}' are at right angles to each other,

$$v = E/B'$$

This is called a “velocity selector”

Problem: A proton travels north at a speed of 4.5×10^6 m/s through a 1.2 T uniform magnetic field that points west. In the same region is a uniform electric field. The proton experiences no acceleration. What is the direction and magnitude of the uniform electric field?

Current Carrying Wire in a Magnetic Field



For a single particle: $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$

For many particles: $\mathbf{F} = (q\mathbf{v} \times \mathbf{B})nV$

where n is the number per unit volume (V)

$$\mathbf{F} = (q\mathbf{v} \times \mathbf{B})nAL$$

$$\mathbf{F} = (\mathbf{L} \times \mathbf{B}) qvnA$$

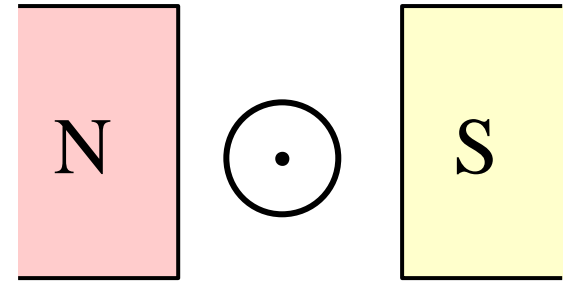
$$\mathbf{F} = I\mathbf{L} \times \mathbf{B}$$

I : current in the wire

\mathbf{L} : length of the wire

Interactive Question

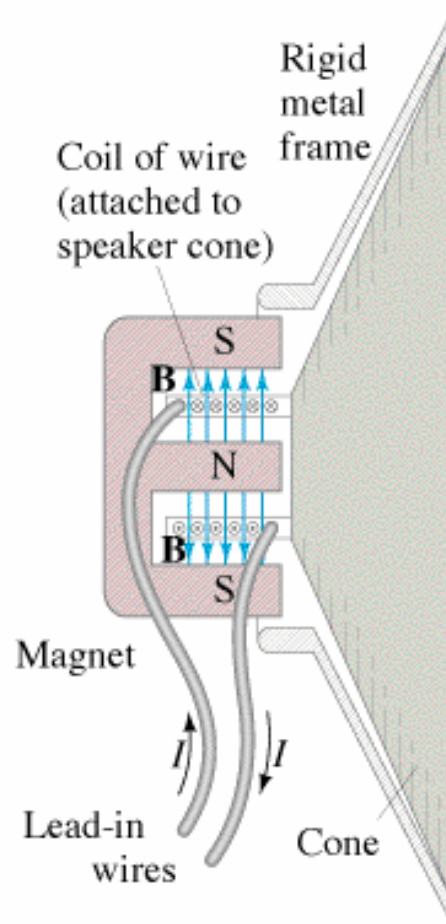
A long straight wire is placed between the poles of a magnet as shown. When a current is flowing out of the page as shown, the direction of the magnetic force on the wire will be:



- A) toward the left
- B) toward the right
- C) toward the top of the page
- D) toward the bottom of the page
- E) out of the page

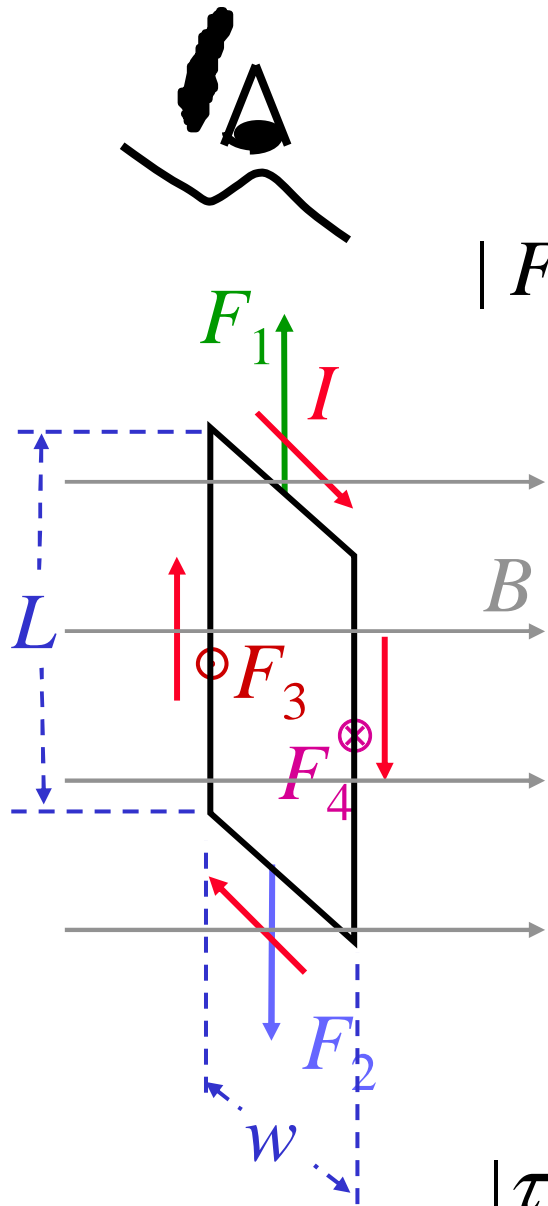
Problem: A wire with a mass of 1.00 g/cm is placed on a horizontal surface with a coefficient of friction of 0.200 . The wire carries a current of 1.50 A eastward and moves horizontally to the north. What are the magnitude and direction of the smallest magnetic field that enables the wire to move in this fashion?

Audio speakers use this principle to create sound



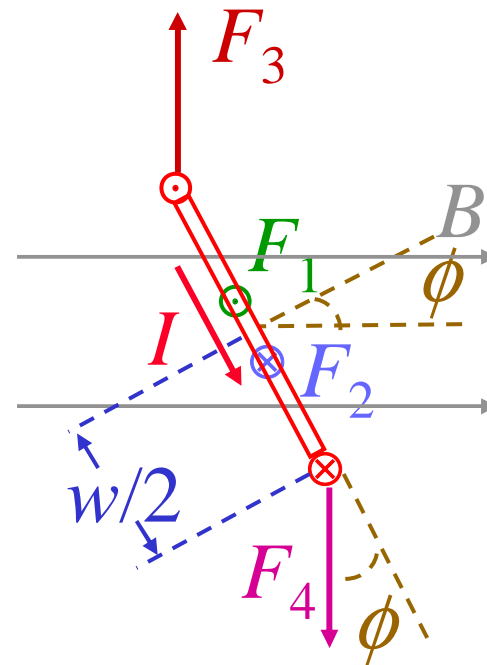
Problem: A square loop of wire sits in the x - y plane with two sides on the x and y axis and two corners at $(0,0)$ and (L,L) . The magnetic field is given by $\mathbf{B} = (B_0 z/L)\mathbf{j} + (B_0 y/L)\mathbf{k}$ where B_0 is a positive constant. The current moves clockwise around the wire. Find the magnitude and direction of the force on each side of the loop and the net magnetic force on the loop.

Magnetic Fields Produce a Torque on Current Carrying Wire Loops

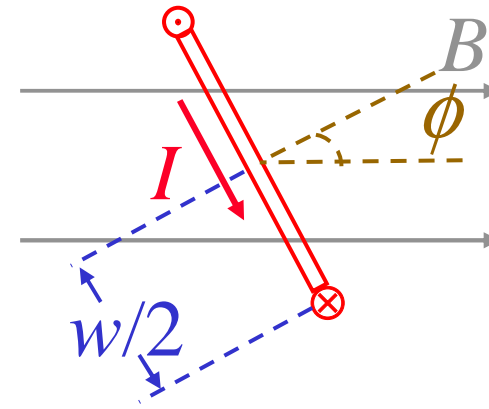
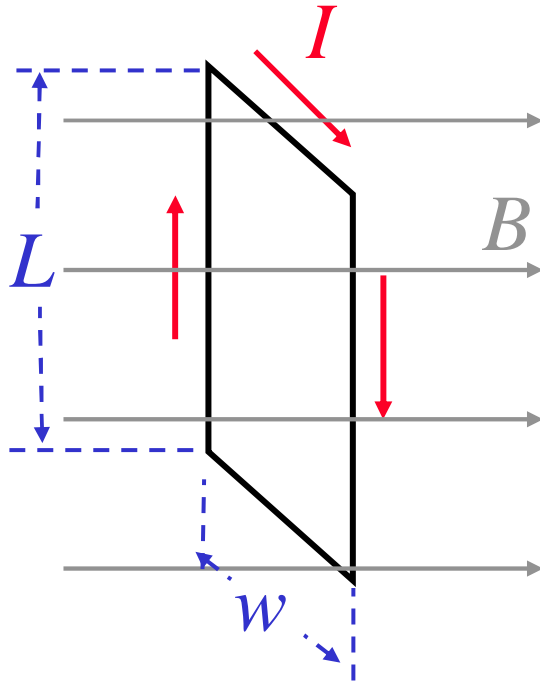


$$|F_1| = |F_2|$$

$$|F_3| = |F_4| = F$$



$$|\tau| = F L \sin \phi = ILB(\sin \theta)(w/2) \sin \phi (\times 2)$$



$$\tau = ILB(\sin \theta)(w/2) \sin \phi (\times 2) = ILBw \sin \phi$$

$$\tau = IAB \sin \phi$$

where A is the area of the loop

For more than one loop of wire:

$$\tau = NIAB \sin \phi = \mu B \sin \phi$$

$$\tau = \mu \times B$$

where $\mu = NIA$ is called the magnetic moment

Potential Energy of a Wire Loop in a Magnetic Field

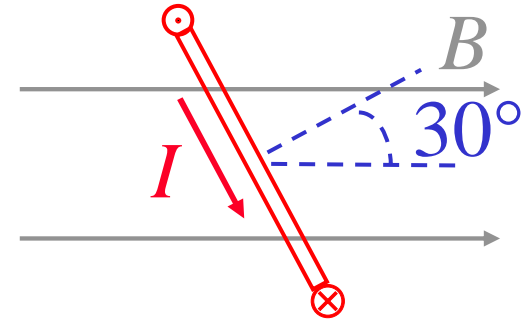
$$U = -W = -\int(-\tau d\theta) = \int \mu B \sin \theta d\theta$$

$$U = -dW = -\mu B \cos \theta + C$$

Chose potential to be zero when $\theta = 90^\circ$.

$$U = -\mu B \cos \theta = -\boldsymbol{\mu} \cdot \mathbf{B}$$

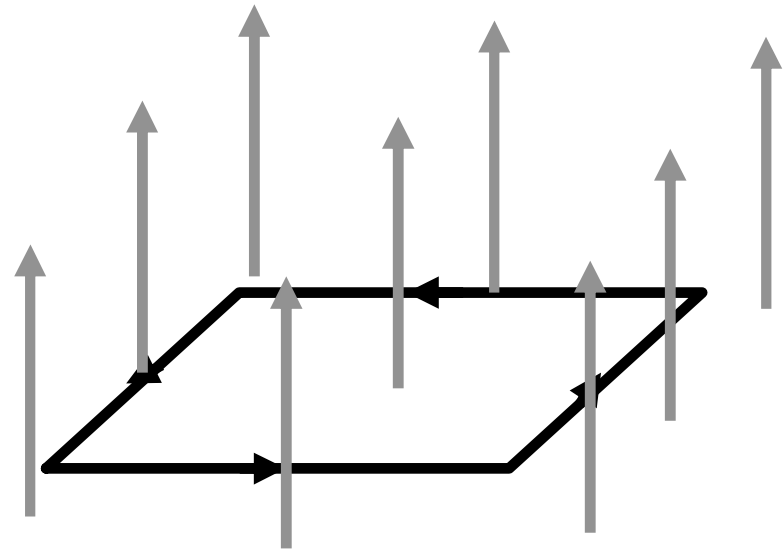
Problem: A circular wire of 15 loops with radius 50.0 cm is oriented at an angle of 30° to a magnetic field of 0.50 T. The current in the loop is 2.0 A in the direction shown.



- Find the magnetic moment of the loop.
- Find the torque at this instance.
- Find the potential energy of the loop.

Interactive Question

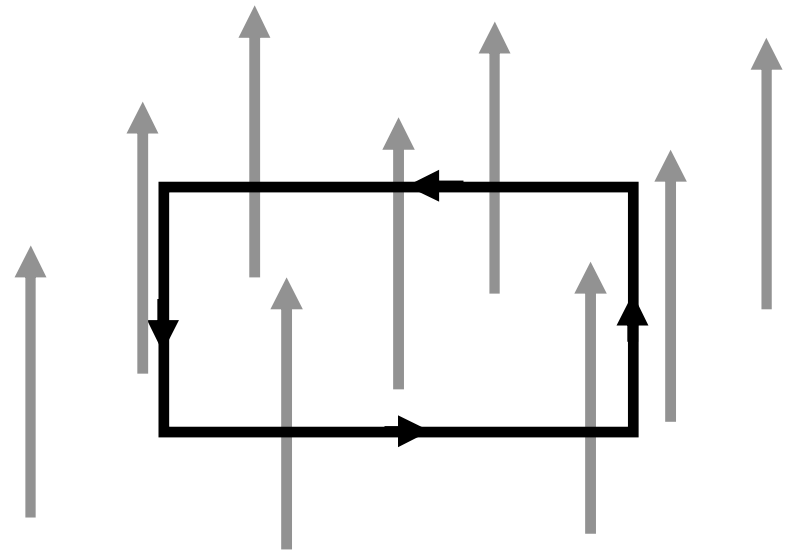
A rectangular loop is placed in a uniform magnetic field with the plane of the loop perpendicular to the direction of the field. If a current is made to flow through the loop in the sense shown by the arrows, the field exerts on the loop:



- A) a net torque
- B) a net force
- C) a net force and a net torque
- D) neither a net force or a net torque

Interactive Question

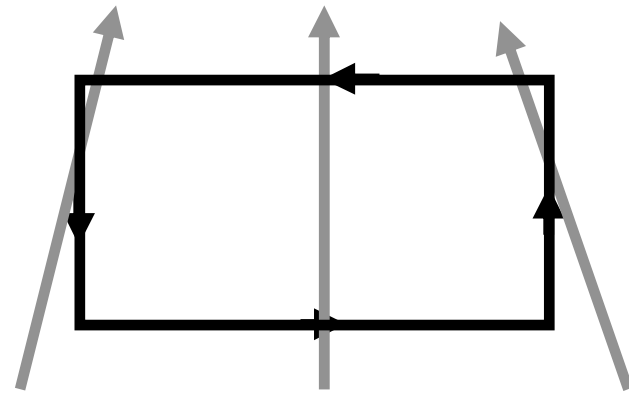
A rectangular loop is placed in a uniform magnetic field with the plane of the loop parallel to the direction of the field. If a current is made to flow through the loop in the sense shown by the arrows, the field exerts on the loop:



- A) a net torque
- B) a net force
- C) a net force and a net torque
- D) neither a net force or a net torque

Interactive Question

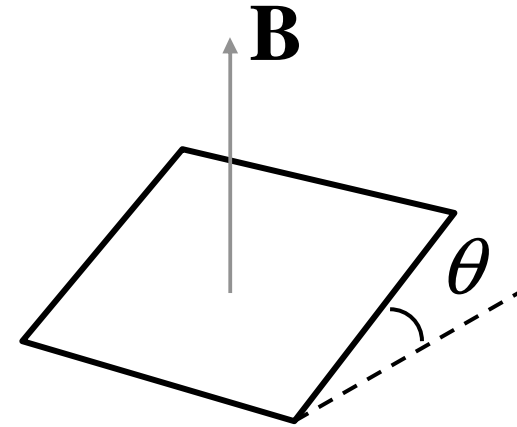
A rectangular loop is placed in a nonuniform magnetic field with the plane of the loop parallel to the direction of the field at its center. If a current is made to flow through the loop in the sense shown by the arrows, the field exerts on the loop:



- A) a net torque
- B) a net force
- C) a net force and a net torque
- D) neither a net force or a net torque

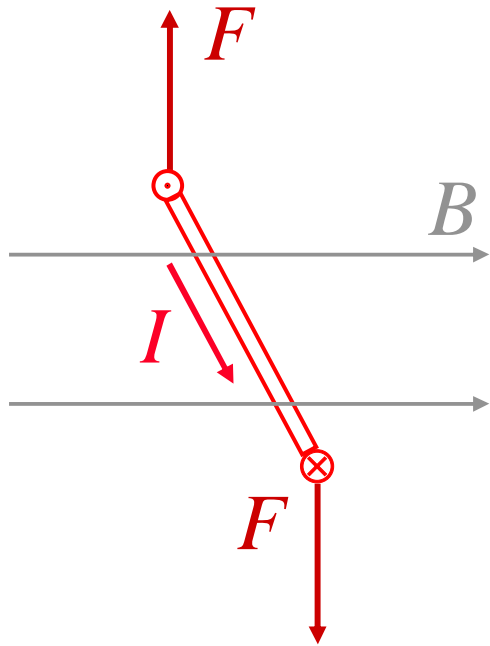
Interactive Question

Consider a square loop of wire that is balancing at an angle θ with only one end on the ground in a constant vertical magnetic field. What can you say about this situation?

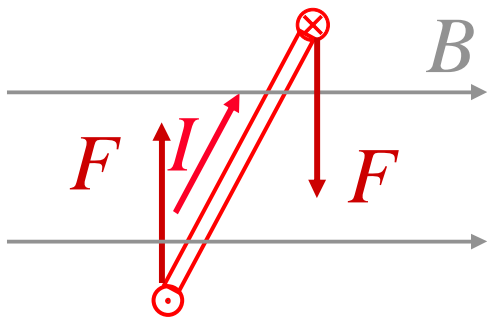


- A) This is an impossible situation
- B) A current is moving clockwise around the loop
- C) A current is moving counterclockwise around the loop
- D) There must also be an electric field
- E) It's all done with smoke and mirrors

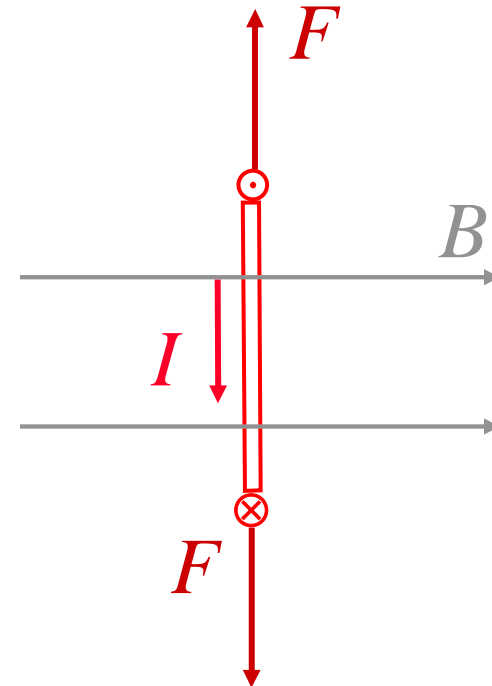
An Electric Motor



Change direction
of current

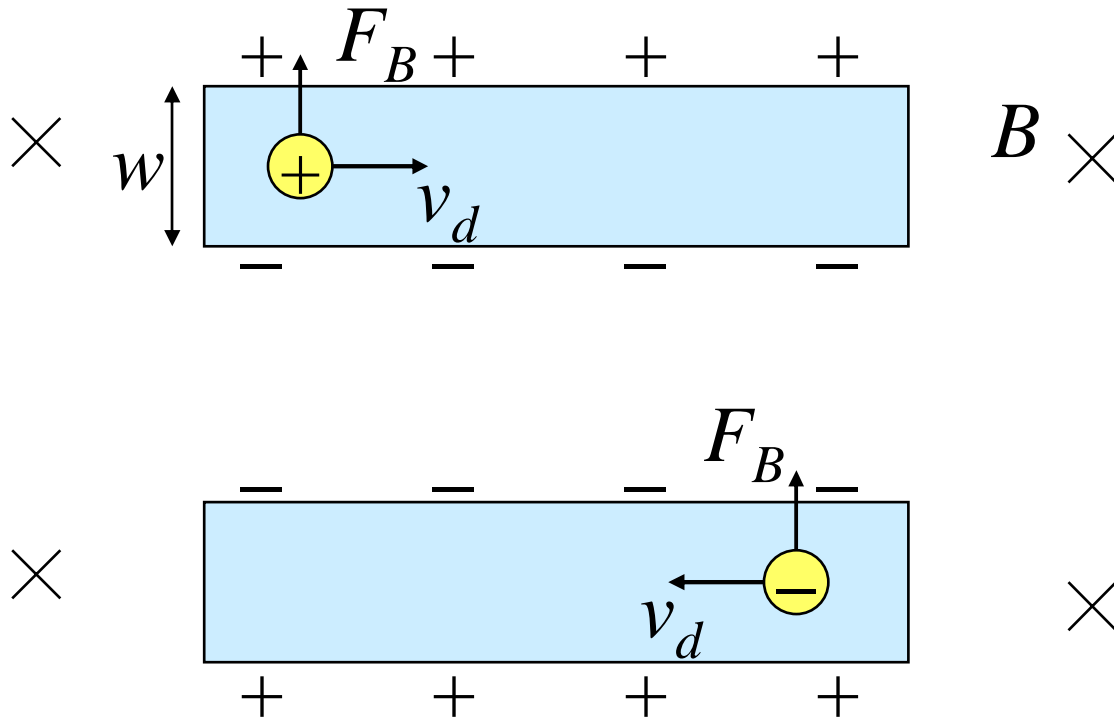


No torque



Repeat the cycle

The Hall Effect



In equilibrium

$$F_B = F_E$$

$$qv_d B = qE$$

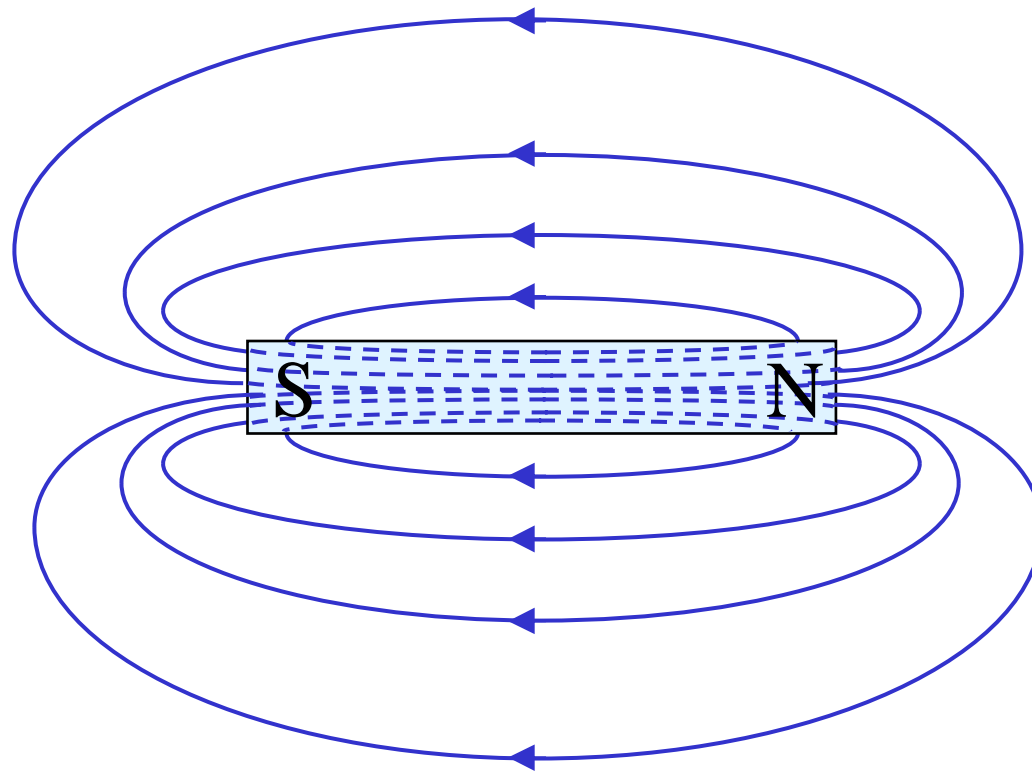
$$E = v_d B$$

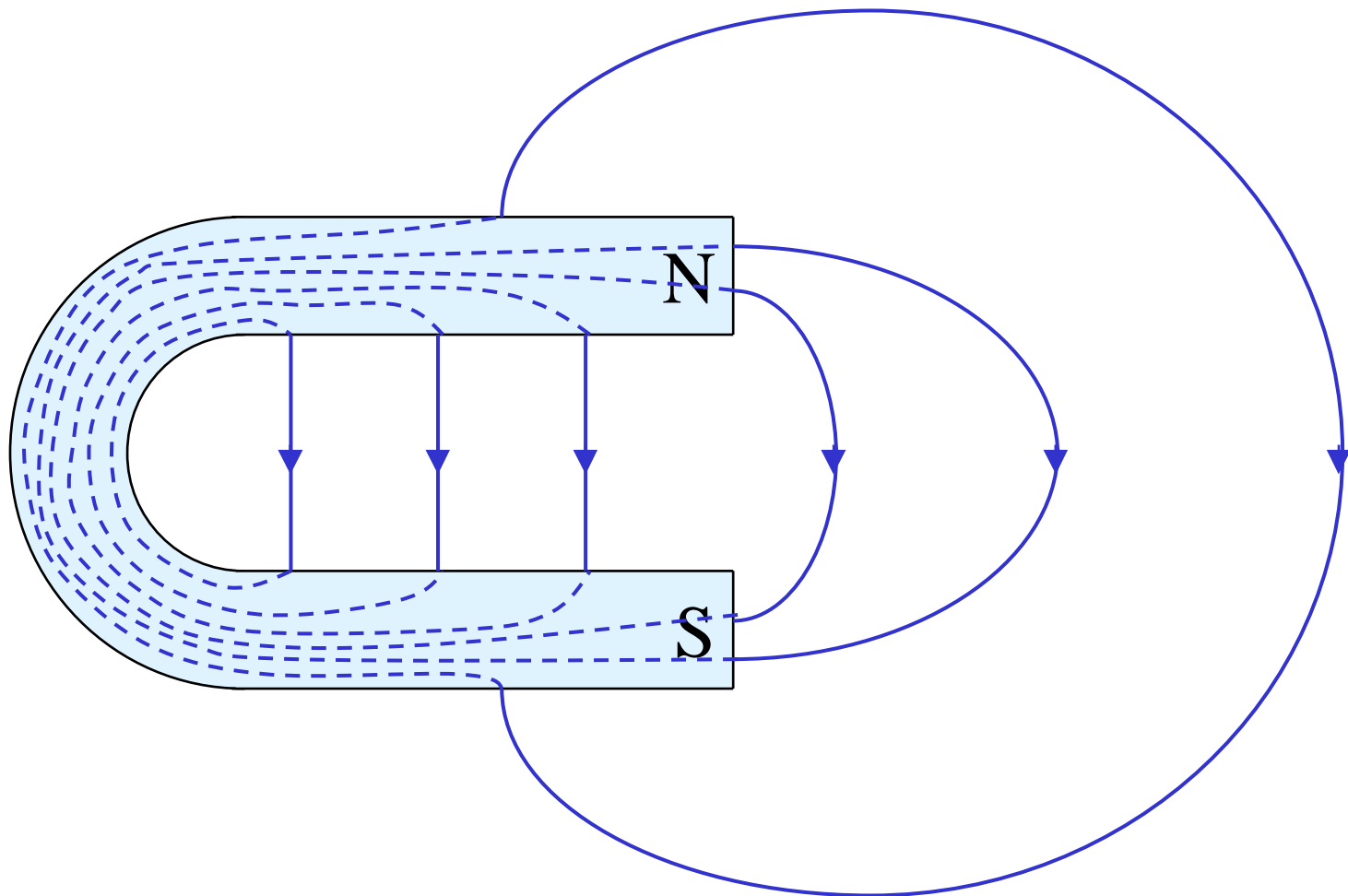
$$V_H = Ew = wv_d B$$

For thickness t ,

$$I = nqv_d A$$

$$n = I/qv_d A = I/ev_d wt = IB/etV_H$$





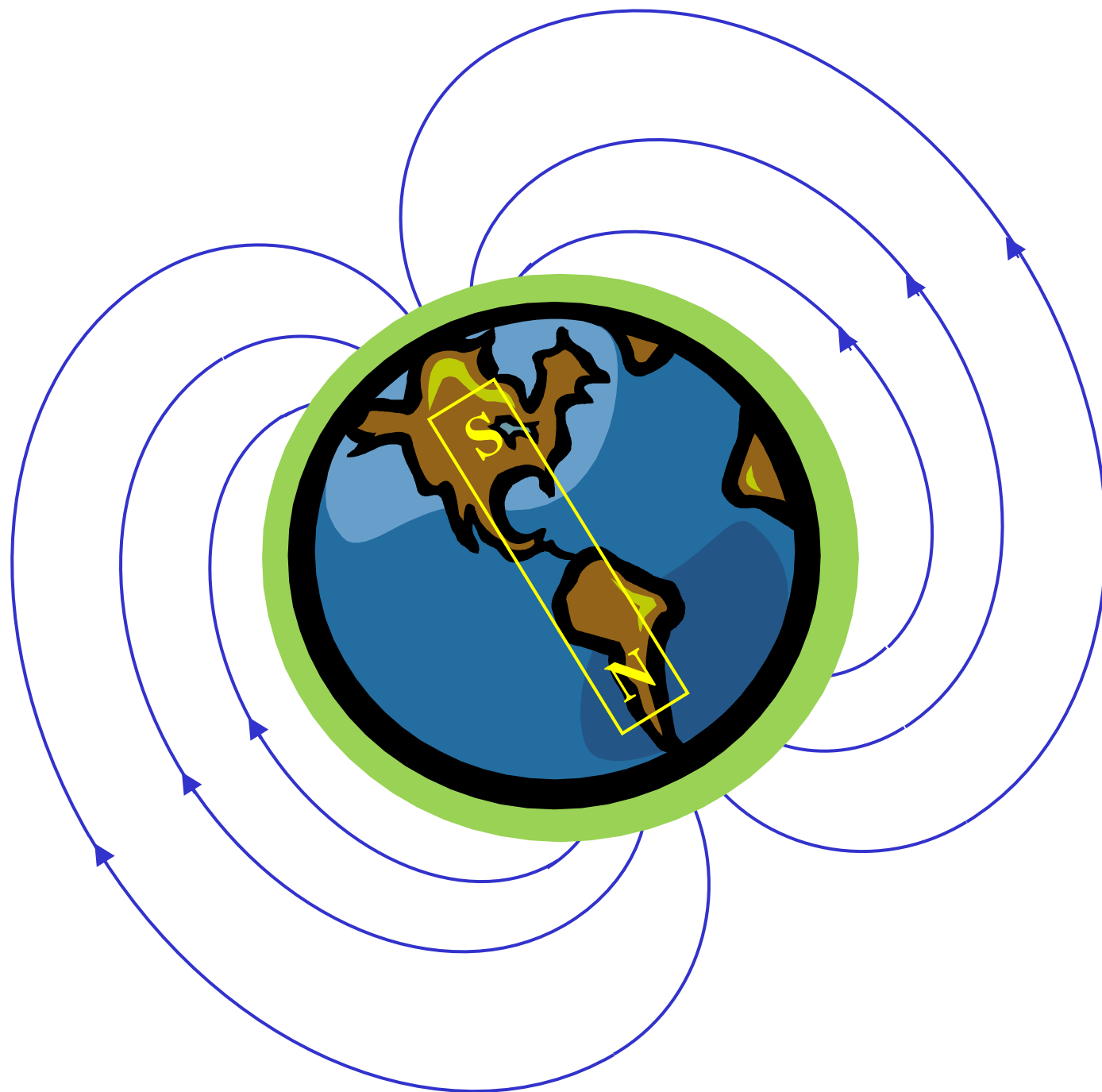
Gauss's Law for Magnet Fields

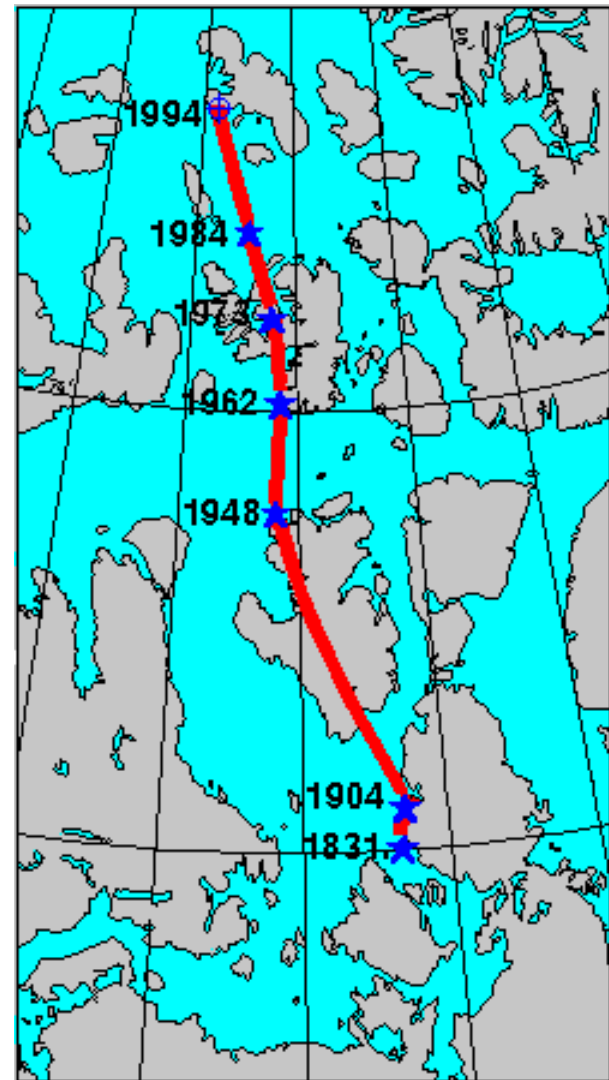
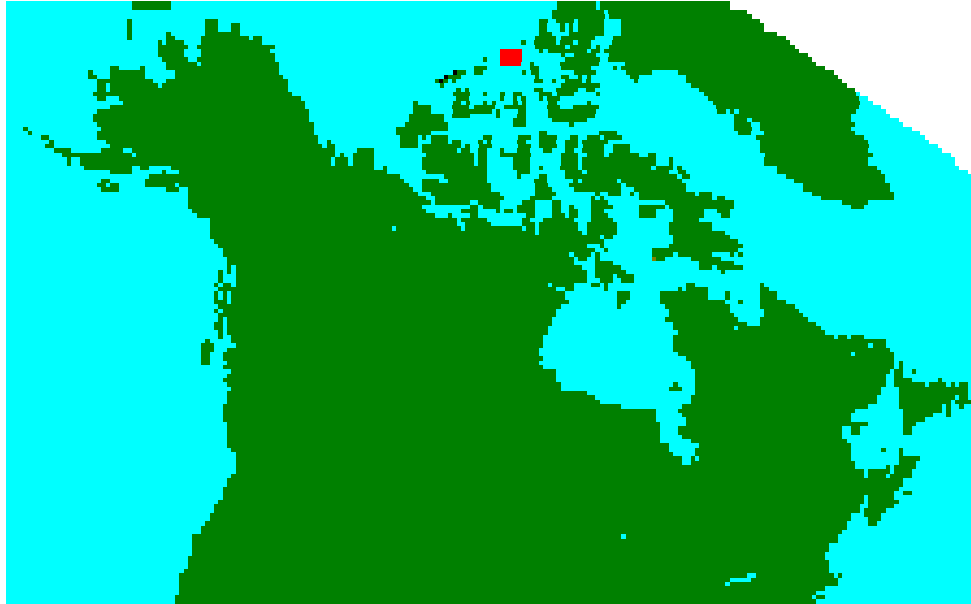
There are no magnetic monopoles. Magnetic field lines are continuous, and do not stop on single poles, since they do not exist.

$$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0$$

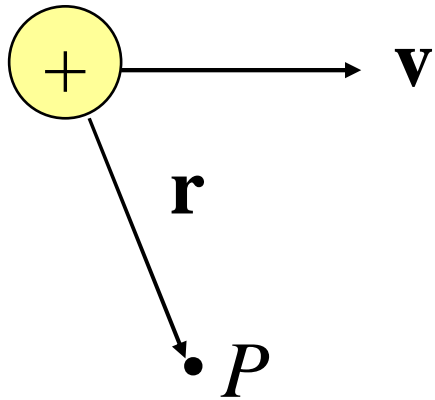
$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$$

Units are $\text{T} \cdot \text{m}^2 = \text{Wb}$ (Weber)





The magnetic field from a moving point charge



The field at P is given by:

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{q\mathbf{v} \times \hat{\mathbf{r}}}{r^2}$$

B : magnetic field

v : velocity of the particle

r : distance from the particle.

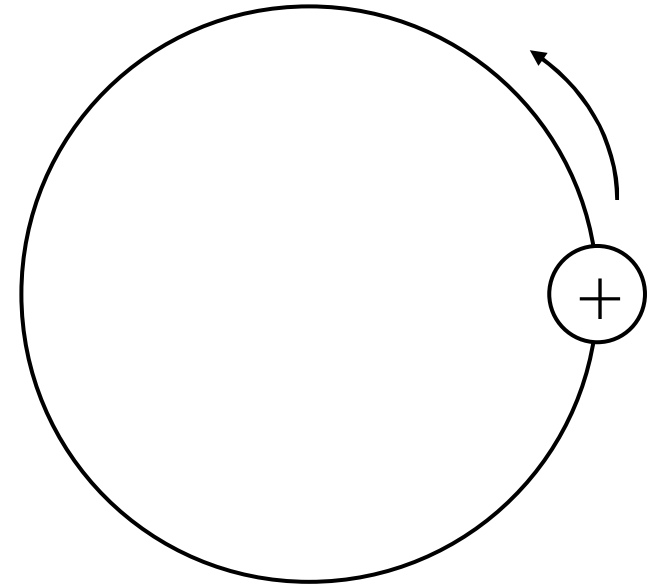
μ_0 : permeability of free space

SI unit for magnetic field: “Telsa” (T).

$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$.

Interactive Question

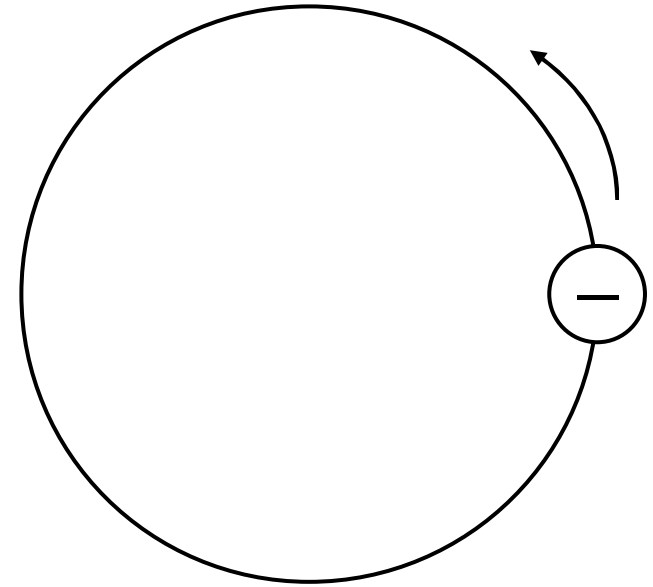
Protons, with a positive electric charge, are going around in a counterclockwise direction as shown. At the center of the circle, they produce a magnetic field that is:



- A) into the page
- B) out of the page
- C) to the left
- D) to the right
- E) zero

Interactive Question

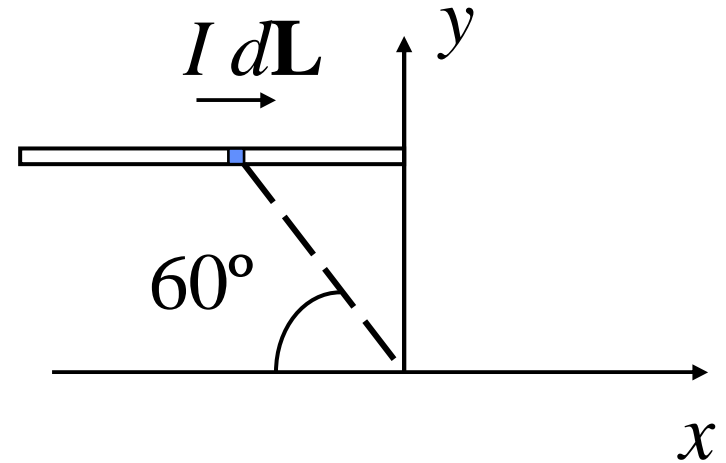
Electrons, with a negative electric charge, are going around in a counterclockwise direction as shown. At the center of the circle, they produce a magnetic field that is:



- A) into the page
- B) out of the page
- C) to the left
- D) to the right
- E) zero

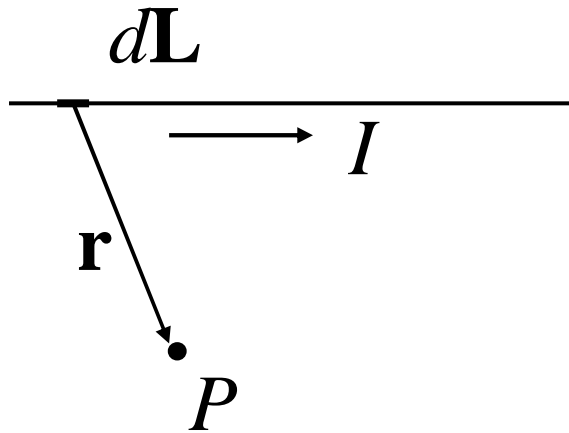
Interactive Question

A wire starts at a point on the y axis and extends in the negative x direction for an infinite distance as shown. What is the direction of the magnetic field ($d\mathbf{B}$) at the origin created by the differential current element $I d\mathbf{L}$ along the negative x axis shown shaded?



- A) Into the slide
- B) Out of the slide
- C) At an angle of 30° below the negative x axis
- D) At an angle of 30° above the positive x axis
- E) None of the above

The magnetic field from a current carrying wire segment

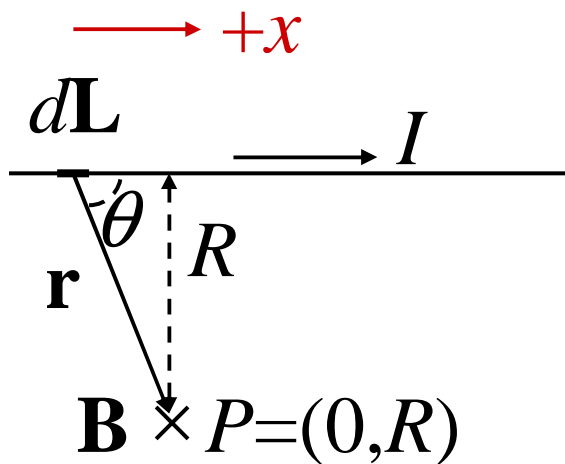


We now have many point charges moving:

$$\begin{aligned}\mathbf{B} &= \frac{\mu_0}{4\pi} \frac{nV}{r^2} q\mathbf{v} \times \hat{\mathbf{r}} \\ &= \frac{\mu_0}{4\pi} \frac{nALq}{r^2} \mathbf{v} \times \hat{\mathbf{r}} \\ &= \frac{\mu_0}{4\pi} \frac{I}{r^2} \mathbf{L} \times \hat{\mathbf{r}} \\ d\mathbf{B} &= \frac{\mu_0}{4\pi} \frac{I}{r^2} d\mathbf{L} \times \hat{\mathbf{r}}\end{aligned}$$

The Biot-Savart Law

The magnetic field from a current carrying wire



$$\mathbf{B} = \frac{\mu_0}{4\pi} \int_{-a}^{+b} \frac{I d\mathbf{L} \times \hat{\mathbf{r}}}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{+b} \frac{\sin \theta dx}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{\theta_1}^{\theta_2} \frac{\sin^2 \theta \sin \theta R d\theta}{\sin^2 \theta R^2}$$

$$= \frac{\mu_0 I}{4\pi R} \int_{\theta_1}^{\theta_2} \sin \theta d\theta = -\frac{\mu_0 I}{4\pi R} \cos \theta \Big|_{\theta_1}^{\theta_2}$$

$$B = \frac{\mu_0 I}{4\pi R} (\cos \theta_1 - \cos \theta_2)$$

$$\tan \theta = -R/x$$

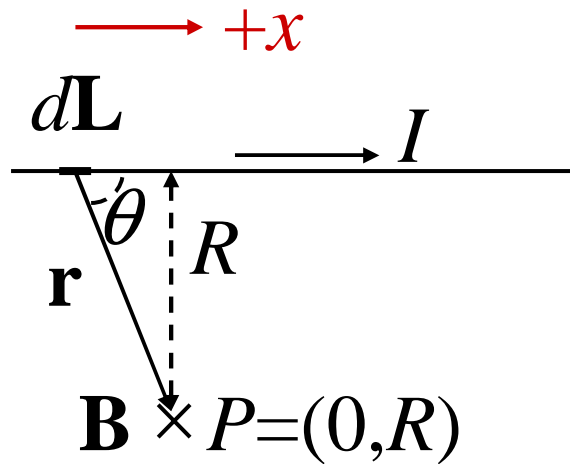
$$x = -R / \tan \theta$$

$$x = -R \cot \theta$$

$$dx = R \csc^2 \theta d\theta$$

$$dx = R d\theta / \sin^2 \theta$$

$$r = R / \sin \theta$$



This can be written:

$$B = \frac{\mu_0 I}{4\pi R} \left(\frac{a}{\sqrt{a^2 + R^2}} + \frac{b}{\sqrt{b^2 + R^2}} \right)$$

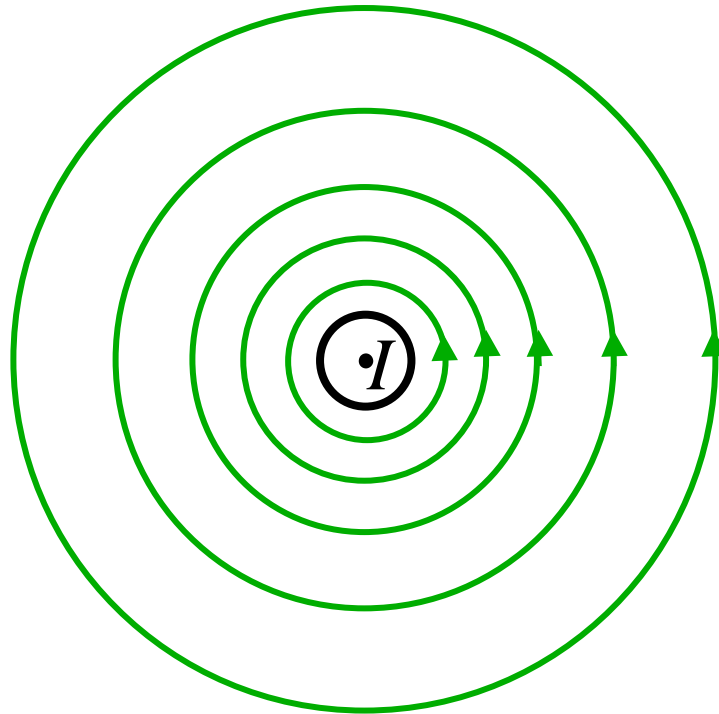
If $a=b$:
$$B = \frac{\mu_0 I}{2\pi R} \frac{a}{\sqrt{a^2 + R^2}}$$

For a very long wire, use $B = (\mu_0 I / 4\pi R)(\cos \theta_1 - \cos \theta_2)$ with $\theta_1 = 0$, $\theta_2 = \pi$, or, use the equation above with $a \gg R$ to get:

$$B = \frac{\mu_0 I}{2\pi R}$$

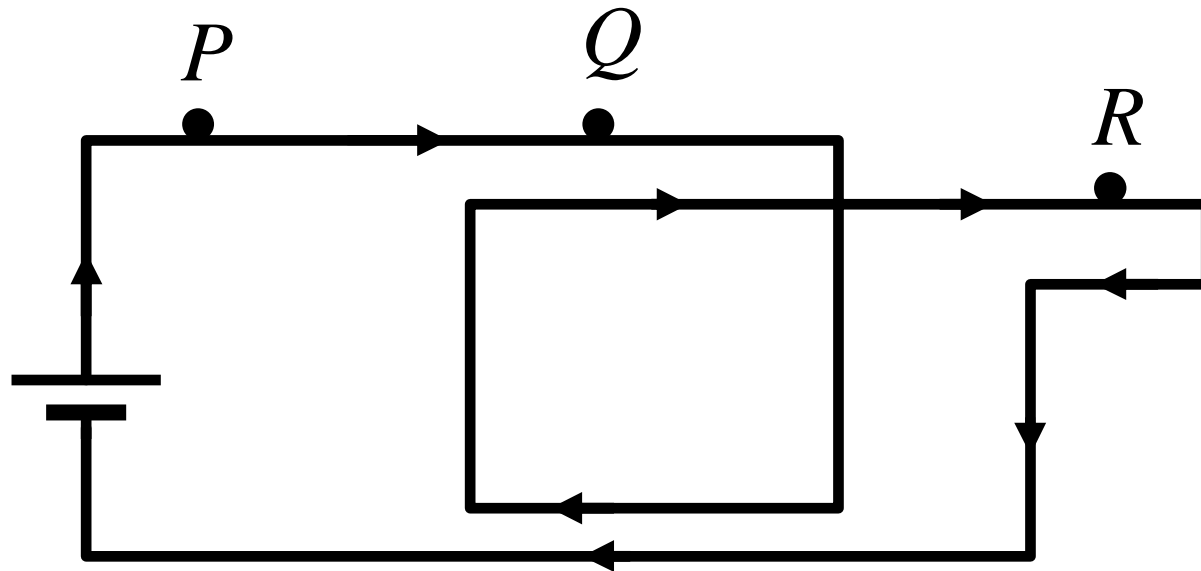
Magnetic Fields are Produced by Currents

$$B = \mu_0 I / 2\pi r$$



Interactive Question

A battery establishes a steady current around the circuit below. A compass needle is placed successively at points P , Q , and R . The relative deflection of the needle in descending order, is



A) P, Q, R

B) Q, R, P

C) R, Q, P

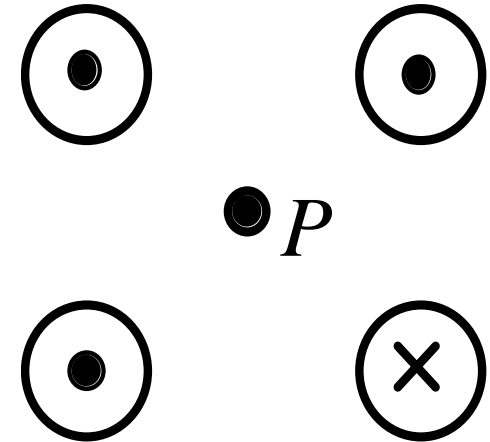
D) P, R, Q

E) Q, P, R

Problem: One straight wire carries a current of 10 A north and another straight wire carries a current of 5.0 A west. What is the direction and magnitude of the magnetic field 0.25 m above the point where the wires intersect?

Interactive Question

Four long straight wires are parallel to each other, their cross sections forming a square of side length 0.02 m. The point P is at the center of the square. Each wire carries a current of 8 A current in the direction shown.

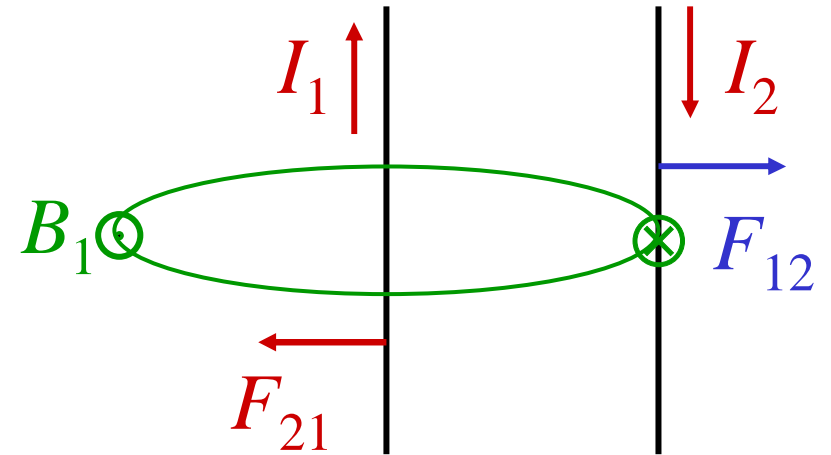
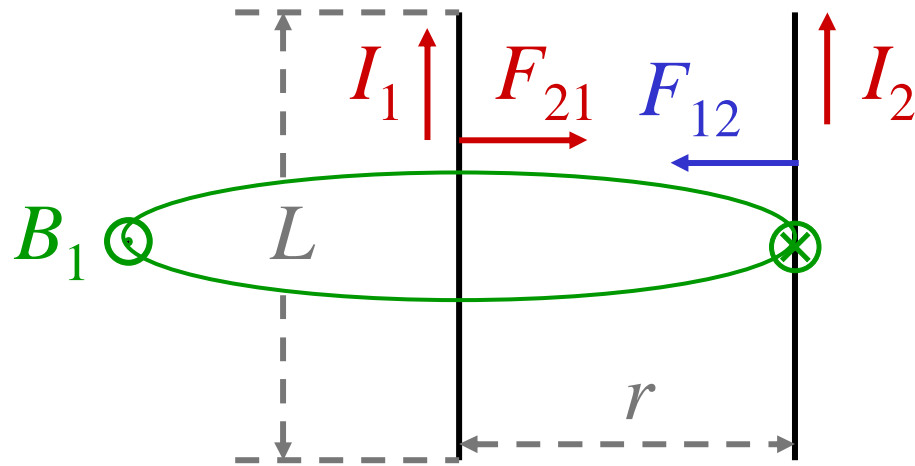


What direction is the magnetic field at P?

- A) Toward the bottom left
- B) Out of the page
- C) Toward the top right
- D) Toward the top left
- E) It is zero

Problem: Two parallel wires, each with a current of 2.0 amps along the x axis are located in the x - y plane. One located at $y=0$ and one at $y=0.40$ m. What is the magnetic field at the point $(0,0,0.3)$?

Two Parallel Wires Carrying Current



$$B_1 = \mu_0 I_1 / 2\pi r$$

$$F_{12} = I_2 L B_1 \sin \theta$$

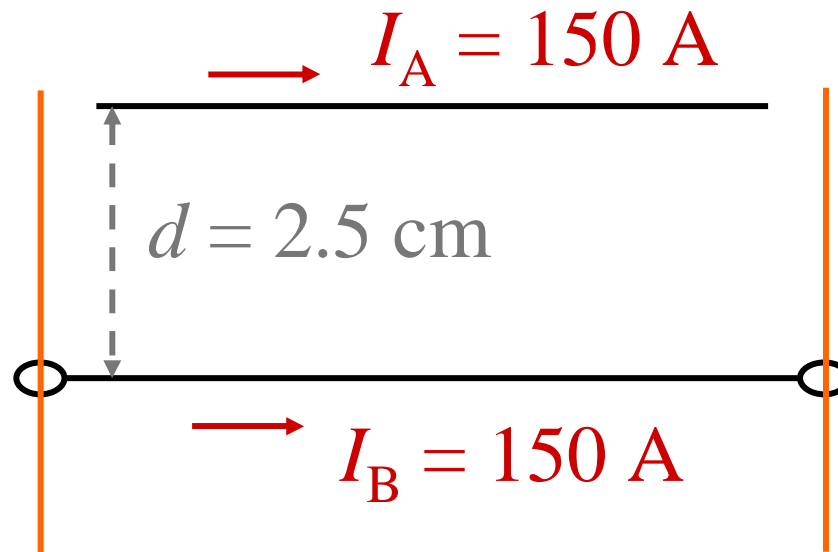
$$F_{12} = I_2 L (\mu_0 I_1 / 2\pi r)$$

$$F_{12} = \mu_0 I_2 I_1 L / (2\pi r)$$

What is F_{21} ?

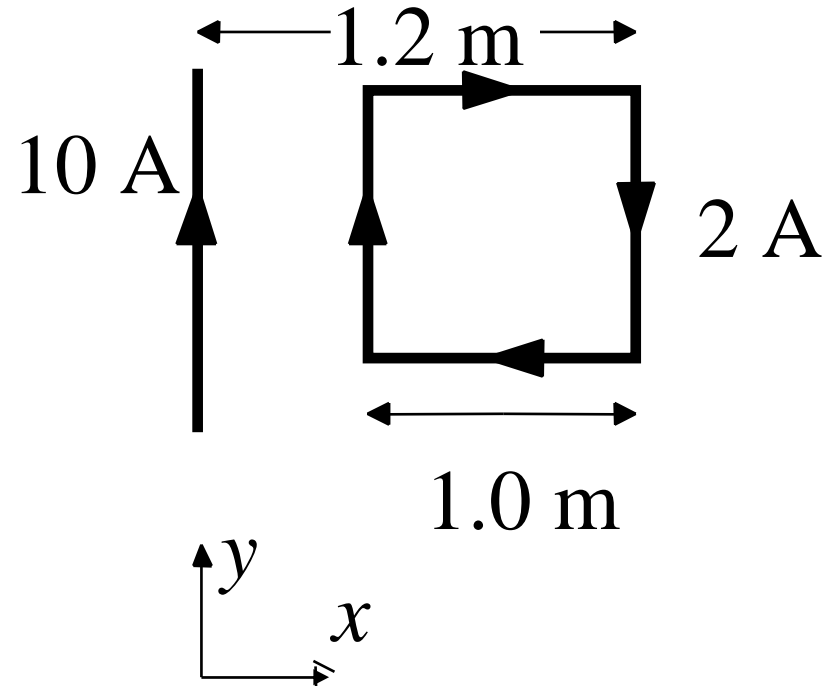
$$F_{21} = F_{12}$$

Problem: Two long parallel conductors are carrying currents in the same direction. Conductor A carries a current of 150 A and is held firmly in position. Conductor B carries I_B and is allowed to slide freely up and down parallel to A between a set of nonconducting guides. If the linear mass density of B is 0.10 g/cm, what value of the current I_B will result in equilibrium when the distance between the conductors is 2.5 cm?



Interactive Question

A long straight wire carries a 10 A current in the direction shown. Next to the wire is a square copper loop (with side length 1 m) that carries a 2 A current in the direction shown.

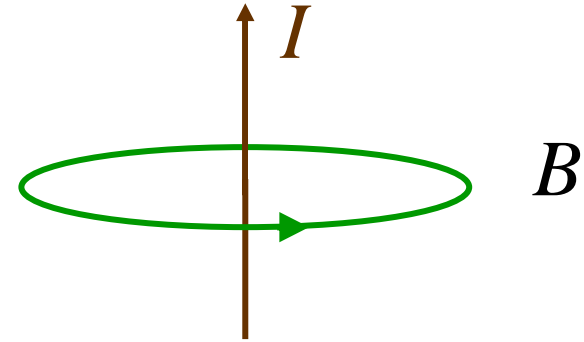


What is the direction of the net force on the loop?

- A) Positive x
- B) Negative x
- C) Positive y
- D) Negative y
- E) 30° with respect to positive x

Magnetic field of a wire loop

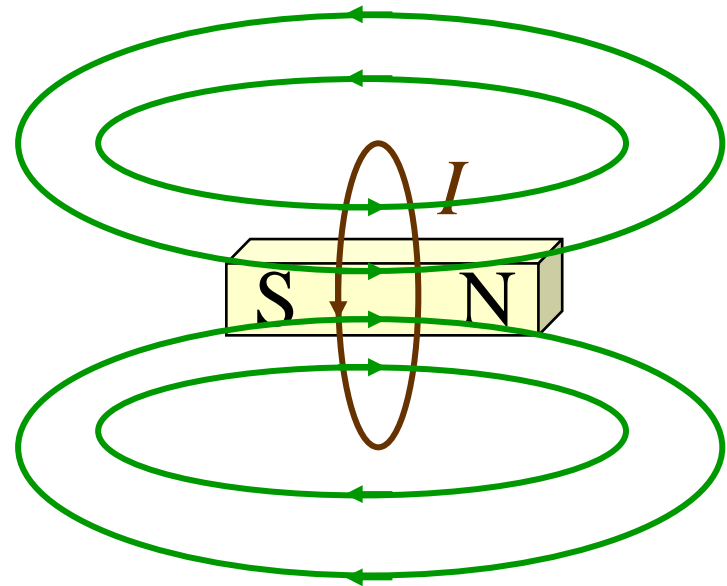
Take a wire with
magnetic field like this:



And twist it in a loop:

To produce a magnetic
field like this:

Which looks like the
field of a bar magnet.



$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{L} \times \hat{\mathbf{r}}}{r^2}$$

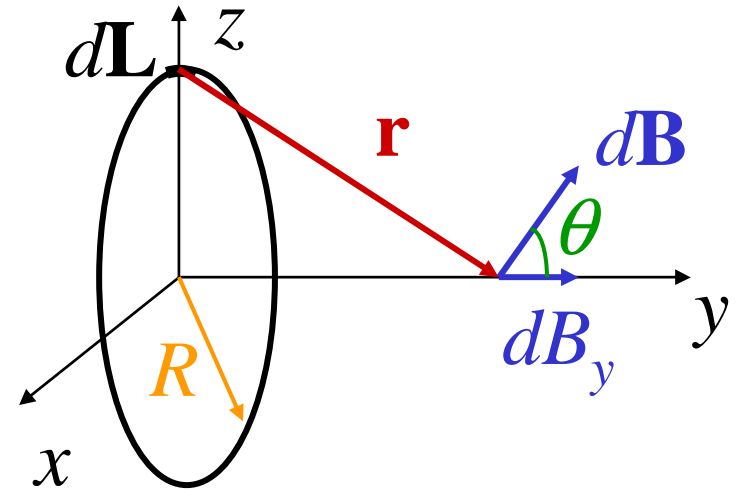
$$dB_y = \frac{\mu_0}{4\pi} \frac{I dL \sin(90)}{r^2} \cos \theta$$

$$dB_y = \frac{\mu_0}{4\pi} \frac{I dL R}{r^2 r} = \frac{\mu_0}{4\pi} \frac{IR dL}{(y^2 + R^2)^{3/2}}$$

$$B_y = \int_0^{2\pi R} \frac{\mu_0}{4\pi} \frac{IR dL}{(y^2 + R^2)^{3/2}}$$

$$= \frac{\mu_0}{4\pi} \frac{IR}{(y^2 + R^2)^{3/2}} \int_0^{2\pi R} dL$$

$$\mathbf{B} = \frac{\mu_0}{2} \frac{IR^2}{(y^2 + R^2)^{3/2}} \hat{\mathbf{j}}$$



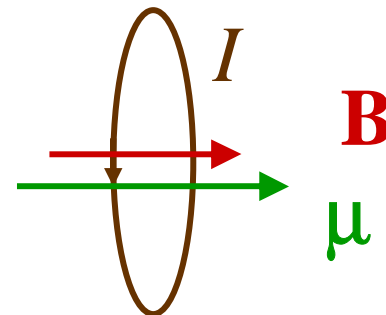
At $y=0$,

$$\mathbf{B} = \frac{\mu_0 I}{2R} \hat{\mathbf{j}}$$

$$\mathbf{B} = \frac{\mu_0}{2} \frac{IR^2}{(y^2 + R^2)^{3/2}} \hat{\mathbf{j}}$$

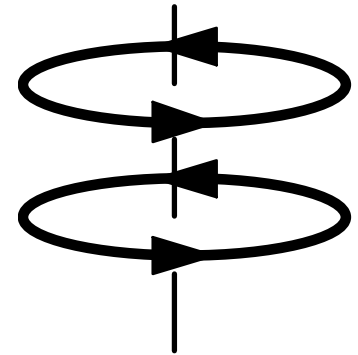
$$\text{At } y=0, \quad \mathbf{B} = \frac{\mu_0 I}{2R} \hat{\mathbf{j}}$$

$$\text{Recall: } \boldsymbol{\mu} = I\mathbf{A} \quad \Rightarrow \quad \boldsymbol{\mu} \propto \mathbf{B}$$



Interactive Question

Two loops carry equal current I in the same direction. They are held in the positions shown in the figure and released. Which statement describes the subsequent behavior of the loops?



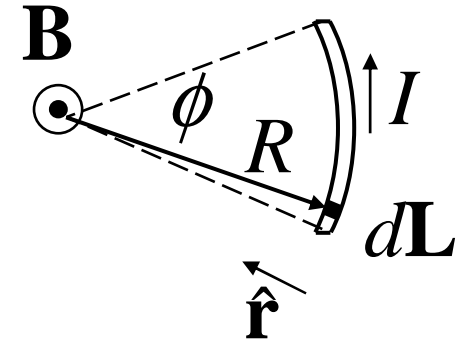
- A) The loops repel each other
- B) The loops attract each other
- C) Both loops move to the left
- D) The loops remain in the position shown
- E) The top loop moves to the right and the bottom loop moves to the left

Magnetic Field of a Circular Arc

$$\mathbf{B} = \frac{\mu_0}{4\pi} \int_0^L \frac{I d\mathbf{L} \times \hat{\mathbf{r}}}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi R^2} \int_0^{R\phi} ds$$

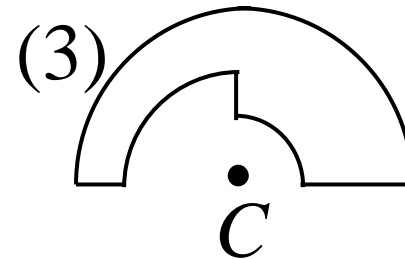
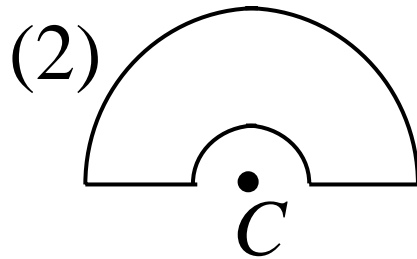
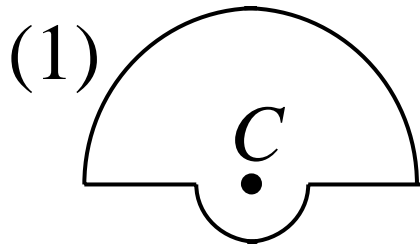
$$B = \frac{\mu_0 I R \phi}{4\pi R^2} = \frac{\mu_0 I \phi}{4\pi R}$$



Note: If $\phi = 2\pi$, we get $B = \mu_0 I / 2R$, which is exactly what we got for the magnetic field at the center of a single loop of wire.

Interactive Question

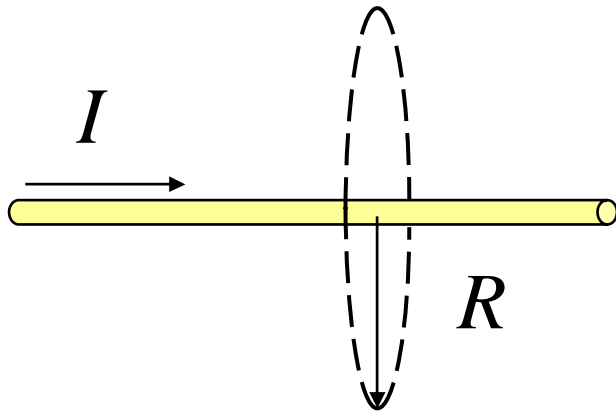
The diagrams show three circuits consisting of concentric circular arcs of radii r , $2r$, or $3r$. Each circuit carries the same current. Rank them according to the magnitude of the magnetic fields produced at point C, least to greatest.



- A) 1, 2, 3
- B) 3, 2, 1
- C) 1, 3, 2
- D) 2, 3, 1
- E) 2, 1, 3

Ampere's Law

$$\oint_C \mathbf{B} \cdot d\mathbf{L} = \mu_0 I_{\text{encl}}$$



$$\oint_C \mathbf{B} \cdot d\mathbf{L} = \mu_0 I_{\text{encl}}$$

$$B \oint_C dL = \mu_0 I$$

$$B 2\pi R = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi R}$$

Same as with the Biot-Savart Law

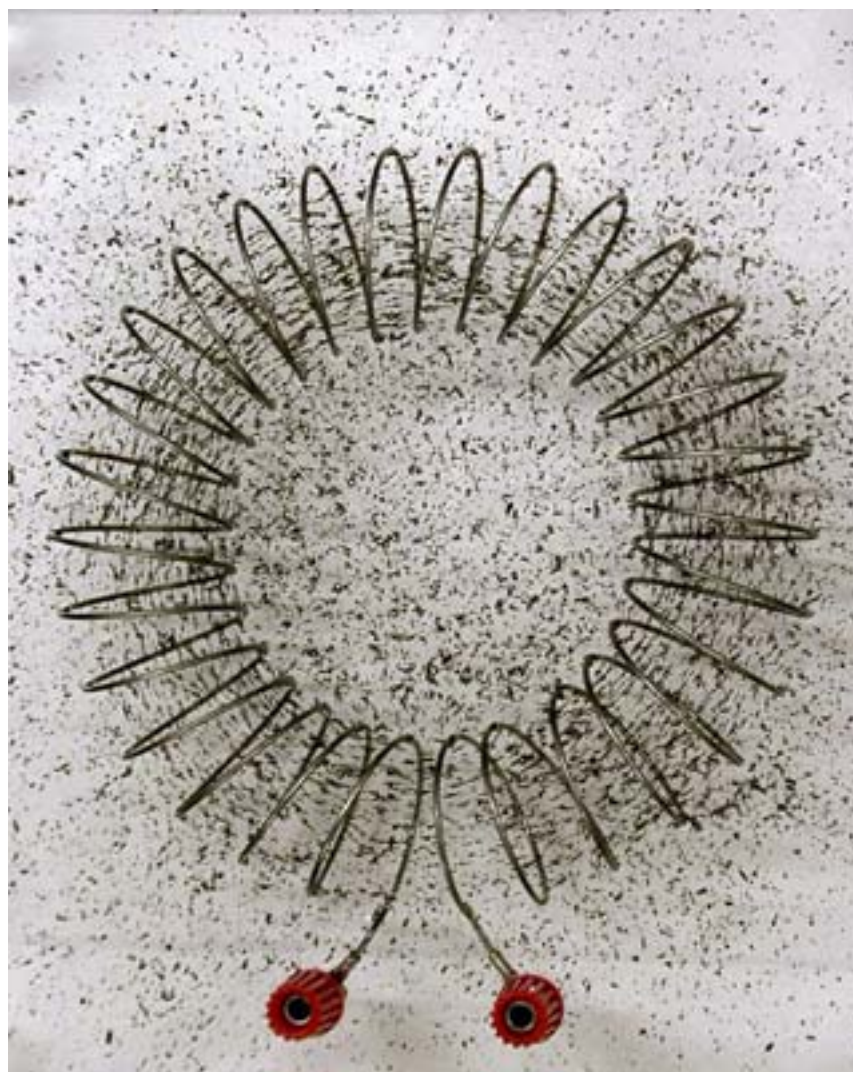
Interactive Question

In Ampere's law, $\oint_C \mathbf{B} \cdot d\mathbf{L} = \mu_0 I_{\text{encl}}$, the symbol $d\mathbf{L}$ is:

- A) an infinitesimal piece of wire that carries current I_{encl} .
- B) in the direction of \mathbf{B} .
- C) perpendicular to \mathbf{B} .
- D) a vector whose magnitude is the length of the wire that carries current I .
- E) none of the above.

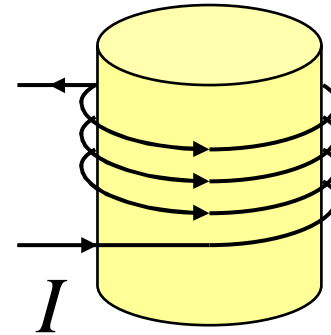
Problem: A long, thick cylindrical conductor with radius a , has a current density given by $J = J_0 r/a$. Find the magnetic field inside and outside the wire.

Problem: What is the magnetic field inside and outside of a toroid that has N total loops of wire?



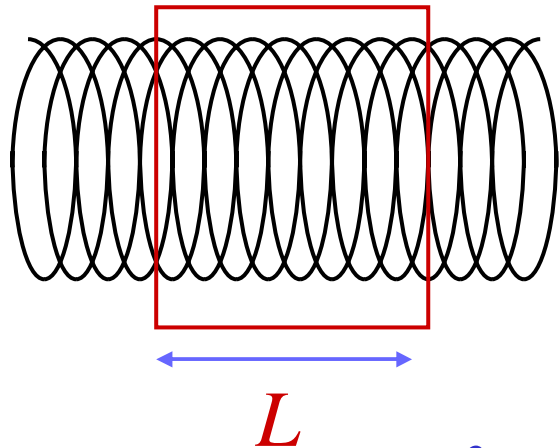
Interactive Question

Magnetic field lines inside the solenoid shown are:



- A) clockwise circles as one looks down the axis from the top of the page.
- B) counterclockwise circles as one looks down the axis from the top of the page.
- C) toward the top of the page.
- D) toward the bottom of the page.
- E) in no direction since $\mathbf{B} = 0$.

The Magnetic Field of a Very Long Solenoid

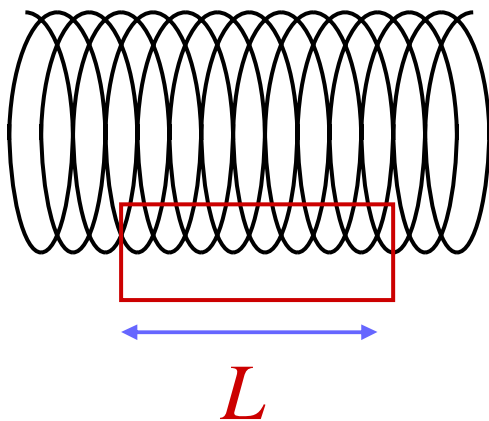


Outside:

$$\oint_C \mathbf{B} \cdot d\mathbf{L} = \mu_0 I_{\text{encl}} = 0$$

$$B = 0$$

Inside:



$$\oint_C \mathbf{B} \cdot d\mathbf{L} = \mu_0 I_{\text{encl}}$$

$$\int_{\text{inside}} \mathbf{B} \cdot d\mathbf{L} + \int_{\text{outside}} \mathbf{B} \cdot d\mathbf{L} + 2 \int_{\text{sides}} \mathbf{B} \cdot d\mathbf{L} = \mu_0 I_{\text{encl}}$$

$$BL + 0 + 0 = \mu_0 NI$$

$$BL = \mu_0 nLI$$

$$B = \mu_0 nI$$

$n = \# \text{ turns/length}$

Changing Magnetic and Electric Fields

Recall Ampere's Law $\oint_C \mathbf{B} \cdot d\mathbf{L} =$



There is a problem in the second case. There is no current, but a magnetic field will be created.

Consider the electric field in the capacitor:

$$Q = CV = (\epsilon_0 A/d)(Ed) = \epsilon_0 AE$$

$$dQ/dt = \epsilon_0 A dE/dt = \epsilon_0 d\Phi_E/dt$$

James Maxwell proposed a “displacement current:”

$$I_D = \epsilon_0 d\Phi_E/dt \text{ where } \Phi_E = \int \mathbf{E} \cdot d\mathbf{A} \text{ is the electric flux.}$$

Then Ampere's law becomes the Ampere-Maxwell law:

$$\oint_C \mathbf{B} \cdot d\mathbf{L} = \mu_0 I + \mu_0 \epsilon_0 d\Phi_E/dt = \mu_0(I + I_D)$$

Recall that Faraday's law states that a changing magnetic flux (or field) creates an electric field.

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{s} = -d\Phi_B/dt$$

Now we also see that a changing electric flux (or field) creates a magnetic field.

Interactive Question

Displacement current exists in the region between the plates of a parallel plate capacitor if:

- A) The capacitor leaks charge across the plates.
- B) The capacitor is being charged.
- C) The capacitor is fully charged.
- D) The capacitor is fully discharged.
- E) More than one of the above is true.

Interactive Question

A magnetic field exists between the plates of a capacitor:

- A) Always
- B) Never
- C) When the capacitor is fully charged.
- D) While the capacitor is being charged.
- E) Only when the capacitor is starting to be charged.

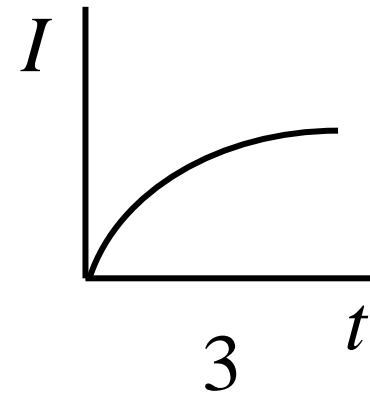
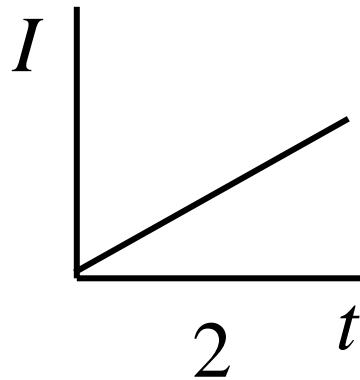
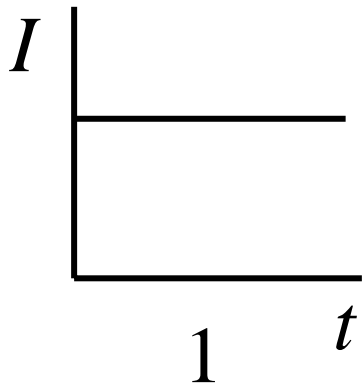
Interactive Question

A sinusoidal emf is connected to a parallel plate capacitor. The magnetic field between the plates

- A) Is zero
- B) Is constant
- C) Is sinusoidal and its amplitude does not depend on the frequency of the source.
- D) Is sinusoidal and its amplitude is proportional to the frequency of the source.
- E) Is sinusoidal and its amplitude is inversely proportional to the frequency of the source

Interactive Question

The curves below show the current (I) charging a capacitor as a function of time. In which case(s) will there be a *changing* electric field and a *changing* magnetic field between the plates of the capacitor?



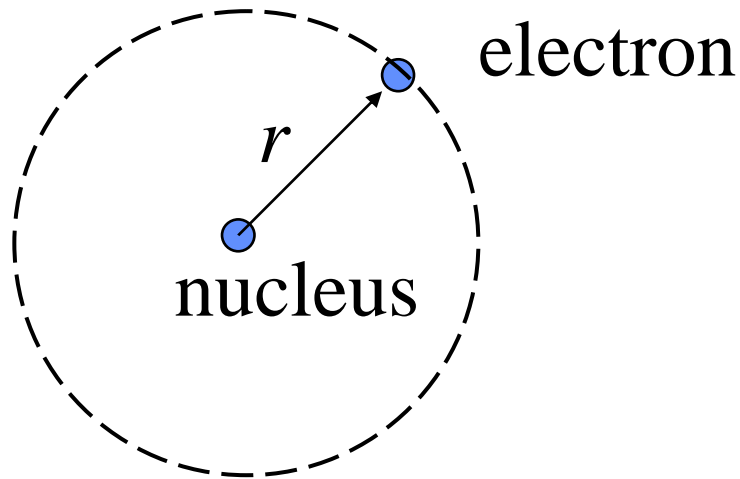
A) 1 only
D) 2 and 3

B) 2 only
E) 1, 2, and 3

C) 3 only

- Problem: A parallel plate $5.00\ \mu\text{F}$ capacitor with $4.00\ \text{cm}$ radius circular plates has a voltage across it that varies sinusoidally with a frequency of $6.00\ \text{kHz}$ with a maximum amplitude of $25.0\ \text{V}$. What is
- The displacement current through the capacitor and
 - The magnetic field at one-half the radius of the plates, assuming the displacement current has a uniform density.

Atomic Look at Magnetism



Angular momentum (L)

$$L = mvr$$

$$\begin{aligned}\mu &= IA = I\pi r^2 \\ &= (q/T) \pi r^2 \\ &= (qv/2\pi r) \pi r^2 \\ &= qvr/2 \\ &= (q/2m)L\end{aligned}$$

$$\mu_l = (q/2m)\mathbf{L}$$

$$\mu_l = (-e/2m)\mathbf{L}$$

There is also an intrinsic (or “Spin”) magnetic moment for the electron which is purely quantum mechanical:

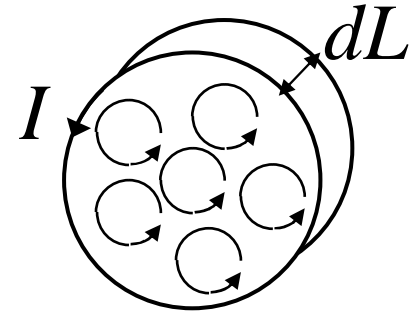
$$\mu_s \propto \mathbf{S}$$

Both of these lead to magnetic effects in material.

Magnetization of Materials

$$\mathbf{M} = d\mathbf{\mu}/dV$$

$$\mathbf{M} = AdI/AdL = dI/dL$$



Place the material in an external magnetic field \mathbf{B}_0

$$\begin{aligned}\mathbf{B} &= \mathbf{B}_0 + \mathbf{B}_M \\ &= \mathbf{B}_0 + \mu_0 \mathbf{M}\end{aligned}$$

In certain kinds of materials, the magnetic field from the magnetic moment remains, even after removing the material from the external magnetic field.

Three Types of Material

- 1) Ferromagnetic: Strong alignment of atomic magnetic moments (fields). This material can have permanent magnetism even when the external field is removed.
- 2) Paramagnetic: Weak alignment of atomic magnetic moments (fields).
- 3) Diamagnetic: Antialignment of atomic magnetic moments (fields).

For Ferromagnetic and Paramagnetic Material

$$\begin{aligned}\mathbf{B} &= \mathbf{B}_0 + \mu_0 \mathbf{M} \\ &= \mathbf{B}_0 + \chi_m \mathbf{B}_0 \\ &= \mathbf{B}_0 (1 + \chi_m) \\ &= K_m \mathbf{B}_0\end{aligned}$$

χ_m is the magnetic susceptibility

$K_m = \mu/\mu_0$ is the relative permeability

Paramagnetic materials have small positive values of K_m , and ferromagnetic materials have large positive values of K_m under certain conditions.