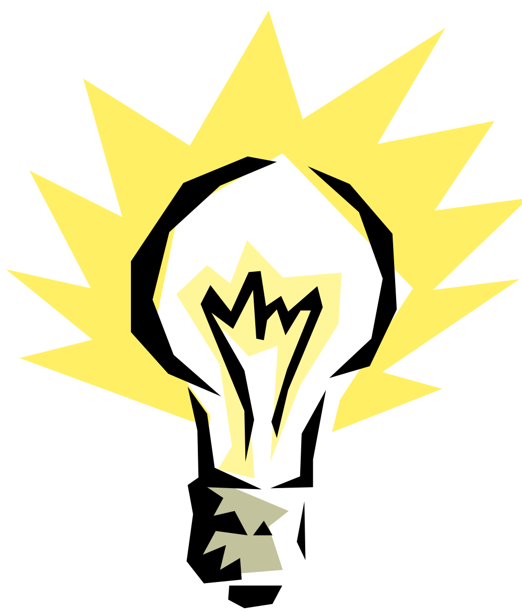
Chapter 19

Electric Current, Resistance, and DC Circuit Analysis

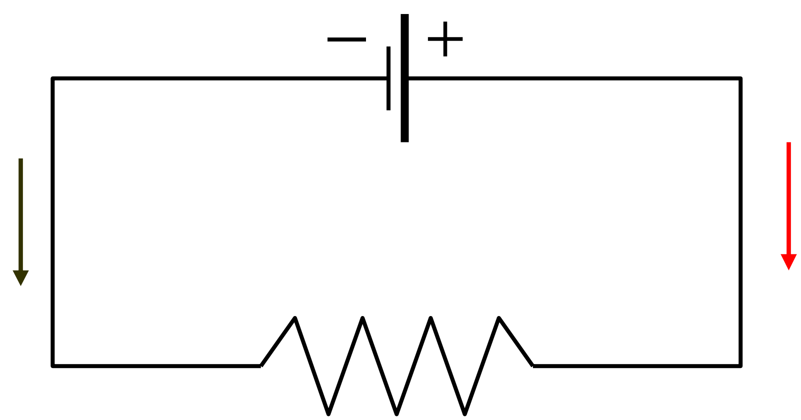


$$I = dq/dt$$

Current is charge per time

SI Units: Coulombs/Second = Amps

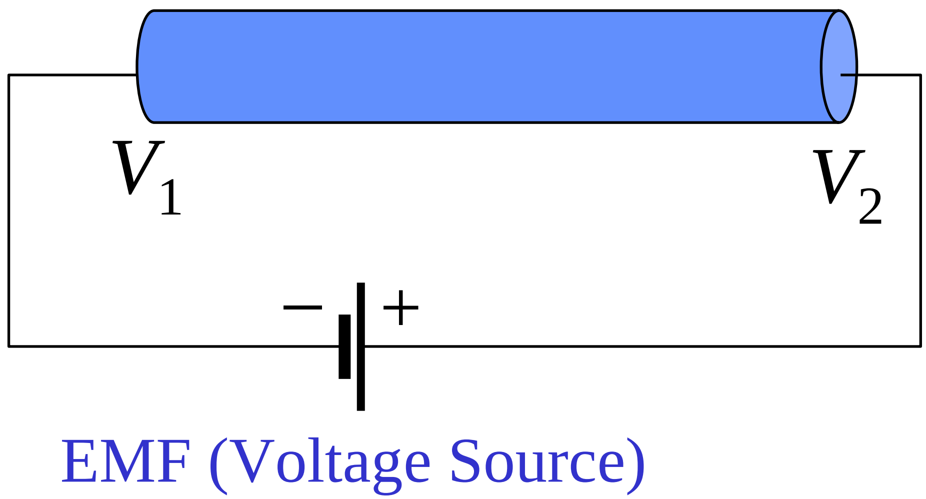
Direction of Electron Flow



Direction of Conventional Current: We will always use this.

Problem: Suppose one trillion electrons flow past a point in one half second. What is the current?

Why does charge move through a wire?



- $V_2 > V_1$
- $dV/dx = -E_x$: An electric field must point from V_2 to V_1 inside the wire!
- Positive charges will move from V_2 to V_1.

Drift Velocity

The speed of an electron is not the same as the average rate that the electron is drifting down a wire (v_d).

In a time Δt, one electron will drift a distance $L = v_d \Delta t$.

The amount of charge that will move by a point in the time Δt will be

$\Delta Q = -neLA = -nev_d A \Delta t$

Where n is the number of free electrons per unit volume e is the electric charge.

$I = dQ/dt = -nev_d A$

Define the current density **(J)** using $I = \int \mathbf{J} \cdot d\mathbf{A}$, then $\mathbf{J} = ne\mathbf{v}_d$

Ohm's Law

Voltage drop is a linear function of resistance and current

$$V = IR$$

SI Unit of resistance is Ohms (Ω)

Using the current density: $J = I/A$ (for constant A)

$V=IR$

$EL=IR(A/A) = J(RA)$

$E= J(RA/L)$

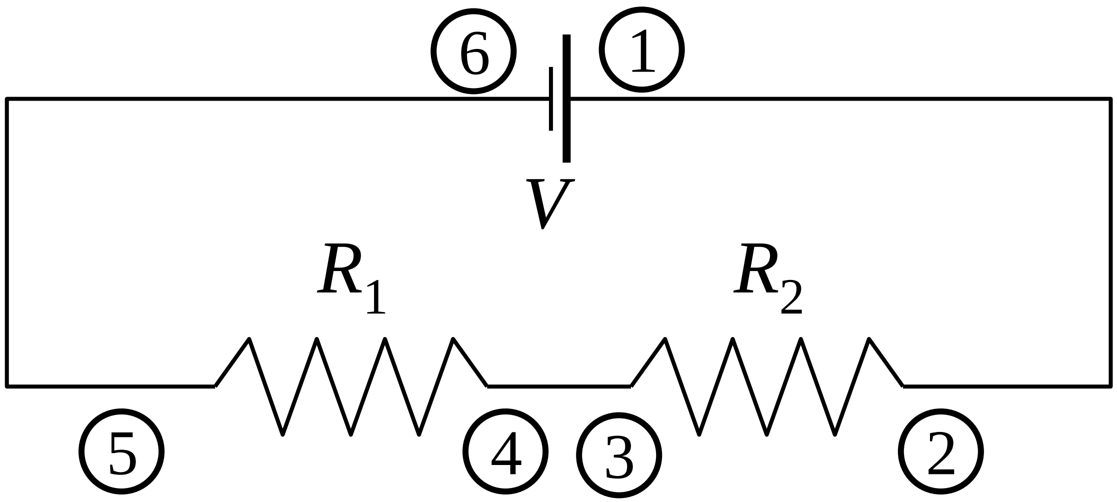
Define $\rho = RA/L = 1/\sigma$

where σ is the conductivity and ρ is the resistivity

$$\mathbf{E}\sigma = \mathbf{J}$$

("physicists" version of Ohm's law)

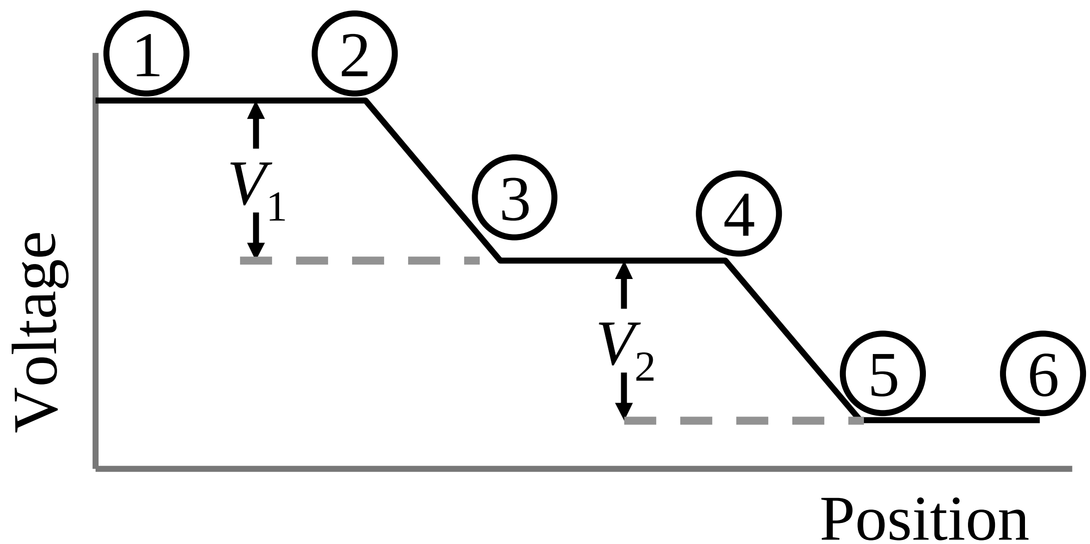
Gravitational Analogy



$V = IR_{eq}$

$V_1 = I_1R_1$

$V_2 = I_2R_2$



Resistance is Related to Resistivity

$$R=\rho L/A$$

R: Resistance
ρ: Resistivity
L: Length
R: Cross-Sectional Area

Resistivity is a property of a material, while resistance also depends on the shape of the material.

Problem: An aluminum wire and a copper wire each have a length of 10 meters. The aluminum has a cross sectional area of 1 mm^2 and the copper has a cross sectional area of 0.5 mm^2. What is the ratio of the resistance of the copper to the aluminum?

(ρ for copper is 1.72×10^{-8} Ω-m and for aluminum 2.82×10^{-8} Ω-m)

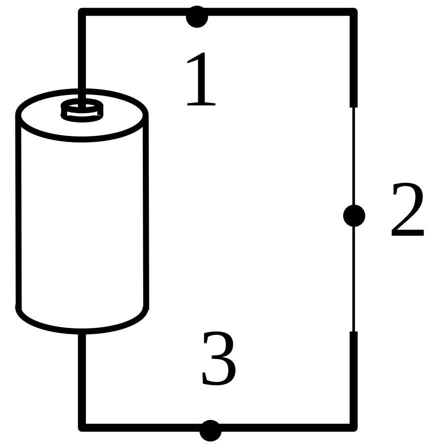
Interactive Question

Consider two copper wires. One has twice the length and twice the cross-sectional area of the other. How do the resistances of these two wires compare?

A) Both have the same resistance
B) The longer wire has twice the resistance
C) The longer wire has four times the resistance
D) The longer wire has half the resistance
E) None of the above

Interactive Question

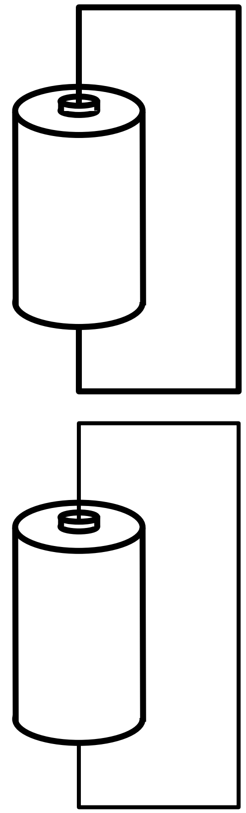
A battery is connected to a wire having two different cross sectional areas as shown. Which entry below is correct?



A) $I_1=I_2=I_3$, $J_1=J_2=J_3$
B) $I_1=I_2=I_3$, $J_1=J_3<J_2$
C) $I_1>I_2>I_3$, $J_1=J_2=J_3$
D) $I_1>I_2>I_3$, $J_1>J_2>J_3$
E) $I_1>I_2>I_3$, $J_1=J_3<J_2$

Interactive Question

Two wires A and B are made of the same material and have the same length. Wire A has twice the cross sectional area of wire B. If each wire is connected to an identical battery, which is true concerning the current in wire A?



Wire A

Wire B

A) It is one-fourth that in B
B) It is four times that in B
C) It is equal to the current in B
D) It is half as much as that in B
E) It is twice as much as that in B

For many metals resistivity is a function of temperature and is given by

$$\rho = \rho_0[1+\alpha(T-T_0)]$$

where α is the coefficient of resistivity for the material, ρ_0 is the resistivity at temperature T_0, and ρ is the resistivity at temperature T where $T > T_0$.

Then by substituting $\rho = RA/L$ into this equation,

$$RA/L = R_0 A/L[1+\alpha(T-T_0)]$$

$$R = R_0[1+\alpha(T-T_0)]$$

Electric Power

Power = Energy/Time

For Electricity:

$P = qV/t = V(q/t) = IV$

And since $V = IR$, we can write electric power as:

$P = IV$ (used for all devices)

$= I^2R$ (only used for resistors)

$= V^2/R$ (only used for resistors)

Interactive Question

A 4 amp current is maintained in a simple circuit with a total resistance of 2 Ω. How much heat is generated in 3 seconds?

A) 3 J
B) 6 J
C) 12 J
D) 24 J
E) 96 J

Electric Energy

Electric energy (not power) is actually purchased from the electric company:

Energy = Power $\times$Time (kW·h)

Power costs about \$0.10 per kWh

A 1000 Watt hair dryer running for 10 minutes costs about (1 kW)(10 min)(1 hour/60 min)(\$0.10/kWh) = \$.017

Problem: How much does it cost to keep your computer on all the time for one month? The computer screen runs at about 4 amps using 120 V.

Interactive Question

If the resistance in a *constant voltage* circuit is doubled, the power dissipated by that circuit will

A) increase by a factor or two.
B) increase by a factor of four.
C) decrease by one-half its original value.
D) decrease to one-fourth its original value.

Interactive Question

At 120 V, which has a larger resistance, a 100 W light bulb or a 60 W light bulb?

A) 100 W
B) 60 W
C) They both have the same resistance

Problem: A light bulb filament is made of tungsten which has a coefficient of resistivity $\alpha = 0.0045\ C^{\circ -1}$. At room temperature of 20° C the filament has a resistance of 10 Ω.

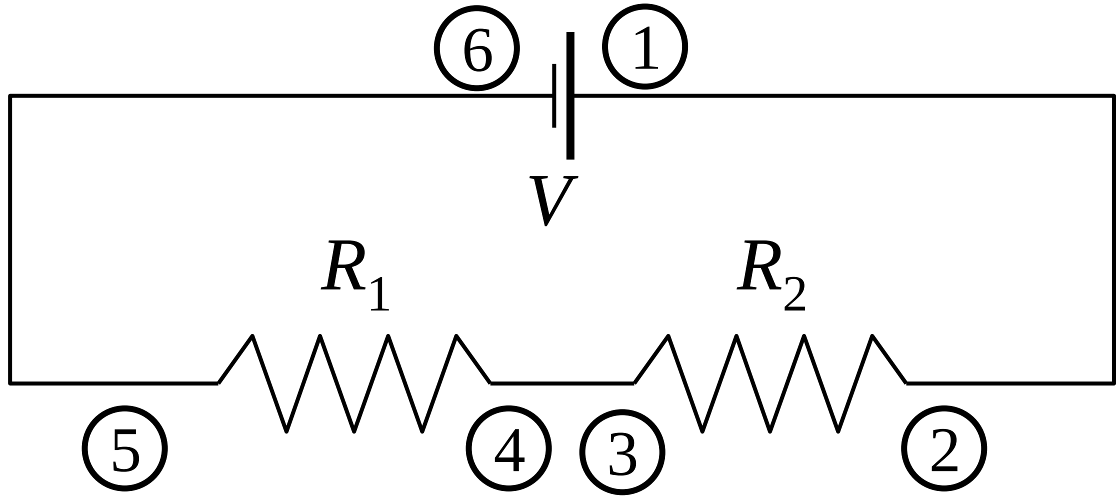
(a) What is the average power of the light bulb immediately after it has been plugged into a 120 V ac outlet?

(b) After a very brief time, the light bulb filament has changed temperature and is glowing brightly, the rms current is measured to be 0.833 amp. What is the resistance of the light bulb now?

(c) What is the average power of the light bulb when it is glowing brightly as in part (b)?

(d) What is the temperature of the filament when it is glowing brightly?

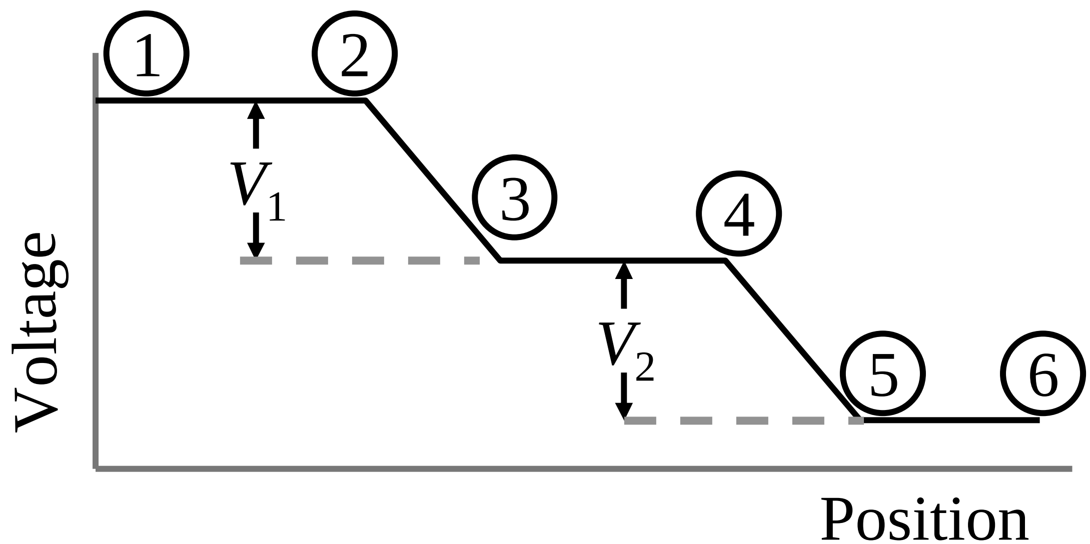
Gravitational Analogy



$$V = IR_{eq}$$

$$V_1 = I_1 R_1$$
$$V_2 = I_2 R_2$$

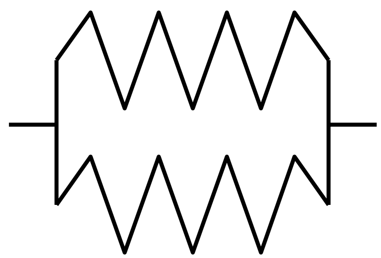
Current can be thought of as water flowing through a pipe



Electronic components like resistors, capacitors, and batteries can be connected in series or parallel:

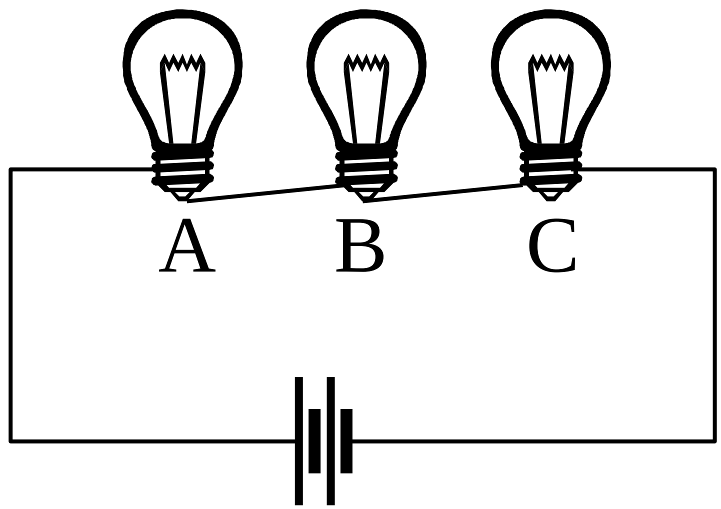
- Components connected in series have the same current flowing through them.

- Components connected in parallel have the same voltage drop across them.



Interactive Question

Three identical light bulbs are arranged as shown. Rank the brightness of each bulb.



A) A>B>C
B) C>B>A
C) A=B=C

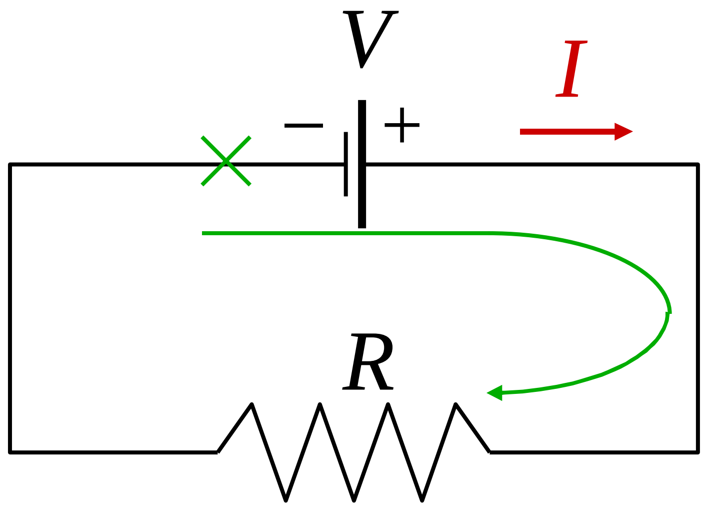
Kirchoff’s Rules

1. Junction Rule. The sum of the magnitudes of the currents directed into a junction equals the sum of the magnitudes of the currents directed out of the junction. (Conservation of charge)
2. Loop Rule. Around any closed loop, the sum of the potential drops equals the sum of the potential rises. (Conservation of energy)

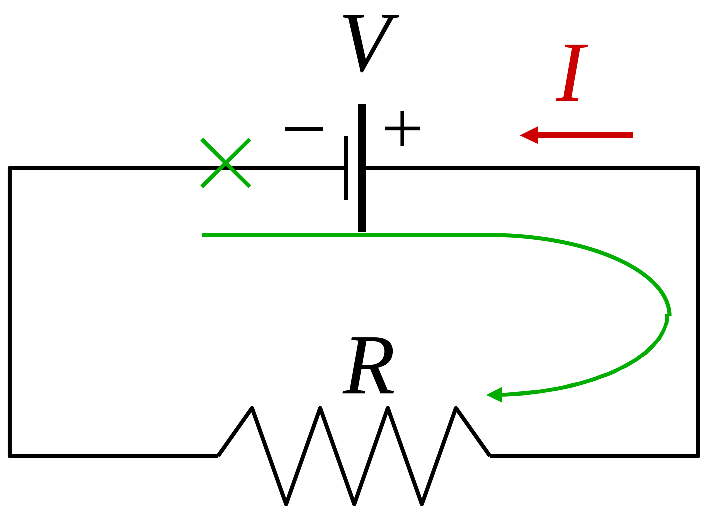
Using Kirchoff's Rules

1. Draw the circuit and draw currents with an arrow in every separate branch of the circuit. (A branch goes from one junction to the next junction.)
2. Apply the junction rule to enough junctions so that every current is used at least once.
3. Apply the loop rule to enough closed loops so that each current appears at least once. Remember the sign convention for the potential changes :
   a. Moving across a battery: plus sign if going from negative to positive terminal.
   b. Moving across a resistor: minus sign if going with the current.

Using Kirchoff's Rules



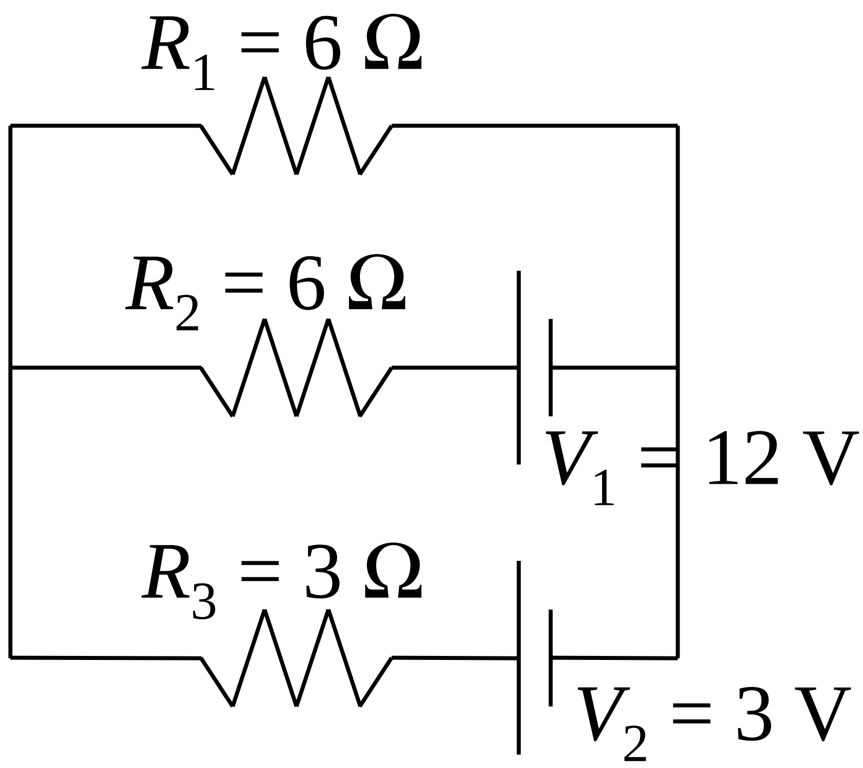
$$+V - IR = 0$$
$$V = IR$$



$$+V + IR = 0$$
$$V = -IR$$

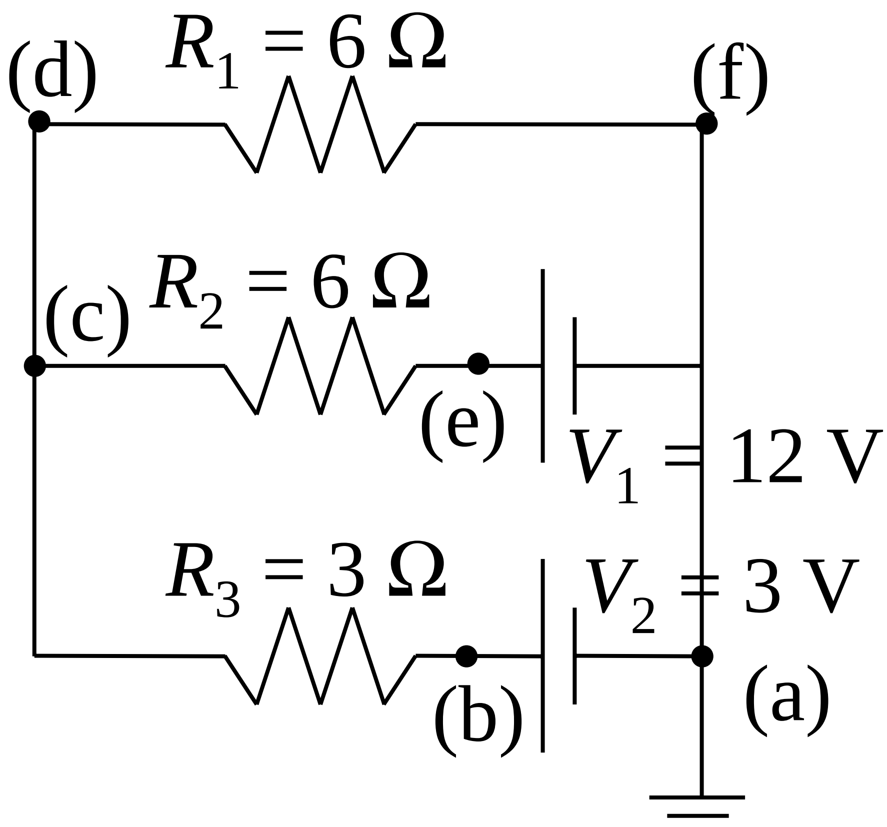
Negative sign indicates the actual current goes the opposite way from drawn.

Problem: Find the current and power in the 3 Ω resistor. What is the current in the other two resistors and the power dissipated in the other two resistors?

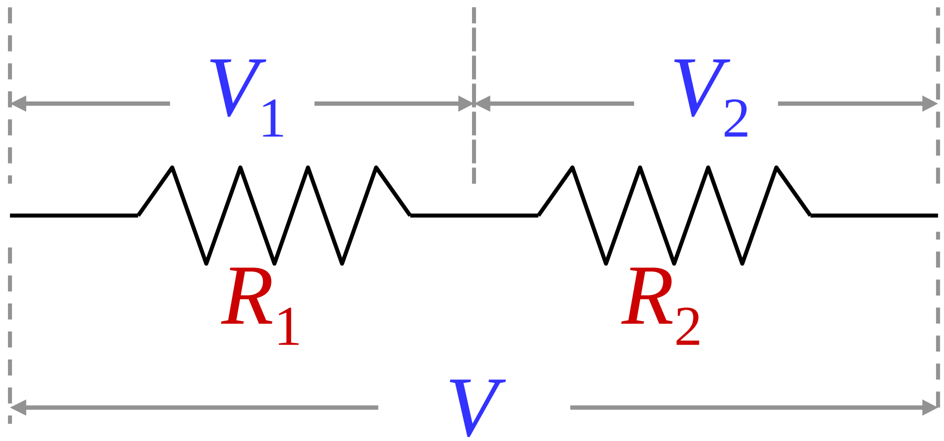


We now have 3 equations, and 3 unknown quantities.

Problem: If you now ground the point shown, what is the absolute potential at each of the indicated points?



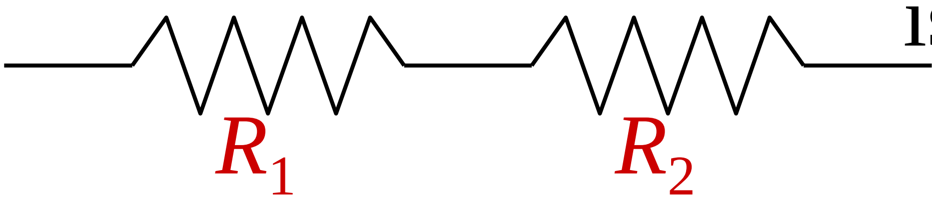
Resistors in Series



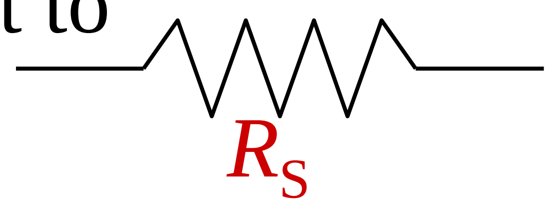
$I_1 = I_2 = I$

$V = V_1 + V_2 = IR_1 + IR_2 = I(R_1 + R_2) = IR_S$

So, $R_S = R_1 + R_2$



is equivalent to



In general, $R_S = R_1 + R_2 + R_3 + \ldots$

Problem: A 3Ω, a 6Ω, and a 9Ω resistor are placed in series and connected to a 12 V battery?

(a) What is the current flowing through each resistor?

(b) What is the voltage drop across each resistor?

Interactive Question

Three resistors, R_1, R_2, and R_3, are connected in series to a battery. Which will result if the resistance of R_3 is increased?

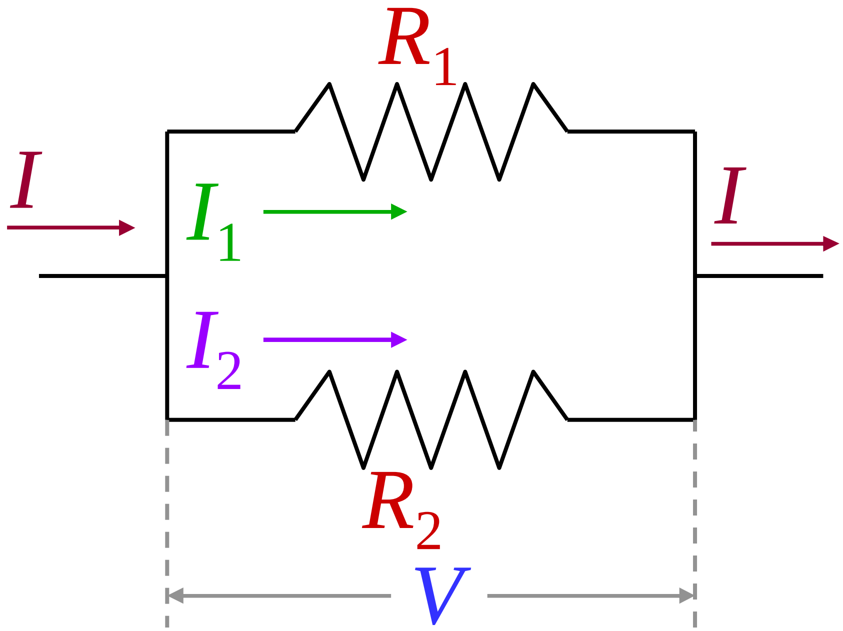
A) The current through R_1 and R_2 will increase.
B) The voltage across R_1 and R_2 will increase.
C) The voltage across the entire circuit will increase.
D) The power used by the entire circuit will increase.
E) The current through R_1 and R_2 will decrease.

Interactive Question

A 100 W light bulb is placed in series with a 60 W light bulb. Given that the brightness is proportional to the power dissipated, which will burn brighter?

A) 60 W
B) 100 W
C) The same

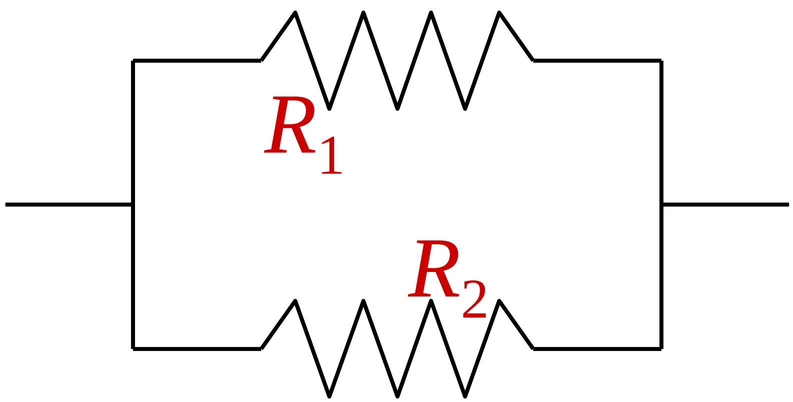
Resistors in Parallel



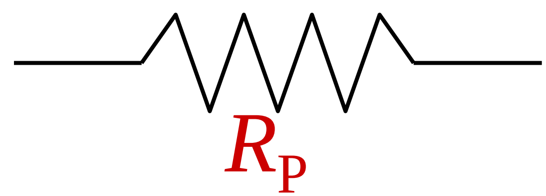
$$V_1 = V_2 = V$$

$$I = I_1 + I_2 = V/R_1 + V/R_2$$
$$= V(1/R_1 + 1/R_2) = V/R_P$$

So, $1/R_P = 1/R_1 + 1/R_2$



is equivalent to



In general, $1/R_P = 1/R_1 + 1/R_2 + 1/R_3 + \ldots$

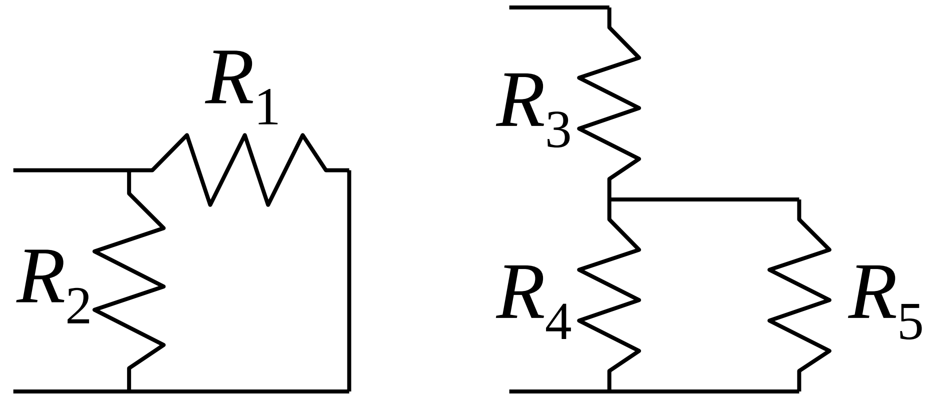
Problem: A 3Ω, a 6Ω, and a 9Ω resistor are placed in parallel and connected to a 12 V battery?

(a) What is the total current flowing through the system?

(b) What is the current in each resistor?

Interactive Question

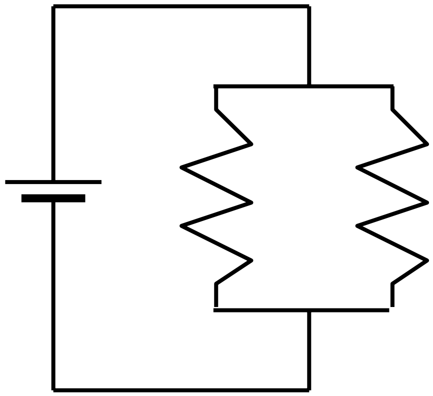
Consider the two circuits on the right. Which of the following statements is true?



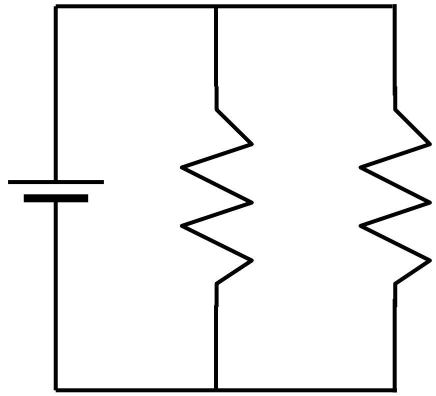
A) R_1 and R_2 are in parallel. R_3 and R_4 are in series.
B) R_4 and R_5 are in parallel. R_3 and R_4 are in series.
C) R_1 and R_2 are in series. R_3 and R_4 are in series.
D) R_1 and R_2 are in parallel. R_4 and R_5 are in parallel.
E) R_1 and R_2 are in series. R_4 and R_5 are in parallel.

Interactive Question

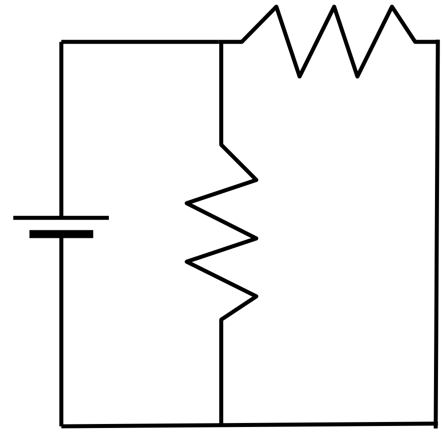
Which of the following circuits are identical?



(A)



(B)

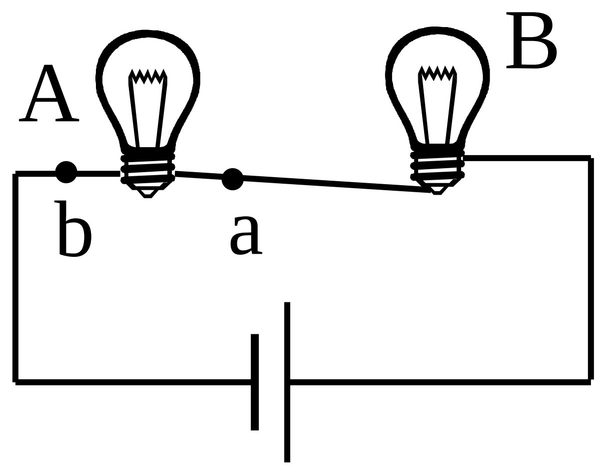


(C)

A) (A) and (B) only
B) (B) and (C) only
C) (A) and (C) only
D) (A), (B), and (C)
E) None of the above

Interactive Question

The circuit below consists of two identical light bulbs burning with equal brightness and a single 120 V battery. Which of the following will be true when bulb A is removed from the circuit?



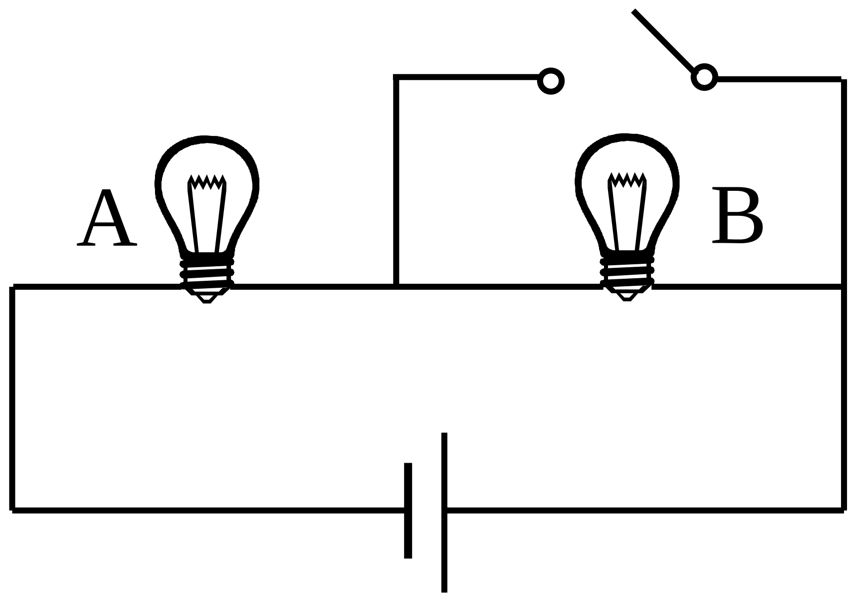
A) The potential difference between points a and b (V_{ab}) will be zero

B) The potential difference between points a and b (V_{ab}) will be 120 V

C) Bulb B will burn brighter

Interactive Question

The circuit below consists of two identical light bulbs burning with equal brightness and a single 12 V battery. When the switch is closed, the brightness of bulb A



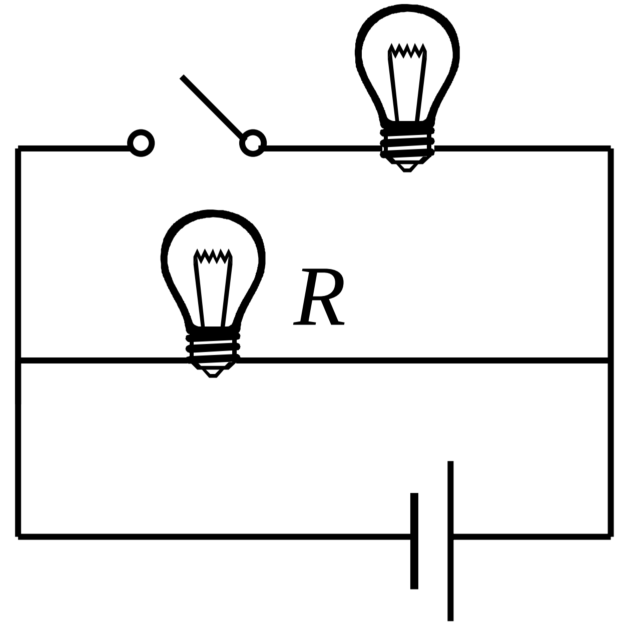
A) increases

B) decreases

C) remains unchanged

Interactive Question

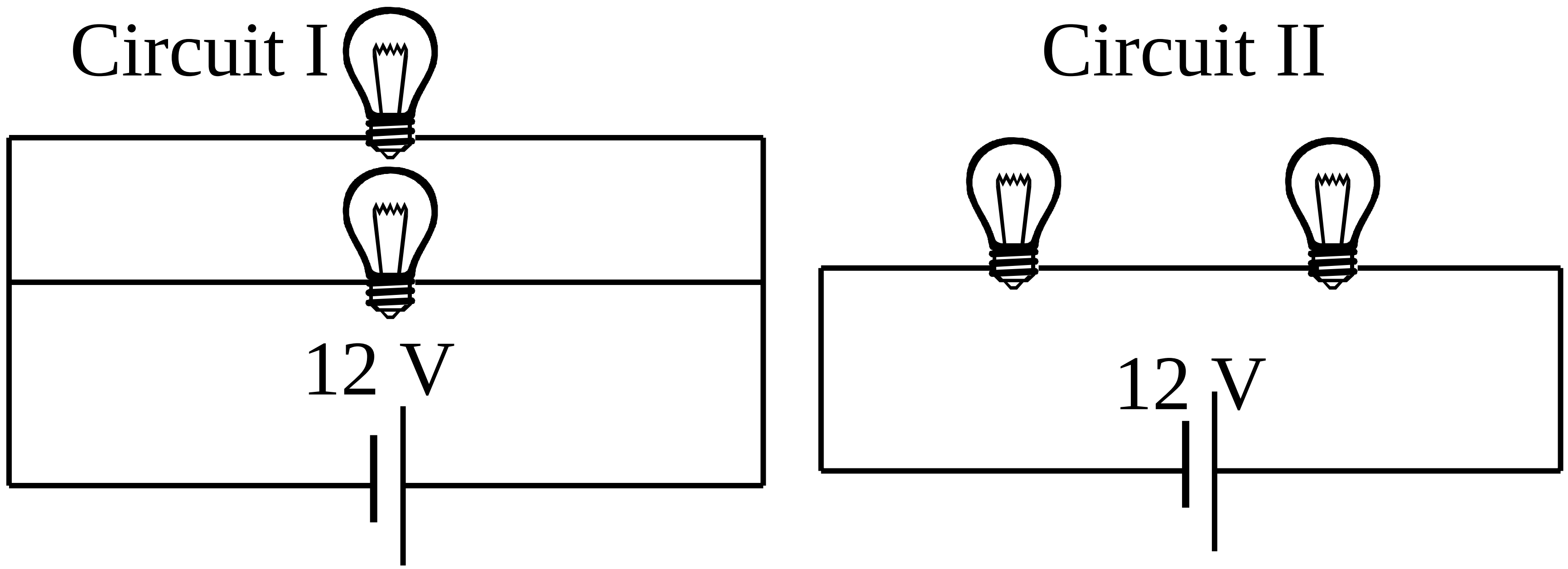
A simple series circuit contains a resistance *R* and an ideal battery. If a second resistor is connected in parallel with *R* by closing the switch,



A) the voltage across *R* will decrease.
B) the voltage across *R* will increase.
C) the total current in the circuit will increase.
D) the rate of heat production in *R* will increase.
E) the equivalent resistance of the circuit will increase.

Interactive Question

If the four light bulbs in the figure are identical, which circuit puts out more light?



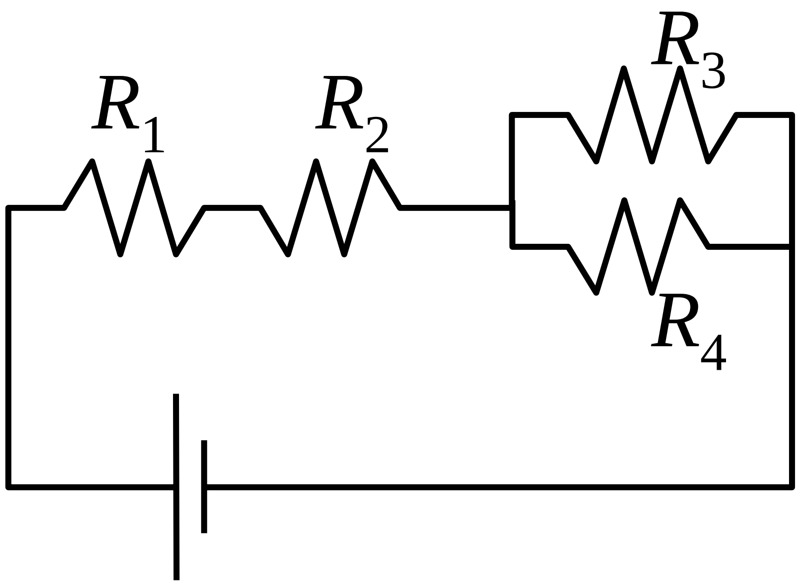
A) I

B) II

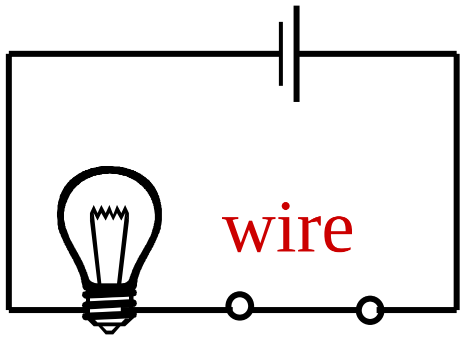
C) The two emit the same amount of light

Problem: What is the equivalent resistance of this circuit?

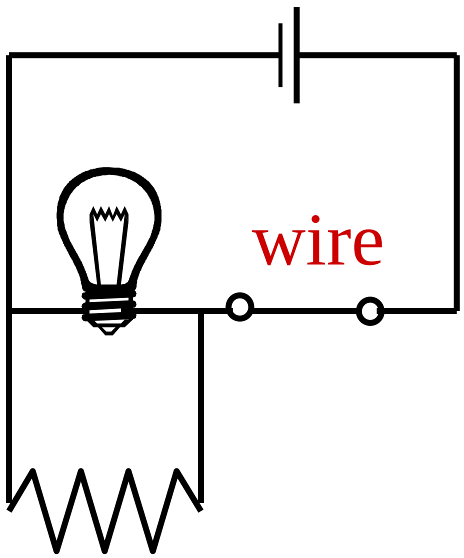
$R_1 = 2\ \Omega$
$R_2 = 4\ \Omega$
$R_3 = 5\ \Omega$
$R_4 = 10\ \Omega$



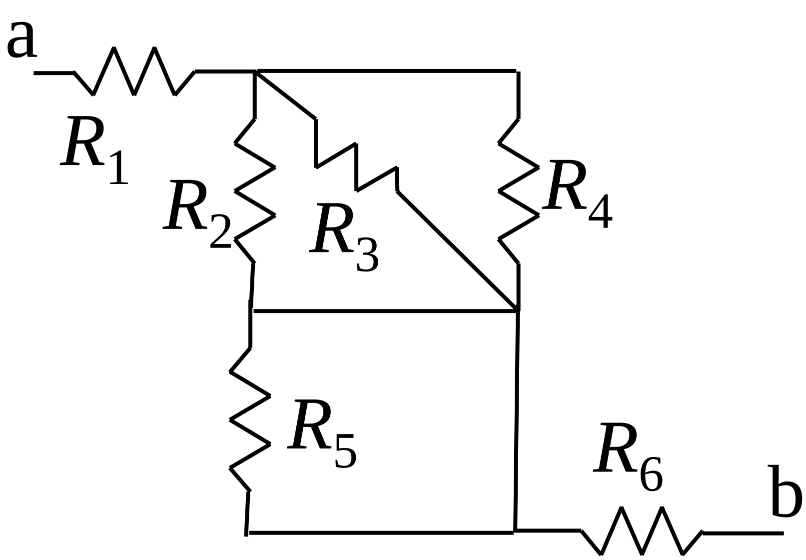
Problem: How many amps run through a tungsten wire when it is hooked up in series to a 100 watt light bulb and plugged into a 120 V electrical outlet?



Problem: How many amps run through a tungsten wire when it is hooked up in series to a 100 watt light bulb that is in parallel with a 600 W heater and plugged into a 120 V electric outlet?

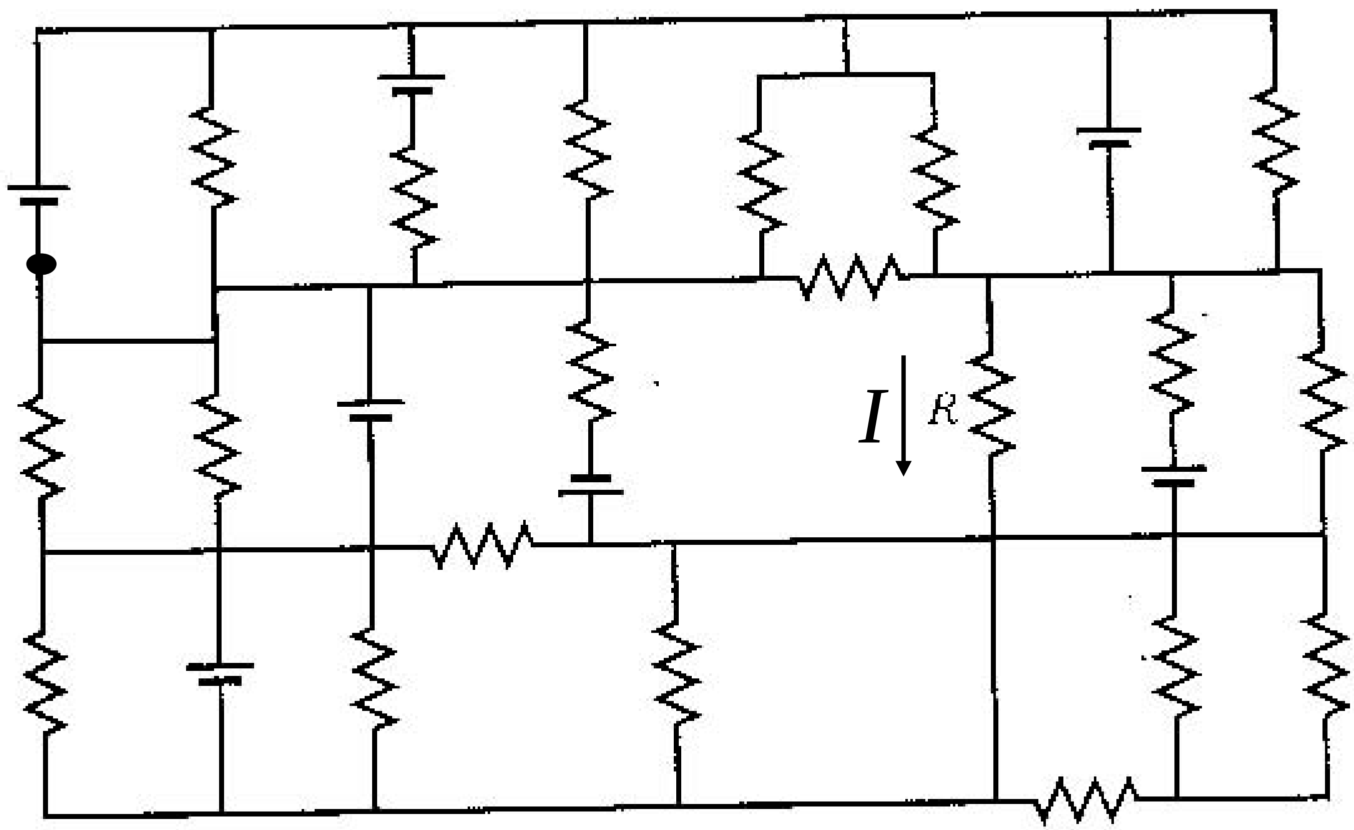


What is the equivalent resistance between points a and b?



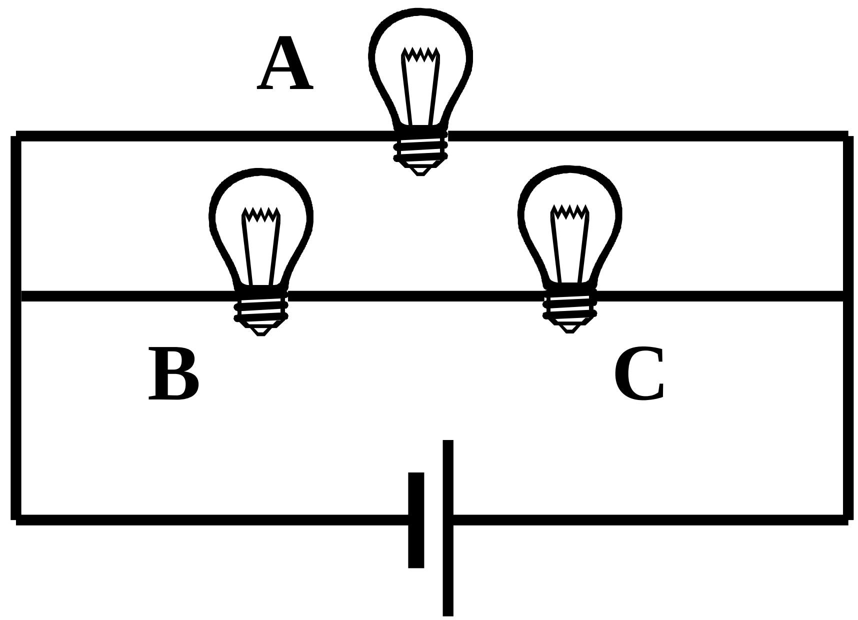
R_1=2 Ω
R_2=6 Ω
R_3=12 Ω
R_4=4 Ω
R_5=10 Ω
R_6=3 Ω

Each of the batteries in the figure has a potential difference V, and each resistor has a resistance R. What is the current through the resistor labeled R?



Interactive Question

The three bulbs in the circuit all have the same resistance. Given that the brightness is proportional to the power dissipated, the brightness of bulbs **B** and **C** together, compared with the brightness of bulb **A**, is



A) four times as much

B) twice as much

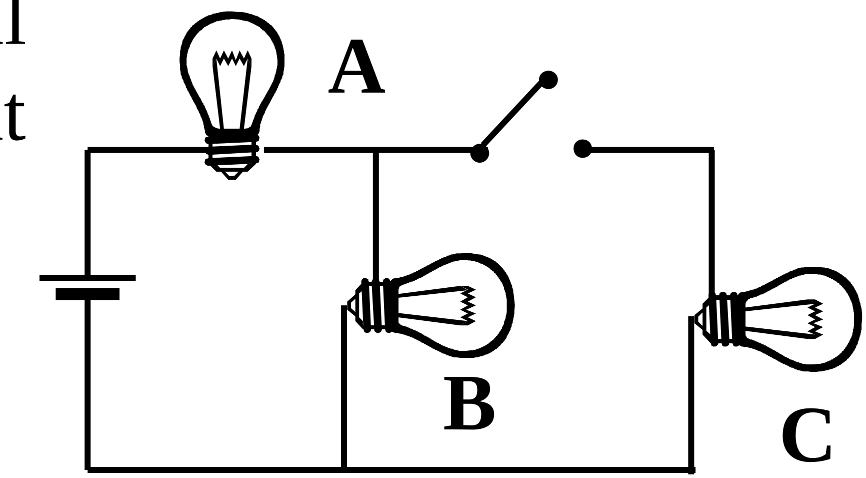
C) the same.

D) half as much

E) one fourth as much

Interactive Question

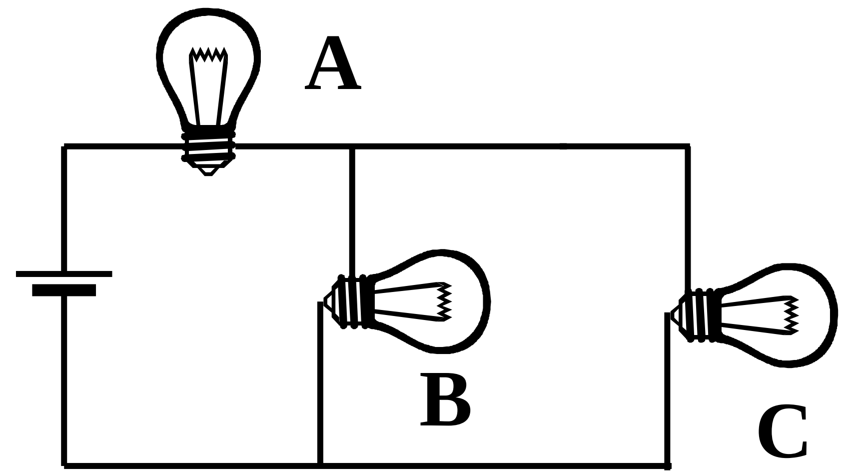
The three bulbs in the circuit all have the same resistance. What happens to the brightness of **A** and **B** when the switch is closed?



	<u>Bulb **A**</u>	<u>Bulb **B**</u>
A)	Stays the same	Increases
B)	Stays the same	Decreases
C)	Stays the same	Stays the same
D)	Increases	Decreases
E)	None of the above	

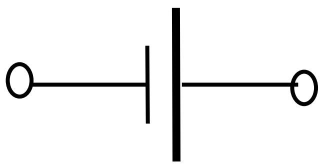
Interactive Question

The three bulbs in the circuit all have the same resistance. If bulb B burns out, what happens to the brightness of bulbs **A** and **C**, and to the total light output?

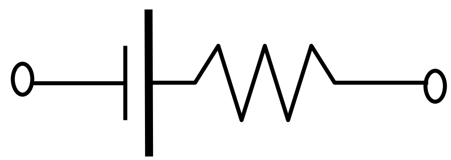


	Bulb **A**	Bulb **C**	Total Light
A)	Stays the same	Stays the same	Stays the same
B)	Increases	Stays the same	Increases
C)	Stays the same	Increases	Increases
D)	Decreases	Increases	Stays the same
E)	Decreases	Increases	Decreases

Terminal Voltage



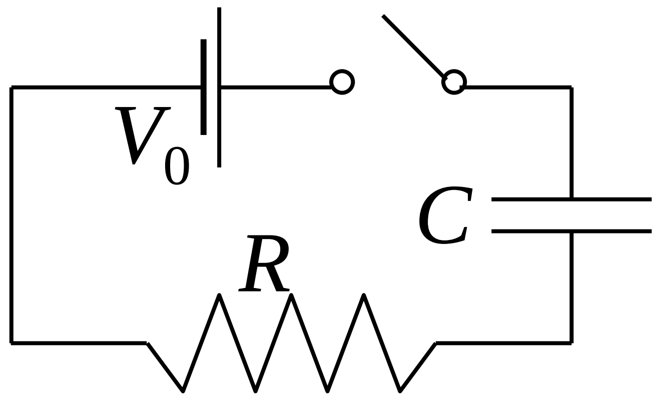
A real battery acts like this:



Internal Resistance

Problem: An ideal battery has a voltage of 9.00 volts and an internal resistance of 0.50 Ω. When it is drawing 0.30 amps, what is the terminal voltage?

Resistors and Capacitors in a Circuit



$V_0 - Q/C - IR = 0$

$V_0 - Q/C - R\, dQ/dt = 0$

$Q = CV_0[1 - e^{-t/(RC)}] = Q_0[1 - e^{-t/(RC)}]$

Q is the charge on the capacitor after a time t.

Q_0 is the total charge the capacitor will hold, $Q_0 = CV_0$.

$RC = \tau$ is the “time constant,” the amount of time for about 63% of the total charge to build up on the capacitor.

$I = dQ/dt = CV_0\, [1 - e^{-t/(RC)}] = (CV_0/RC)e^{-t/(RC)}$

$= (V_0/R)\, e^{-t/(RC)}$

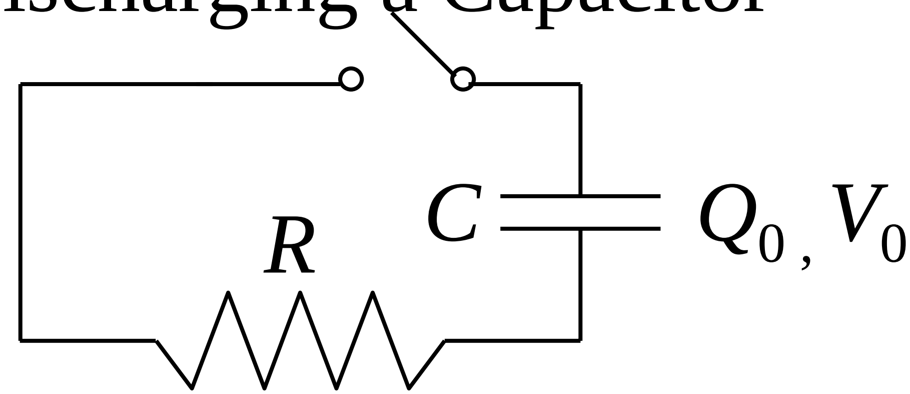
Resistors and Capacitors in a Circuit

$$Q = Q_0[1 - e^{-t/(RC)}]$$
$$CV = CV_0[1 - e^{-t/(RC)}]$$
$$V = V_0[1 - e^{-t/(RC)}]$$

V is the charge on the capacitor after a time t.
V_0 is the maximum voltage (set by the battery).

Limits: When t is 0, $V = 0$
When t is ∞ , $V = V_0$

Discharging a Capacitor



$$Q/C + IR = 0$$

$$Q/C + R\,dQ/dt = 0$$

$$Q = Q_0 e^{-t/(RC)} \qquad \text{or} \qquad V = V_0 e^{-t/(RC)}$$

Q_0 and V_0 are the initial charge and voltage.

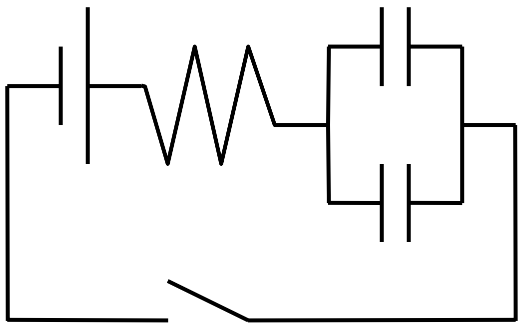
Q and V are the charge and voltage after a time t.

$RC = \tau$ is the "time constant," the amount of time for the capacitor to lose about 63% its initial charge.

$$I = dQ/dt = -(CV_0/RC)\mathrm{e}^{-t/(RC)}$$

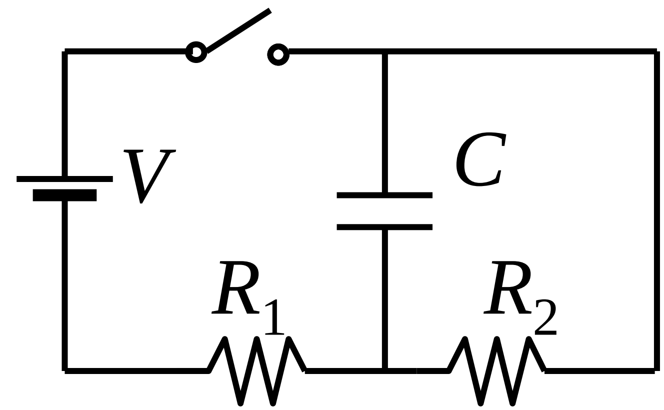
$$= -\,(V_0/R)\,\mathrm{e}^{-t/(RC)}$$

Problem: How long does it take for 50% of the maximum charge to be deposited on this circuit when the switch is closed. The resistor is 2 MΩ and each capacitor is 10 nF.



Interactive Question

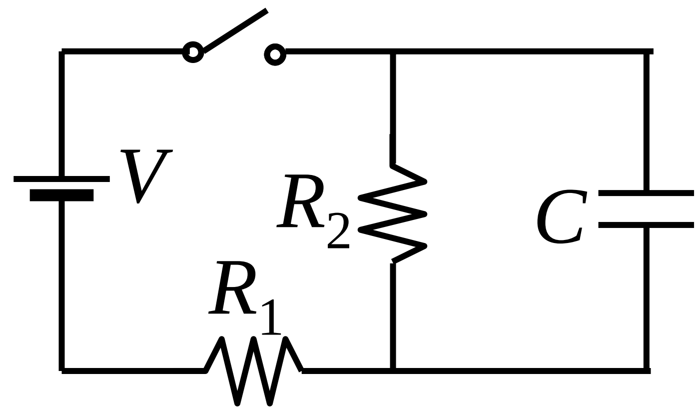
In the circuit shown $R_1 = R_2$. What is the initial voltage across R_1, R_2,and C when the switch is first closed?



	$\underline{C}$	$\underline{R_1}$	$\underline{R_2}$
A)	0	½V	½V
B)	V	0	0
C)	0	V	0
D)	½V	½V	½V
E)	0	0	0

Interactive Question

In the circuit shown $R_1 = R_2$. What is the voltage across R_1, R_2, and C after the switch has been closed for a long time?



	$\underline{C}$	$\underline{R_1}$	$\underline{R_2}$
A)	0	½V	½V
B)	V	0	0
C)	0	V	0
D)	½V	½V	½V
E)	0	0	0